GCSE ONLINE TUTORING

RATIO, PROPORTION & RATES / TOPIC GUIDE 34

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Reverse percentages

Shared GCSE core with extension problems

Recover original amounts from changed values. The central idea is to identify what percentage the known amount represents, then divide by that multiplier. Extend to chains of changes, profit calculations and rounded information.

Original = final ÷ change multiplier

Reverse multiplication by dividing, not subtracting.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Find percentage multipliers, calculate with decimals and rearrange simple equations.

Your route through this notebook

1. Recovering a whole from a known percentage
2. Reversing an increase
3. Reversing a decrease
4. Reversing successive changes
5. Profit bases, losses and rounded final amounts

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Recovering a whole from a known percentage

If p% of an original amount is known, that known amount is p/100 times the original. Recovering the original reverses multiplication, so divide by p/100. The unitary method gives the same answer: divide by p to find 1%, then multiply by 100.

The known quantity may be a part, a discount amount or a remaining balance. Identify the wording before choosing p. If £18 is the discount at 15%, then £18 represents 15%; if £102 is the sale price after that discount, it represents 85%. Both can reveal the same original price, but they require different divisors.

A reliable method

1. State what percentage the known amount represents.

2. Write known = multiplier × original.

3. Divide to recover the original and check forwards.

Worked example 1

35% of an amount is 84. Find the amount.

STEP-BY-STEP SOLUTION

0.35A = 84, so A = 84/0.35 = 240. Check: 35% of 240 is 84.

Worked example 2

A 12% discount saves £27. Find the original price.

STEP-BY-STEP SOLUTION

The saving itself is 12% of the original. Original = 27/0.12 = £225.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Do not assume the known value is 100%. In a reverse question, the important first sentence is often “this amount represents ___% of the original”.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 20% of an amount is 46. Find the whole.

1.2 7.5% of a mass is 18 kg. Find the mass.

1.3 A 25% discount saves £16. Find the original price.

1.4 140% of a value is 98. Find the value.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Reversing an increase

After a p% increase, the new amount is (100 + p)% of the original. Its multiplier is 1 + p/100. The original is therefore new divided by this multiplier. Subtracting p% of the new amount uses a different base and usually gives the wrong result.

A tax or surcharge calculation is a mathematical model here, not a statement about any current legal tax rate. If a problem explicitly specifies a 20% charge added to a base price, the final price is 120% of the base. The charge itself is 20/120 of the final price. Keep base amount, added charge and final amount distinct.

A reliable method

1. Convert the stated increase into the full new multiplier.

2. Divide the final amount by that factor.

3. Check by applying the increase to your recovered amount.

Worked example 1

A value is 276 after a 15% increase. Find the original.

STEP-BY-STEP SOLUTION

New = 1.15 × original. Original = 276/1.15 = 240.

Worked example 2

A bill is £180 including a hypothetical 20% added charge. Find the base and charge.

STEP-BY-STEP SOLUTION

Base = 180/1.2 = £150. Charge = £30, which is one sixth of the final total, not 20% of it.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Reducing a 120% final amount by 20% gives 96% of the original. The exact undoing factor is 1/1.2, which corresponds to a 16 2/3% decrease from the new amount.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 After a 10% rise a price is £77. Find the original.

2.2 A mass is 486 kg after an 8% rise. Find the original.

2.3 A final £345 includes a specified 15% added charge. Find the base.

2.4 A value doubles. What multiplier reverses the increase?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Reversing a decrease

After a p% decrease, the final amount is (100 − p)% of the original. Divide the known final value by 1 − p/100. For a positive original amount and a decrease below 100%, the recovered original should be larger than the final. That provides a useful sense check.

Adding the same percentage to the sale price does not undo a discount, because the added percentage is calculated on a smaller base. At a 100% decrease, the multiplier is zero and every original amount becomes zero; the original cannot be recovered from the final zero alone. This illustrates why division requires a non-zero multiplier.

A reliable method

1. Find the percentage remaining.

2. Convert it to a decimal and divide the final amount.

3. Check the original is sensible and reproduces the final amount.

Worked example 1

A coat costs £68 after a 15% discount. Find the original.

STEP-BY-STEP SOLUTION

Remaining percentage is 85%, multiplier 0.85. Original = 68/0.85 = £80.

Worked example 2

A population model gives 736 after an 8% decrease. Find the original.

STEP-BY-STEP SOLUTION

736 represents 92% of the original. Original = 736/0.92 = 800.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For a 30% discount, divide by 0.70, not 0.30. The sale price represents the part left, while the discount represents the part removed.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A sale price is £54 after 25% off. Find the original.

3.2 An amount becomes 357 after a 15% decrease. Find the original.

3.3 A value becomes 96 after a 40% decrease. Find the original.

3.4 Explain why adding 25% to a 25%-discounted price does not restore the original.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Reversing successive changes

Each forward percentage change contributes a multiplier. Their product is the overall factor between original and final. To reverse the chain, divide by that product, or undo the individual multipliers in reverse order. Do not simply add the rates and divide by an invented net percentage.

If a change amount is given rather than the final value, relate that change to the original through the difference between the overall factor and 1. If a sequence contains a fixed cash adjustment as well as percentage changes, it must be undone in the appropriate reverse order: addition and multiplication do not generally commute.

A reliable method

1. Write the forward chain explicitly, including any fixed additions.
2. Combine pure percentage factors or reverse mixed operations one by one.
3. Check by replaying the original chain.

Worked example 1

After a 20% rise then a 10% fall, an amount is 216. Find the original.

STEP-BY-STEP SOLUTION

Overall factor $1.2 \times 0.9 = 1.08$. Original $= 216/1.08 = 200$.

Worked example 2

An item is discounted 20%, then a £6 delivery fee makes the total £70. Find the original item price.

STEP-BY-STEP SOLUTION

Undo the final addition first: item sale price = £64. Original = 64/0.8 = £80.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

If delivery had been added before the discount, the equation would be $0.8(P + 6) = 70$ rather than $0.8P + 6 = 70$. Write the model before reversing it.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 After two 10% increases a value is 242. Find original.

4.2 After 20% off then another 25% off, a price is £90. Find original.

4.3 A price rises 5%, then £8 is added to give £92. Find original.

4.4 A 10% rise then a 20% fall reduces an amount by £36. Find original.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Profit bases, losses and rounded final amounts

A profit percentage needs a named reference. A 25% markup on cost means selling price = 1.25 × cost. A 25% profit margin on selling price means cost is 75% of selling price. These are different definitions and can lead to different original costs from the same selling price. State the base rather than assuming all profit percentages mean the same thing.

If a final amount has been rounded, the original may lie in an interval rather than being uniquely determined. Use the final rounding interval and reverse the positive multiplier on both bounds. A usual school problem may intend an exact displayed price, so follow its wording; when it explicitly says “rounded”, acknowledge the uncertainty.

A reliable method

1. Identify whether the reference is cost or selling price.

2. Use the correct equation for the stated definition.

3. If rounding is explicit, reverse the interval rather than claiming an exact original.

Worked example 1

An item sells for £150 at a 25% profit on cost. Find cost.

STEP-BY-STEP SOLUTION

150 = 1.25C, so C = £120. Profit is £30, which is 25% of cost but 20% of selling price.

Worked example 2

After a 20% increase, a positive value rounds to 60 to the nearest whole number. Bound the original.

STEP-BY-STEP SOLUTION

$59.5 \leq 1.2A < 60.5$. Divide by 1.2: $595/12 \leq A < 605/12$, approximately $49.5833 \leq A < 50.4167$. The fractions are the exact bounds.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

If a selling price includes a 20% margin on selling price, cost is 80% of selling price. If it includes a 20% markup on cost, cost is selling price/1.2. Label the denominator in every profit-rate calculation.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 Selling price is £84 at 20% profit on cost. Find cost.

5.2 Selling price is £84 with a 20% profit margin on selling price. Find cost.

5.3 An item sells for £126 at a 10% loss on cost. Find cost.

5.4 A final value after a 25% decrease rounds to 30 to the nearest integer. Give exact original bounds.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / MAKE THE RELATIONSHIP VISIBLE

Connect the representations

Reverse the multiplier, not the percentage label



READ THE FIGURE

After a 15% discount the known £68 represents 85%, so original = 68/0.85 = £80. Adding 15% to £68 would instead give £78.20, because that percentage uses the smaller sale price as its base. Check the recovery by applying the original discount forwards.

Discuss: identify the reference whole, fixed quantity or rate in this picture. Recalculate one labelled result without looking at the explanation. Say which assumptions make the comparison valid.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. 18% of a value is 45. Find the value. [2 marks]

R2. A price is £96 after a 20% increase. Find original. [3 marks]

R3. A price is £96 after a 20% decrease. Find original. [3 marks]

R4. A hypothetical bill of £230 includes a specified 15% added charge. Find base and charge. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. A price becomes £198 after two successive 10% discounts. Find original to nearest penny. [3 marks]

R6. An item sells at £180 with 20% profit on cost. Find cost and profit. [3 marks]

R7. A price is discounted 25%, then £9 delivery makes £81. Find original item price. [3 marks]

R8. After a 10% rise then a 20% fall, a value is £48 lower than originally. Find original. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Recovering a whole from a known percentage

1.1 20% of an amount is 46. Find the whole.

46/0.20 = 230.

1.2 7.5% of a mass is 18 kg. Find the mass.

18/0.075 = 240 kg.

1.3 A 25% discount saves £16. Find the original price.

16/0.25 = £64.

1.4 140% of a value is 98. Find the value.

98/1.4 = 70.

Lesson 2 • Reversing an increase

2.1 After a 10% rise a price is £77. Find the original.

77/1.10 = £70.

2.2 A mass is 486 kg after an 8% rise. Find the original.

486/1.08 = 450 kg.

2.3 A final £345 includes a specified 15% added charge. Find the base.

345/1.15 = £300.

2.4 A value doubles. What multiplier reverses the increase?

Divide by 2, or multiply by 0.5. Doubling is a 100% increase; its reversal is a 50% decrease.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Reversing a decrease

3.1 A sale price is £54 after 25% off. Find the original.

54/0.75 = £72.

3.2 An amount becomes 357 after a 15% decrease. Find the original.

357/0.85 = 420.

3.3 A value becomes 96 after a 40% decrease. Find the original.

96/0.60 = 160.

3.4 Explain why adding 25% to a 25%-discounted price does not restore the original.

0.75 × 1.25 = 0.9375, so it restores only 93.75% of the original. Reversal requires division by 0.75.

Lesson 4 • Reversing successive changes

4.1 After two 10% increases a value is 242. Find original.

$242/(1.1^2) = 200$.

4.2 After 20% off then another 25% off, a price is £90. Find original.

90/(0.8 × 0.75) = £150.

4.3 A price rises 5%, then £8 is added to give £92. Find original.

(92 − 8)/1.05 = £80.

4.4 A 10% rise then a 20% fall reduces an amount by £36. Find original.

Overall factor 0.88, so decrease is 12% of original. Original = 36/0.12 = £300.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Profit bases, losses and rounded final amounts

5.1 Selling price is £84 at 20% profit on cost. Find cost.

84/1.2 = £70.

5.2 Selling price is £84 with a 20% profit margin on selling price. Find cost.

Cost is 80% of £84: £67.20.

5.3 An item sells for £126 at a 10% loss on cost. Find cost.

126 = 0.9C, so C = £140.

5.4 A final value after a 25% decrease rounds to 30 to the nearest integer. Give exact original bounds.

29.5 ≤ 0.75A < 30.5; divide by 3/4: 118/3 ≤ A < 122/3.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. 18% of a value is 45. Find the value. [2 marks]

45/0.18 = 250. [1 for divisor; 1 for result.]

R2. A price is £96 after a 20% increase. Find original. [3 marks]

96/1.2 = £80. [1 for 120%; 1 for division; 1 for answer.]

R3. A price is £96 after a 20% decrease. Find original. [3 marks]

96/0.8 = £120. [1 for 80%; 1 for division; 1 for answer.]

R4. A hypothetical bill of £230 includes a specified 15% added charge. Find base and charge. [3 marks]

Base 230/1.15 = £200; charge £30. [1 for multiplier; 1 for base; 1 for charge.]

R5. A price becomes £198 after two successive 10% discounts. Find original to nearest penny. [3 marks]

$198/0.9^2 = 244.444\ldots$, so £244.44. [1 for product; 1 for division; 1 for rounding.]

R6. An item sells at £180 with 20% profit on cost. Find cost and profit. [3 marks]

Cost 180/1.2 = £150; profit £30. [1 for relationship; 1 for cost; 1 for profit.]

R7. A price is discounted 25%, then £9 delivery makes £81. Find original item price. [3 marks]

(81 − 9)/0.75 = £96. [1 for subtracting delivery; 1 for reverse multiplier; 1 for result.]

R8. After a 10% rise then a 20% fall, a value is £48 lower than originally. Find original. [4 marks]

Overall multiplier 1.1 × 0.8 = 0.88. Loss is 12%, so 48/0.12 = £400. [1 for factor; 1 for loss percentage; 1 for division; 1 for result.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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