GCSE ONLINE TUTORING

RATIO, PROPORTION & RATES / TOPIC GUIDE 35

35

Interest, growth and decay

Shared core with Higher extensions

Distinguish fixed additions from repeated percentage changes. Explore hypothetical interest, depreciation and population models, reverse multi-period changes and use recurrence relations. Rates are supplied mathematical assumptions, not current financial offers.

$$A = P \times q^n$$

Repeated percentage change uses powers of a multiplier.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Use percentage multipliers, powers, reverse percentages and calculator arithmetic.

Your route through this notebook

1. Simple interest and linear growth
2. Compound interest and repeated growth
3. Depreciation and repeated decay
4. Reverse growth, rates and time thresholds
5. Changing rates and recurrence models — Higher extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Simple interest and linear growth

Simple interest calculates each period's interest from the original principal. If the principal is P and the rate is r% per period, each period adds Pr/100. After n periods the interest is I = P(r/100)n and the balance is P + I. Equal additions create a linear pattern rather than exponential growth.

The rate and time must use compatible periods. At an annual simple rate, six months is half a year if the problem uses a 12-month year. In real contracts the day-count or rounding convention may differ; here use the convention explicitly supplied. Distinguish interest earned from final balance, and do not calculate a new period's simple interest on an already increased balance.

A reliable method

1. Identify principal, rate per period and number of matching periods.

2. Calculate interest from the original principal only.

3. Add the interest if a final balance is requested.

Worked example 1

Find simple interest and final balance on £800 at 4% per year for 3 years.

STEP-BY-STEP SOLUTION

Yearly interest = 800 × 0.04 = £32. Total interest £96; final balance £896.

Worked example 2

Find simple interest on £1200 at 5% per year for 9 months, using a 12-month year.

STEP-BY-STEP SOLUTION

Time 9/12 = 0.75 year. Interest = 1200 × 0.05 × 0.75 = £45. Final balance would be £1245.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A balance growing by the same number of pounds each year is not necessarily growing by the same percentage of its current value. The base for simple interest is fixed.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find simple interest on £600 at 3% annually for 4 years.

1.2 Find final balance for £2500 at 2% simple interest for 3 years.

1.3 Find simple interest on £900 at 6% annually for 6 months, using half a year.

1.4 Simple interest of £180 is earned on £1500 in 3 years. Find annual rate.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Compound interest and repeated growth

Compound growth applies the rate to the current amount each period. A rate r% gives multiplier $q = 1 + r/100$; after n equal periods, $A = Pq^n$. The newly added amount can grow in later periods, so equal percentages do not create equal additions.

The model assumes the rate stays fixed and no deposits or withdrawals occur. Unless the task specifies rounding after each period, retain full precision and round the final money answer to the nearest penny. Total growth or interest is $A - P$, not A itself. A 3-year compound rate is not normally three times a 1-year rate because the multipliers combine.

A reliable method

1. Convert the rate to a per-period multiplier.

2. Raise that multiplier to the number of periods.

3. Multiply by the starting amount and subtract it if only the growth is requested.

Worked example 1

£1000 grows by 5% each year for 3 years. Find balance and interest.

STEP-BY-STEP SOLUTION

$A = 1000 \times 1.05^3 = £1157.625$, rounded to £1157.63. Unrounded interest is £157.625, or £157.63 to the nearest penny.

Worked example 2

A population starts at 800 and grows by 10% per year for 2 years. Find the model prediction.

STEP-BY-STEP SOLUTION

$800 \times 1.1^2 = 968$. Actual counts are discrete, but this model happens to give an integer.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A 5% compound rate over 3 years gives overall factor 1.157625, an increase of 15.7625%. Adding 5 + 5 + 5 gives only the simple-interest comparison.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find £500 after 2 years at 4% compound growth.

2.2 Find £2000 after 3 years at 3% compound growth, nearest penny.

2.3 A population of 1500 grows 2% per year for 2 years. Find the model value.

2.4 Compare 2-year simple and compound interest on £1000 at 10% annually.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Depreciation and repeated decay

A fixed percentage decrease each period uses multiplier $q = 1 - r/100$. For $0 < r < 100$, this lies between 0 and 1, so $A = Pq^n$ decreases but remains positive for every finite n. The absolute losses become smaller because each percentage acts on a smaller current amount.

Do not use a negative multiplier for ordinary depreciation or subtract the same cash amount each period unless the model says to do so. Mathematical decay may continue indefinitely, but real values such as resale prices may have floors or changing rates. Interpret the formula as a model within its stated range.

A reliable method

1. Find the remaining fraction per period.
2. Use Pq^n with a positive decay factor below 1.
3. Distinguish the remaining value from total loss.

Worked example 1

A machine worth £8000 depreciates 15% annually for 3 years. Find value.

STEP-BY-STEP SOLUTION

Remaining multiplier 0.85. Value = $8000 \times 0.85^3 = £4913$. Total loss = £3087.

Worked example 2

A substance has 200 g and halves every 6 hours. Find mass after 18 hours.

STEP-BY-STEP SOLUTION

There are 3 half-life periods. Mass = $200 \times (1/2)^3 = 25$ g. The supplied half-life is a hypothetical mathematical model.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A 20% annual decrease for 5 years does not mean 100% disappears. The factor is $0.8^5 = 0.32768$, leaving 32.768% of the original.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 £1200 decreases 10% yearly for 2 years. Find remaining value.

3.2 A 500 g quantity decays 8% per hour for 3 hours. Find model mass to 2 decimal places.

3.3 A value retains 70% each year. State percentage depreciation.

3.4 A population falls from 4000 by 5% annually for 2 years. Find model value.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Reverse growth, rates and time thresholds

If final value A, multiplier q and period count n are known, reverse the model by $P = A/q^n$. If P,A,n are known instead, $q^n = A/P$, so q is the positive nth root of A/P. Convert q into an increase or decrease percentage. For these positive-amount models use a positive multiplier.

To find the first whole period above or below a target, calculate successive values or a table near the crossing. Checking only one side is incomplete: verify the previous period still fails and the selected period meets the condition. “Exceeds” is strict, whereas “at least” includes equality.

A reliable method

1. Choose whether the unknown is initial amount, multiplier or period count.

2. Use division, roots or a table as appropriate.

3. For a first-period answer, test the period immediately before it.

Worked example 1

After 2 years at 10% growth, a value is 605. Find original.

STEP-BY-STEP SOLUTION

$P = 605/1.1^2 = 500$.

Worked example 2

Find the first whole year when 100 grows above 150 at 10% annually.

STEP-BY-STEP SOLUTION

Year 4: $100 \times 1.1^4 = 146.41$, below 150. Year 5: 161.051, above 150. First year is 5.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A quotient such as 605/500 = 1.21 is the overall 2-period factor, not the per-period factor. The yearly multiplier is √1.21 = 1.1, giving 10%, not 21%.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 After 3 years at 20% depreciation, a value is £2560. Find original.

4.2 An amount grows from 800 to 968 over 2 equal annual percentage rises. Find annual rate.

4.3 An amount falls from 500 to 320 over 2 equal percentage decreases. Find rate.

4.4 A value starts at 100 and halves each period. Find the first whole period below 10.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Changing rates and recurrence models — Higher extension

Different period rates require different multipliers. Multiply them individually, grouping equal rates with powers when convenient. A deposit or withdrawal introduces an additive term. If an amount grows 5% and then £20 is withdrawn each period, its recurrence is $A_{n+1} = 1.05A_n - 20$.

Order matters with addition: withdrawing first gives $1.05(A_n - 20)$, a different model. Record whether A_0 is the initial amount and retain full precision between updates. A recurrence can have a fixed balance where growth exactly offsets withdrawal; solving $A = qA - d$ locates it, but the behaviour of nearby balances still needs interpretation.

A reliable method

1. Write the exact order of each period’s operations.

2. Use a product of factors for rate-only changes, or a recurrence when cash amounts are added or removed.

3. Check the first update and interpret any fixed point.

Worked example 1

£2000 grows by 3% in year 1 and 5% in year 2. Find final balance.

STEP-BY-STEP SOLUTION

$2000 \times 1.03 \times 1.05 = £2163$. Overall growth is 8.15%, not 8%.

Worked example 2

Start with £500. Each year add 10% growth then withdraw £30. Find A_1 and A_2.

STEP-BY-STEP SOLUTION

$A_{n+1} = 1.1A_n - 30$. $A_1 = 550 - 30 = £520$. $A_2 = 572 - 30 = £542$.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The recurrence uses the previous balance, not always the original one. Writing subscripts keeps the updates clear and prevents repeated use of A_0.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 Start £800; each period grows 5%, then £25 is added. Find first two updated balances.

5.2 Write a recurrence for a population losing 4% then gaining 60 members each year.

5.3 Find the fixed balance for $A_{n+1} = 1.1A_n - 30$.

5.4 Compare withdrawing £20 before versus after 5% growth from £400.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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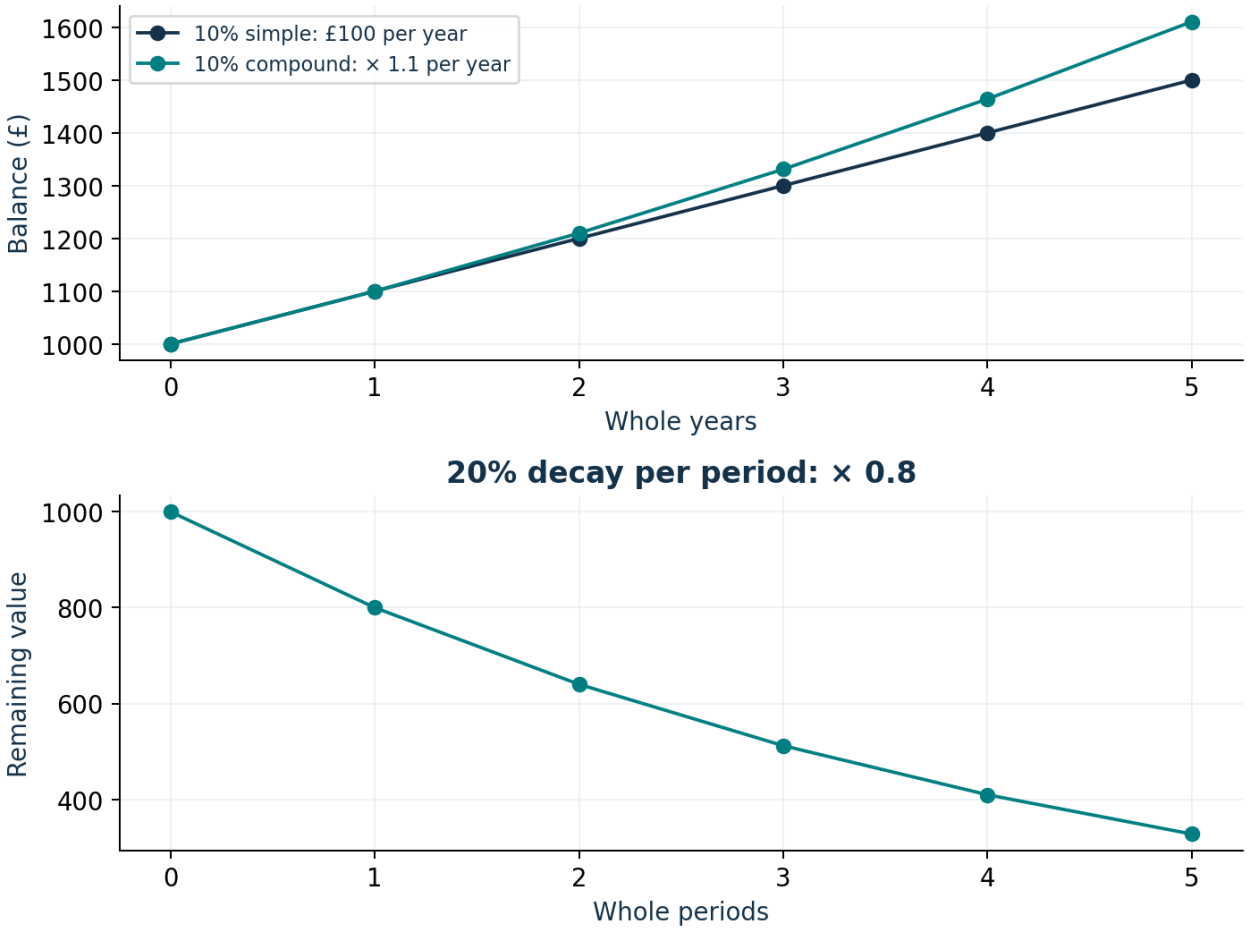
VISUAL EXPLAINER / MAKE THE RELATIONSHIP VISIBLE

Connect the representations

Equal additions differ from equal multipliers



READ THE FIGURE

The top model starts at £1000. Simple interest adds £100 each year, while compound growth multiplies each current balance by 1.1. After 3 years the balances are £1300 and £1331. The lower model retains 80% each period; its values approach zero without becoming zero in any finite number of periods. Lines connect whole-period values to guide the eye.

Discuss: identify the reference whole, fixed quantity or rate in this picture. Recalculate one labelled result without looking at the explanation. Say which assumptions make the comparison valid.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find simple interest on £1400 at 3% yearly for 5 years. [3 marks]

R2. Find £1500 after 2 years of 4% compound growth. [3 marks]

R3. A £6000 value depreciates 12% yearly for 3 years. Find value to nearest penny. [3 marks]

R4. An amount is 1452 after 2 years of 10% growth. Find original. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. A value rises from 400 to 484 in 2 equal annual percentage rises. Find rate. [3 marks]

R6. A quantity starts at 200 and retains 80% each period. Find first whole period below 100. [4 marks]

R7. Higher: $A_0 = 1000$ and $A_{n+1} = 1.02A_n + 50$. Find A_1,A_2. [3 marks]

R8. Higher: write and solve the fixed-point equation for $P_{n+1} = 0.9P_n + 80$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Simple interest and linear growth

1.1 Find simple interest on £600 at 3% annually for 4 years.

600 × 0.03 × 4 = £72.

1.2 Find final balance for £2500 at 2% simple interest for 3 years.

Interest £150; balance £2650.

1.3 Find simple interest on £900 at 6% annually for 6 months, using half a year.

900 × 0.06 × 0.5 = £27.

1.4 Simple interest of £180 is earned on £1500 in 3 years. Find annual rate.

180 = 1500 × (r/100) × 3; r = 4%.

Lesson 2 • Compound interest and repeated growth

2.1 Find £500 after 2 years at 4% compound growth.

500 × 1.04^2 = £540.80.

2.2 Find £2000 after 3 years at 3% compound growth, nearest penny.

2000 × 1.03^3 = £2185.454, so £2185.45.

2.3 A population of 1500 grows 2% per year for 2 years. Find the model value.

1500 × 1.02^2 = 1560.6, or about 1561 people if a whole-person estimate is requested.

2.4 Compare 2-year simple and compound interest on £1000 at 10% annually.

Simple interest £200; compound interest £210. Compound earns £10 more under these assumptions.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Depreciation and repeated decay

3.1 £1200 decreases 10% yearly for 2 years. Find remaining value.

$1200 \times 0.9^2 = £972$.

3.2 A 500 g quantity decays 8% per hour for 3 hours. Find model mass to 2 decimal places.

$500 \times 0.92^3 = 389.344$ g, so 389.34 g.

3.3 A value retains 70% each year. State percentage depreciation.

30% per year.

3.4 A population falls from 4000 by 5% annually for 2 years. Find model value.

$4000 \times 0.95^2 = 3610$.

Lesson 4 • Reverse growth, rates and time thresholds

4.1 After 3 years at 20% depreciation, a value is £2560. Find original.

$2560/0.8^3 = £5000$.

4.2 An amount grows from 800 to 968 over 2 equal annual percentage rises. Find annual rate.

$q^2 = 968/800 = 1.21$; $q = 1.1$, so 10%.

4.3 An amount falls from 500 to 320 over 2 equal percentage decreases. Find rate.

$q^2 = 0.64$, so $q = 0.8$: 20% decrease per period.

4.4 A value starts at 100 and halves each period. Find the first whole period below 10.

Period 3 gives 12.5; period 4 gives 6.25. First is 4.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Changing rates and recurrence models — Higher extension

5.1 Start £800; each period grows 5%, then £25 is added. Find first two updated balances.

$A_1 = 1.05 \times 800 + 25 = £865$. $A_2 = 1.05 \times 865 + 25 = £933.25$.

5.2 Write a recurrence for a population losing 4% then gaining 60 members each year.

$P_{n+1} = 0.96P_n + 60$, with an initial P_0 supplied.

5.3 Find the fixed balance for $A_{n+1} = 1.1A_n - 30$.

$A = 1.1A - 30$ gives $0.1A = 30$, so $A = 300$.

5.4 Compare withdrawing £20 before versus after 5% growth from £400.

Before: $1.05(400 - 20) = £399$. After: $1.05 \times 400 - 20 = £400$. The outcomes differ by £1.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find simple interest on £1400 at 3% yearly for 5 years. [3 marks]

1400 × 0.03 × 5 = £210. [1 for per-year interest; 1 for periods; 1 for total.]

R2. Find £1500 after 2 years of 4% compound growth. [3 marks]

$1500 \times 1.04^2 = £1622.40$. [1 for multiplier; 1 for power; 1 for result.]

R3. A £6000 value depreciates 12% yearly for 3 years. Find value to nearest penny. [3 marks]

$6000 \times 0.88^3 = £4088.832$, so £4088.83. [1 for factor; 1 for power; 1 for result.]

R4. An amount is 1452 after 2 years of 10% growth. Find original. [3 marks]

$1452/1.1^2 = 1200$. [1 for total factor; 1 for reverse division; 1 for result.]

R5. A value rises from 400 to 484 in 2 equal annual percentage rises. Find rate. [3 marks]

$q = \sqrt{(484/400)} = 1.1$, so 10%. [1 for quotient; 1 for root; 1 for rate.]

R6. A quantity starts at 200 and retains 80% each period. Find first whole period below 100. [4 marks]

Period 3 gives 102.4; period 4 gives 81.92. First is 4. [1 for model; 1 for preceding value; 1 for crossing value; 1 for conclusion.]

R7. Higher: $A_0 = 1000$ and $A_{n+1} = 1.02A_n + 50$. Find A_1,A_2. [3 marks]

$A_1 = 1070$; $A_2 = 1.02 \times 1070 + 50 = 1141.40$. [1 for first update; 1 for second method; 1 for result.]

R8. Higher: write and solve the fixed-point equation for $P_{n+1} = 0.9P_n + 80$. [3 marks]

$P = 0.9P + 80$ gives $0.1P = 80$, so $P = 800$. [1 for fixed-point equation; 1 for isolation; 1 for value.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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