GCSE ONLINE TUTORING

RATIO, PROPORTION & RATES / TOPIC GUIDE 36

36

Direct and inverse proportion

Shared reasoning with Higher algebraic models

Turn proportional relationships into equations, tables and graphs. Distinguish a constant quotient from a constant product, then extend to powers, roots and combined models. State positivity and constant-condition assumptions where they matter.

Direct: $y = kx$ Inverse: $y = k/x$

Constant quotient and constant product are different.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Use ratios, rearrange formulae, calculate powers and interpret coordinates.

Your route through this notebook

1. Direct proportion and the constant k
2. Inverse proportion and a fixed product
3. Proportion to powers and roots — Higher
4. Reading proportional graphs — Higher extension
5. Combined variation and choosing a model — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Direct proportion and the constant k

If y is directly proportional to x, then y = kx for a constant k. For non-zero x, the quotient y/x is fixed. When x is multiplied by a factor, y is multiplied by the same factor. The graph is a straight line through the origin, although a contextual domain may show only part of it.

The symbol ∝ states a proportional relationship but does not supply k. Use a known pair to find k, then write a complete equation before calculating other values. A straight line with a non-zero intercept is not direct proportion. Check whether a relationship is stated as a model or merely suggested by a few observed values.

A reliable method

1. Translate y ∝ x into y = kx.

2. Find k from a known pair using y/x.

3. Use the complete equation and verify constant quotient.

Worked example 1

y ∝ x and y = 18 when x = 4. Find y when x = 10.

STEP-BY-STEP SOLUTION

18 = 4k, so k = 4.5. Equation y = 4.5x; at x = 10, y = 45.

Worked example 2

Decide whether y values 6,15,24 at x values 2,5,8 fit direct proportion.

STEP-BY-STEP SOLUTION

Quotients are 3,3,3, so the supplied pairs fit y = 3x. This is a consistent model for the table.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“Both values increase” is not enough. For y = x + 5, both may increase, but y/x changes and the graph misses the origin.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 $y \propto x$, $y = 28$ at $x = 7$. Find k and equation.

1.2 Use $y = 4x$ to find x when $y = 50$.

1.3 Is $y = 3x + 2$ directly proportional to x?

1.4 For $y \propto x$, what happens to y when x is multiplied by 2.5?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Inverse proportion and a fixed product

If y is inversely proportional to x, then $y = k/x$ for non-zero x. The product xy is constant. Multiplying x by a positive factor divides y by that factor in a positive-quantity model. A fixed job shared between identical workers is a common example, provided individual productivity stays constant.

Inverse proportion is not any decreasing relationship. The line $y = 10 - x$ decreases but does not have a constant product. For positive k and positive inputs, the inverse curve falls and approaches the axes without reaching them. Negative inputs may be mathematically permitted, but a physical model such as worker numbers usually restricts $x > 0$.

A reliable method

1. Translate $y \propto 1/x$ into $y = k/x$.

2. Find k using the product of a known pair.

3. Use the equation, remembering x cannot be zero.

Worked example 1

y is inversely proportional to x, and $y = 12$ at $x = 5$. Find y at $x = 8$.

STEP-BY-STEP SOLUTION

$k = 12 \times 5 = 60$. Thus $y = 60/x$, so $y = 7.5$ at $x = 8$.

Worked example 2

A fixed job takes 9 workers 8 days. Find days for 12 equivalent workers.

STEP-BY-STEP SOLUTION

Worker-days are constant: $k = 72$. Time $= 72/12 = 6$ days, under the stated constant-productivity model.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

When x rises from 4 to 8 in an inverse model, its doubling halves y. Subtracting 4 from y has no general justification: the relationship is multiplicative.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 $y \propto 1/x$, $y = 8$ at $x = 3$. Find equation.

2.2 Use $y = 24/x$ at $x = 10$.

2.3 For $xy = 90$, find x when $y = 12$.

2.4 A decreasing table has pairs (1,9),(2,8),(3,7). Is it inverse proportion?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Proportion to powers and roots — Higher

The quantity on the right of a proportional statement may be x^2, x^3, $\sqrt{x}$ or a reciprocal power. If $y \propto x^2$, write $y = kx^2$ and calculate $k = y/x^2$ from a known pair. Doubling x then multiplies y by four. If $y \propto 1/x^2$, doubling x divides y by four.

For positive quantities, root relationships produce different scaling again: $y \propto \sqrt{x}$ means multiplying x by 9 multiplies y by 3. When solving for x after squaring, take account of the stated domain. A length is positive, while a purely algebraic square relationship may allow two signs.

A reliable method

1. Copy the exact power or root in the proportional statement.

2. Find k using that transformed input.

3. Use roots or powers carefully when recovering an input.

Worked example 1

$y \propto x^2$; $y = 45$ when $x = 3$. Find y at $x = 5$.

STEP-BY-STEP SOLUTION

$k = 45/9 = 5$. Thus $y = 5x^2$ and $y = 125$ at $x = 5$.

Worked example 2

$y \propto 1/\sqrt{x}$ for $x > 0$, and $y = 6$ at $x = 4$. Find y at $x = 9$.

STEP-BY-STEP SOLUTION

$6 = k/2$, so $k = 12$. At $x = 9$, $y = 12/3 = 4$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

In $y = kx^2$, y/x is not normally constant, but y/x^2 is. The transformation named in the statement determines which ratio to test.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 $y \propto x^3$ and $y = 16$ at $x = 2$. Find k.

3.2 $y \propto \sqrt{x}$ and $y = 15$ at $x = 9$. Find y at $x = 25$.

3.3 $y \propto 1/x^2$ and $y = 18$ at $x = 2$. Find y at $x = 6$.

3.4 $y = 3x^2$ with $x > 0$. Find x when $y = 75$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Reading proportional graphs — Higher extension

The shape of a graph can support a proportional model, but axis labels are crucial. A plot of y against x is a line through the origin for $y = kx$. For $y = kx^2$, a plot of y against x^2 is a line through the origin even though y against x is curved. A plot of y against $1/x$ similarly linearises $y = k/x$.

The gradient of the transformed straight line is k, using the plotted horizontal quantity. At the origin of a transformed graph, the physical input may not have a finite valid value: $1/x = 0$ does not correspond to any finite non-zero x. Do not extend a model outside its domain merely because the visual axes allow it.

A reliable method

1. Read what the horizontal axis represents: x, x^2, $1/x$ or another transformation.

2. Use a straight line through the origin to identify the proportional relation.

3. Calculate k as vertical change/horizontal transformed change.

Worked example 1

A y versus x^2 line goes through (4,12) and the origin. Find the model.

STEP-BY-STEP SOLUTION

The horizontal coordinate is x^2. Gradient $12/4 = 3$, so $y = 3x^2$. The point corresponds to $x = 2$ or -2 unless the domain is restricted.

Worked example 2

A y versus $1/x$ line has gradient 20. Find y when $x = 5$.

STEP-BY-STEP SOLUTION

The model is $y = 20(1/x) = 20/x$. Thus $y = 4$, with $x \neq 0$.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A straight graph labelled y against x^2 does not imply $y \propto x$. Read the axis variable before interpreting a gradient.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 A y against x line through the origin has gradient 7. Write the model.

4.2 A y against x^3 line has gradient 2. Write the model.

4.3 A y against $1/x^2$ line passes through (0.25,5). Find k.

4.4 On y against x^2, a plotted point is (9,36). For $x > 0$, find x and k.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Combined variation and choosing a model — Higher

A quantity can depend on more than one variable. “y is directly proportional to a and inversely proportional to b” means $y = ka/b$. A known triple determines k; then changes in either input can be modelled while the others are held fixed. Record every exponent explicitly for models such as $y = ka^2/b$.

The phrase “all other conditions unchanged” is essential. A measured association alone does not prove an exact proportional law. When comparing models, test fixed quotients or products using data and consider whether a fixed charge, offset or limiting process makes the simple model unsuitable. Interpret the mathematics within the problem’s assumptions.

A reliable method

1. Translate each direct factor into a numerator factor and each inverse factor into a denominator factor.

2. Find k using a complete known set of inputs.

3. Substitute new values or combine their scale factors.

Worked example 1

$y \propto a/b$. When $a = 6, b = 4, y = 9$. Find y for $a = 10, b = 5$.

STEP-BY-STEP SOLUTION

$9 = k \times 6/4$ gives $k = 6$. New $y = 6 \times 10/5 = 12$.

Worked example 2

$y = ka^2/b$. If a triples and b doubles, how does y change?

STEP-BY-STEP SOLUTION

The a^2 factor multiplies by 9; dividing by doubled b adds factor 1/2. Overall factor $9/2 = 4.5$.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For a combined model, changing one input while silently changing another can disguise the relationship. State which variables are held constant before saying “doubling x doubles y”.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 $y = ka/b$ and $y = 15$ at $a = 10, b = 2$. Find k.

5.2 Use $y = 3a/b$ at $a = 8, b = 6$.

5.3 $y \propto a/b^2$. If both a and b double, find the output factor.

5.4 Cost is $C = 4n + 12$. Explain why a direct-proportion model fails.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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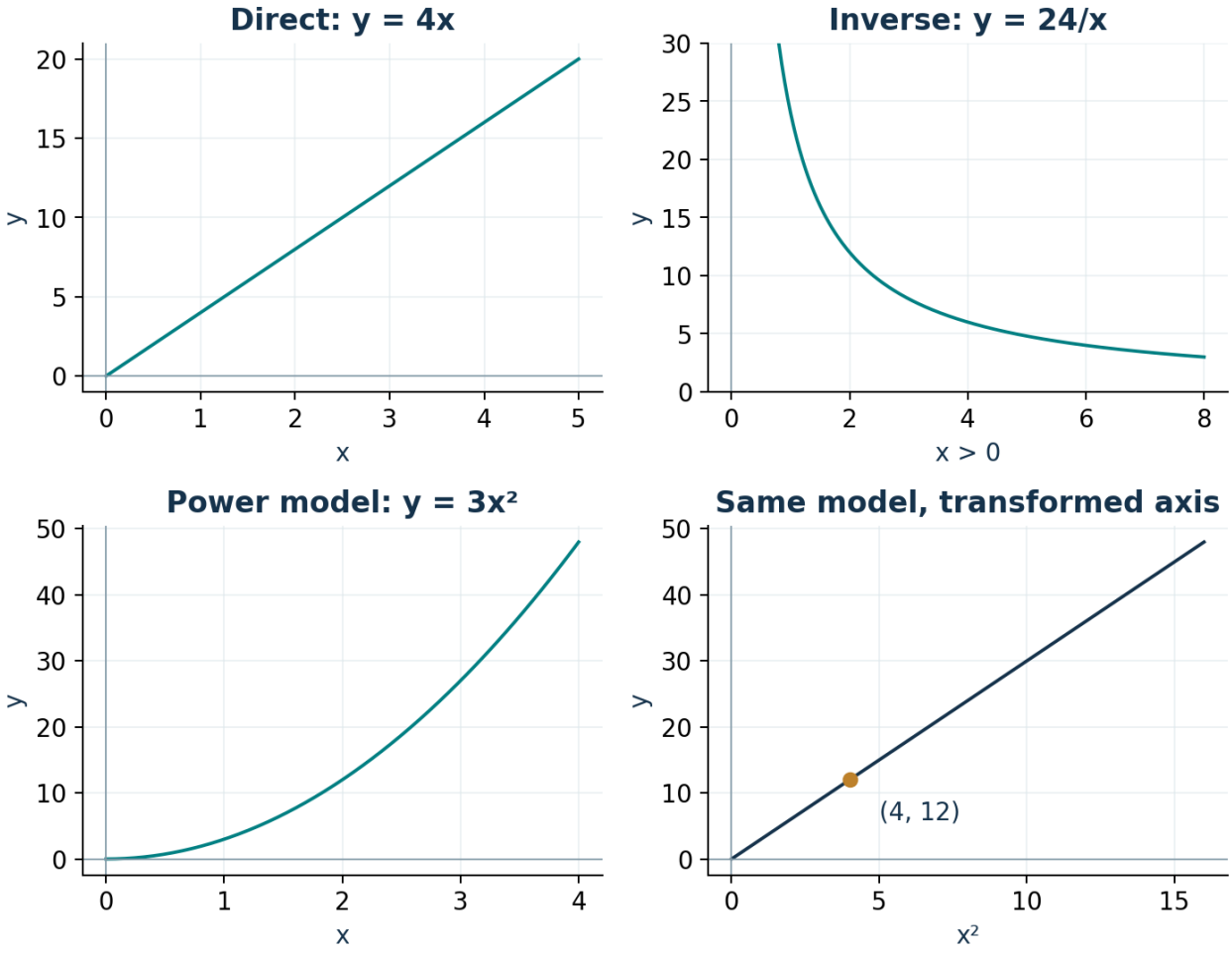
VISUAL EXPLAINER / MAKE THE RELATIONSHIP VISIBLE

Connect the representations

Read both the rule and the horizontal variable



READ THE FIGURE

Direct proportion has constant y/x and passes through the origin. Inverse proportion has constant xy and excludes x = 0. The bottom graphs show the same power model $y = 3x^2$: it is curved against x but straight against x^2. On the transformed graph, (4,12) means $x^2 = 4$, not x = 4.

Discuss: identify the reference whole, fixed quantity or rate in this picture. Recalculate one labelled result without looking at the explanation. Say which assumptions make the comparison valid.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. $y \propto x$ and $y = 35$ at $x = 7$. Find y at $x = 12$. [3 marks]

R2. $y \propto 1/x$ and $y = 9$ at $x = 8$. Find y at $x = 6$. [3 marks]

R3. State whether $y = 5x - 1$ represents direct proportion. Explain. [2 marks]

R4. Higher: $y \propto x^2$ and $y = 48$ at $x = 4$. Find y at $x = 7$. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Higher: $y \propto 1/x^3$ and $y = 16$ at $x = 2$. Find y at $x = 4$. [3 marks]

R6. Higher: $y \propto \sqrt{x}$ and $y = 20$ at $x = 16$. Find positive x when $y = 35$. [4 marks]

R7. Higher: a line of y against 1/x passes through (0.2,6). Write y in terms of x. [3 marks]

R8. Higher: $y \propto a^2/b$, and $y = 12$ at $a = 2, b = 3$. Find y at $a = 5, b = 6$. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Direct proportion and the constant k

1.1 $y \propto x$, $y = 28$ at $x = 7$. Find k and equation.

$k = 4$; $y = 4x$.

1.2 Use $y = 4x$ to find x when $y = 50$.

$x = 50/4 = 12.5$.

1.3 Is $y = 3x + 2$ directly proportional to x?

No, because of the non-zero constant term.

1.4 For $y \propto x$, what happens to y when x is multiplied by 2.5?

y is also multiplied by 2.5, with k fixed.

Lesson 2 • Inverse proportion and a fixed product

2.1 $y \propto 1/x$, $y = 8$ at $x = 3$. Find equation.

$k = 24$; $y = 24/x$.

2.2 Use $y = 24/x$ at $x = 10$.

$y = 2.4$.

2.3 For $xy = 90$, find x when $y = 12$.

$x = 90/12 = 7.5$.

2.4 A decreasing table has pairs (1,9),(2,8),(3,7). Is it inverse proportion?

No: products 9,16,21 are not constant. The table fits $y = 10 - x$ instead.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Proportion to powers and roots — Higher

3.1 $y \propto x^3$ and $y = 16$ at $x = 2$. Find k.

$k = 16/8 = 2$.

3.2 $y \propto \sqrt{x}$ and $y = 15$ at $x = 9$. Find y at $x = 25$.

$k = 5$; $y = 25$.

3.3 $y \propto 1/x^2$ and $y = 18$ at $x = 2$. Find y at $x = 6$.

$k = 72$; $y = 72/36 = 2$.

3.4 $y = 3x^2$ with $x > 0$. Find x when $y = 75$.

$x^2 = 25$; positive $x = 5$.

Lesson 4 • Reading proportional graphs — Higher extension

4.1 A y against x line through the origin has gradient 7. Write the model.

$y = 7x$.

4.2 A y against x^3 line has gradient 2. Write the model.

$y = 2x^3$.

4.3 A y against $1/x^2$ line passes through $(0.25,5)$. Find k.

$k = 5/0.25 = 20$, so $y = 20/x^2$.

4.4 On y against x^2, a plotted point is $(9,36)$. For $x > 0$, find x and k.

$x = 3$; $k = 36/9 = 4$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Combined variation and choosing a model — Higher

5.1 $y = ka/b$ and $y = 15$ at $a = 10, b = 2$. Find k.

$15 = 5k$, so $k = 3$.

5.2 Use $y = 3a/b$ at $a = 8, b = 6$.

$y = 4$.

5.3 $y \propto a/b^2$. If both a and b double, find the output factor.

Factor $2/2^2 = 1/2$: y halves.

5.4 Cost is $C = 4n + 12$. Explain why a direct-proportion model fails.

$C/n = 4 + 12/n$ is not constant. There is a fixed charge of 12.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. $y \propto x$ and $y = 35$ at $x = 7$. Find y at $x = 12$. [3 marks]

$k = 5$; $y = 5x$; new $y = 60$. [1 each stage.]

R2. $y \propto 1/x$ and $y = 9$ at $x = 8$. Find y at $x = 6$. [3 marks]

$k = 72$; $y = 72/x$; new $y = 12$. [1 each stage.]

R3. State whether $y = 5x - 1$ represents direct proportion. Explain. [2 marks]

No: it has a non-zero intercept and y/x is not constant. [1 for conclusion; 1 for reason.]

R4. Higher: $y \propto x^2$ and $y = 48$ at $x = 4$. Find y at $x = 7$. [3 marks]

$k = 3$; $y = 3x^2$; new $y = 147$. [1 each stage.]

R5. Higher: $y \propto 1/x^3$ and $y = 16$ at $x = 2$. Find y at $x = 4$. [3 marks]

$k = 128$; $y = 128/x^3$; new $y = 2$. [1 each stage.]

R6. Higher: $y \propto \sqrt{x}$ and $y = 20$ at $x = 16$. Find positive x when $y = 35$. [4 marks]

$k = 5$. Then $35 = 5\sqrt{x}$ gives $\sqrt{x} = 7$ and $x = 49$. [1 for k; 1 for equation; 1 for root; 1 for x.]

R7. Higher: a line of y against $1/x$ passes through $(0.2,6)$. Write y in terms of x. [3 marks]

Gradient $6/0.2 = 30$, so $y = 30/x$. [1 for transformed ratio; 1 for k; 1 for equation.]

R8. Higher: $y \propto a^2/b$, and $y = 12$ at $a = 2, b = 3$. Find y at $a = 5, b = 6$. [4 marks]

$12 = 4k/3$ gives $k = 9$. New $y = 9 \times 25/6 = 37.5$. [1 for model; 1 for k; 1 for substitution; 1 for result.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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