GCSE ONLINE TUTORING

RATIO, PROPORTION & RATES / TOPIC GUIDE 37

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Compound measures and rates

Shared GCSE core with extension problems

Compound measures describe one quantity per another. Use units to organise speed, density, pressure, flow, pay and unit pricing. Learn to rearrange rate relationships, convert compound units and combine stages without averaging rates incorrectly.

Rate = numerator quantity / denominator quantity

Use compatible units and combine totals carefully.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Convert metric units, calculate with decimals and fractions, and rearrange simple formulae.

Your route through this notebook

1. Reading per and converting compound units
2. Speed, time and whole-journey averages
3. Density, mass, volume and mixtures
4. Pressure, force and contact area
5. Flow, pay, unit prices and combining rates

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Reading per and converting compound units

The word per means for each or divided by. A speed in m/s divides distance in metres by time in seconds; a density in g/cm³ divides mass in grams by volume in cubic centimetres. Write the formula from the meaning before using a memorised rearrangement. Units can confirm whether a calculation is appropriate.

Converting compound units may require changing numerator and denominator in opposite directions. To change km/h to m/s, multiply by 1000 metres per kilometre and divide by 3600 seconds per hour, so divide by 3.6 overall. Area and volume units also carry squared and cubed scale factors: 1 m³ = 1,000,000 cm³, not 100 cm³. Work one unit conversion at a time.

A reliable method

1. Identify the numerator quantity and denominator quantity.

2. Convert each to the required units, including any powers.

3. Check the final ratio of units matches the requested measure.

Worked example 1

Convert 72 km/h to m/s.

STEP-BY-STEP SOLUTION

72 × 1000/3600 = 20 m/s. Equivalently 72/3.6 = 20.

Worked example 2

Convert 1.2 g/cm³ to kg/m³.

STEP-BY-STEP SOLUTION

1.2 g = 0.0012 kg and 1 cm³ = 0.000001 m³. Ratio = 0.0012/0.000001 = 1200 kg/m³.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A cubic centimetre is a cube 1 cm on each side. One cubic metre contains 100 × 100 × 100 such cubes. Linear conversion factors cannot be copied unchanged into volume units.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Convert 15 m/s to km/h.

1.2 Convert 90 km/h to m/s.

1.3 Convert 2500 cm^3 to litres.

1.4 Convert 850 kg/m^3 to g/cm^3.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Speed, time and whole-journey averages

Average speed equals total distance divided by total elapsed time. Rearranging gives distance = speed × time and time = distance/speed. Use matching time units: 1 hour 30 minutes is 1.5 hours, not 1.30 hours. Include stops if the question asks for an average over the entire journey.

For multiple stages, calculate their distances and times before combining. The arithmetic mean of two speeds is valid for equal travel times, not generally for equal distances. Average speed has no direction; displacement divided by time is average velocity, which is a different measure. Read the quantity requested rather than substituting a rate mechanically.

A reliable method

1. Convert each stage to consistent units.

2. Add all travelled distances and relevant elapsed times.

3. Divide totals, then interpret stops and direction correctly.

Worked example 1

A car travels 150 km at 60 km/h. Find time.

STEP-BY-STEP SOLUTION

Time = 150/60 = 2.5 hours = 2 hours 30 minutes.

Worked example 2

Travel 60 km at 30 km/h and another 60 km at 60 km/h. Find average speed.

STEP-BY-STEP SOLUTION

Times are 2 hours and 1 hour. Total 120 km in 3 hours gives 40 km/h, not (30 + 60)/2 = 45 km/h.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Equal-distance stages spend longer at the slower speed, so the simple mean overweights the faster stage. Average using total distance and total time to avoid this trap.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Travel 84 km in 1 hour 12 minutes. Find average speed.

2.2 At 18 m/s, how far in 2 minutes?

2.3 A 30 km ride takes 1 hour cycling and a 15-minute stop. Find average over the full outing.

2.4 Travel at 40 km/h for 30 minutes and 60 km/h for 30 minutes. Find average speed.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Density, mass, volume and mixtures

Density is mass per unit volume: $\rho = m/V$. Rearranging gives $m = \rho V$ and $V = m/\rho$. Ensure mass and volume match the density units. A density of 2.7 g/cm³ means each cubic centimetre has mass 2.7 g under the model's uniform-density assumption.

For a mixture or composite object, add masses and volumes before dividing. The arithmetic mean of two densities is only appropriate when the component volumes are equal and volumes add as assumed. Real liquid mixtures may change volume when combined; a school model must specify or assume additive volumes. For a hollow object, distinguish material density from average density including empty space.

A reliable method

1. Use $\rho = m/V$ with compatible units.

2. For components, find each mass and add totals.

3. Divide total mass by total volume under the stated volume assumptions.

Worked example 1

Find mass of 80 cm³ of a material with density 2.7 g/cm³.

STEP-BY-STEP SOLUTION

$m = 2.7 \times 80 = 216$ g.

Worked example 2

Mix 100 cm³ at 1.2 g/cm³ and 300 cm³ at 0.8 g/cm³, assuming volumes add. Find density.

STEP-BY-STEP SOLUTION

Masses 120 g and 240 g, total 360 g. Total volume 400 cm³. Density = 360/400 = 0.9 g/cm³.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A denser material need not have a larger total mass: a very small dense sample can weigh less than a large low-density sample. Density compares equal volumes.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find density of mass 540 g and volume 200 cm^3.

3.2 Find volume of mass 780 g at density 7.8 g/cm^3.

3.3 Find mass of 0.004 m^3 at density 2500 kg/m^3.

3.4 Combine 200 cm^3 at 3 g/cm^3 with 100 cm^3 at 6 g/cm^3. Assume volumes add. Find density.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Pressure, force and contact area

Pressure is force per unit area: P = F/A. With force in newtons and area in square metres, the unit is N/m², called the pascal (Pa). A fixed force spread over a larger area gives smaller pressure. Rearrangements are F = PA and A = F/P, where denominators must be non-zero.

The area conversion is often the difficult step. Since 1 m² = 10,000 cm², 250 cm² = 0.025 m². A quantity in N/cm² can be a valid pressure, but it is not numerically the same as pascals. Use the actual contact area; the total surface area of an object is usually irrelevant to how its weight is spread on the ground.

A reliable method

1. Identify the force and the area actually carrying it.

2. Convert to square metres if pascals are requested.

3. Calculate and check whether a larger area should increase or reduce the answer.

Worked example 1

A force of 600 N acts over 0.03 m². Find pressure.

STEP-BY-STEP SOLUTION

P = 600/0.03 = 20,000 Pa.

Worked example 2

A force of 400 N acts on 200 cm². Find pressure in Pa.

STEP-BY-STEP SOLUTION

200 cm² = 200/10,000 = 0.02 m². Pressure = 400/0.02 = 20,000 Pa.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Dividing square centimetres by 100 converts neither correctly to square metres nor by a valid area factor. The linear metre-to-centimetre factor must be squared.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Find pressure for 900 N on 0.15 m^2.

4.2 Find force at 12,000 Pa over 0.04 m^2.

4.3 Find area supporting 750 N at 15,000 Pa.

4.4 A fixed force has its contact area halved. What happens to pressure?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Flow, pay, unit prices and combining rates

Rates occur whenever one quantity changes per unit of another: litres per minute, pounds per hour, kilometres per litre and items per machine-hour. Choose multiplication or division from the meaning and verify the units. “£ per hour × hours” gives £; “litres ÷ litres per minute” gives minutes.

Rates can be added when they refer to the same output and time basis. Two independent inlet taps contribute their volumes per minute; a drain is subtracted. If the net rate is zero or negative, an initially unfilled tank cannot reach a higher target under the same model. For pay problems, separate hours paid at different rates, and treat overtime multipliers according to the explicit question.

A reliable method

1. Identify the rate’s units and the required output.

2. Convert rates to a common unit before combining.

3. Use a net rate or calculate stages separately; check feasibility.

Worked example 1

Two taps supply 6 and 9 litres/minute. A drain removes 3 litres/minute. Time to add 180 litres?

STEP-BY-STEP SOLUTION

Net inflow 6 + 9 − 3 = 12 litres/minute. Time = 180/12 = 15 minutes, assuming all rates remain constant.

Worked example 2

A worker has 30 hours at £12/hour and 4 overtime hours at 1.5 times that rate. Find gross pay.

STEP-BY-STEP SOLUTION

Normal pay 30 × 12 = £360. Overtime rate £18/hour, pay £72. Total gross pay £432, before any deductions.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

If one machine completes a job in 6 hours and another in 3, their combined time is not 9 hours or 1.5 hours. Add job rates: 1/6 + 1/3 = 1/2 job per hour, so together they need 2 hours under independent constant-rate assumptions.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 A car travels 540 km using 45 litres. Find km per litre.

5.2 A tap supplies 2.4 litres/minute. Find litres in 25 seconds.

5.3 One machine takes 8 hours for a job, another 12. Find combined time at constant independent rates.

5.4 A worker earns £14/hour for 35 hours and twice that rate for 3 extra hours. Find gross pay.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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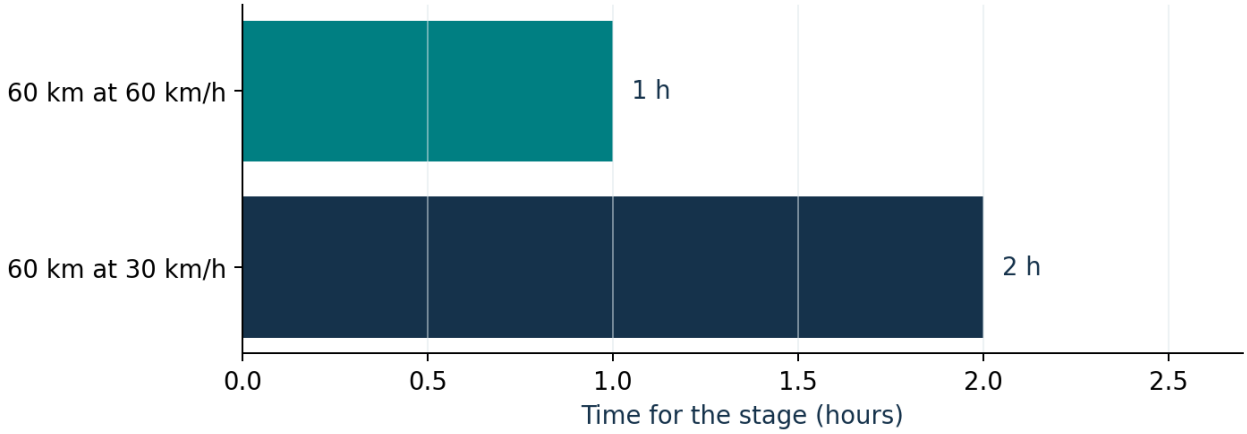
VISUAL EXPLAINER / MAKE THE RELATIONSHIP VISIBLE

Connect the representations

Total distance ÷ total time gives average speed



120 km ÷ 3 h = 40 km/h

Not the unweighted mean (30 + 60)/2 = 45 km/h

Speed: distance / time

Density: mass / volume

Pressure: force / area

Flow: volume / time

READ THE FIGURE

Equal-distance stages take different times. The slower stage lasts twice as long, so the whole-journey speed is 120/3 = 40 km/h. The formulas below all express a numerator quantity per denominator quantity; their units show which multiplication or division is needed when rearranging.

Discuss: identify the reference whole, fixed quantity or rate in this picture. Recalculate one labelled result without looking at the explanation. Say which assumptions make the comparison valid.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Convert 108 km/h to m/s. [2 marks]

R2. A vehicle travels 180 km in 2 hours 15 minutes. Find average speed. [3 marks]

R3. Travel 40 km at 20 km/h then 40 km at 40 km/h. Find average speed. [4 marks]

R4. A material has volume 350 cm^3 and density 2.4 g/cm^3. Find mass. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Combine 300 cm^3 at 2 g/cm^3 and 200 cm^3 at 5 g/cm^3. Assume volumes add. Find density. [4 marks]

R6. A force of 720 N acts on 240 cm^2. Find pressure in Pa. [3 marks]

R7. Two inlets supply 8 and 5 litres/minute; a drain removes 4. Find time to add 270 litres. [3 marks]

R8. A worker earns £13/hour for 32 hours and 1.5 times that rate for 6 overtime hours. Find gross pay. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Reading per and converting compound units

1.1 Convert 15 m/s to km/h.

15 × 3.6 = 54 km/h.

1.2 Convert 90 km/h to m/s.

90/3.6 = 25 m/s.

1.3 Convert 2500 cm^3 to litres.

1000 cm^3 = 1 litre, so 2.5 litres.

1.4 Convert 850 kg/m^3 to g/cm^3.

Divide by 1000: 0.85 g/cm^3.

Lesson 2 • Speed, time and whole-journey averages

2.1 Travel 84 km in 1 hour 12 minutes. Find average speed.

Time 1.2 hours; speed 84/1.2 = 70 km/h.

2.2 At 18 m/s, how far in 2 minutes?

Time 120 s; distance 18 × 120 = 2160 m.

2.3 A 30 km ride takes 1 hour cycling and a 15-minute stop. Find average over the full outing.

30/1.25 = 24 km/h.

2.4 Travel at 40 km/h for 30 minutes and 60 km/h for 30 minutes. Find average speed.

Distances 20 and 30 km. Total 50 km in 1 hour, so 50 km/h. Equal-time weighting makes the simple mean valid here.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Density, mass, volume and mixtures

3.1 Find density of mass 540 g and volume 200 cm³.

540/200 = 2.7 g/cm³.

3.2 Find volume of mass 780 g at density 7.8 g/cm³.

780/7.8 = 100 cm³.

3.3 Find mass of 0.004 m³ at density 2500 kg/m³.

2500 × 0.004 = 10 kg.

3.4 Combine 200 cm³ at 3 g/cm³ with 100 cm³ at 6 g/cm³. Assume volumes add. Find density.

Masses 600 g and 600 g; total 1200 g in 300 cm³ gives 4 g/cm³.

Lesson 4 • Pressure, force and contact area

4.1 Find pressure for 900 N on 0.15 m².

6000 Pa.

4.2 Find force at 12,000 Pa over 0.04 m².

F = 12,000 × 0.04 = 480 N.

4.3 Find area supporting 750 N at 15,000 Pa.

A = 750/15,000 = 0.05 m².

4.4 A fixed force has its contact area halved. What happens to pressure?

Pressure doubles because P = F/A with F fixed.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Flow, pay, unit prices and combining rates

5.1 A car travels 540 km using 45 litres. Find km per litre.

540/45 = 12 km/litre.

5.2 A tap supplies 2.4 litres/minute. Find litres in 25 seconds.

Rate 2.4/60 = 0.04 litre/s; volume 1 litre.

5.3 One machine takes 8 hours for a job, another 12. Find combined time at constant independent rates.

Rate 1/8 + 1/12 = 5/24 job/hour. Time = 24/5 = 4.8 hours = 4 hours 48 minutes.

5.4 A worker earns £14/hour for 35 hours and twice that rate for 3 extra hours. Find gross pay.

35 × 14 + 3 × 28 = £490 + £84 = £574.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Convert 108 km/h to m/s. [2 marks]

108/3.6 = 30 m/s. [1 for conversion method; 1 for result.]

R2. A vehicle travels 180 km in 2 hours 15 minutes. Find average speed. [3 marks]

Time 2.25 hours; speed 180/2.25 = 80 km/h. [1 for time; 1 for quotient; 1 for unit/result.]

R3. Travel 40 km at 20 km/h then 40 km at 40 km/h. Find average speed. [4 marks]

Times 2 and 1 hours. Total 80 km in 3 hours gives 80/3 ≈ 26.67 km/h. [1 for each time; 1 for totals; 1 for quotient.]

R4. A material has volume 350 cm^3 and density 2.4 g/cm^3. Find mass. [2 marks]

2.4 × 350 = 840 g. [1 for multiplication; 1 for result/unit.]

R5. Combine 300 cm^3 at 2 g/cm^3 and 200 cm^3 at 5 g/cm^3. Assume volumes add. Find density. [4 marks]

Mass 600 + 1000 = 1600 g; volume 500 cm^3; density 3.2 g/cm^3. [1 for each mass; 1 for totals; 1 for density.]

R6. A force of 720 N acts on 240 cm^2. Find pressure in Pa. [3 marks]

Area 0.024 m^2; pressure 720/0.024 = 30,000 Pa. [1 for area conversion; 1 for division; 1 for unit/result.]

R7. Two inlets supply 8 and 5 litres/minute; a drain removes 4. Find time to add 270 litres. [3 marks]

Net 9 litres/minute; time 270/9 = 30 minutes. [1 for net rate; 1 for division; 1 for result.]

R8. A worker earns £13/hour for 32 hours and 1.5 times that rate for 6 overtime hours. Find gross pay. [4 marks]

Normal £416; overtime rate £19.50/hour, pay £117; total £533. [1 for normal; 1 for overtime rate; 1 for overtime pay; 1 for total.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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