GCSE ONLINE TUTORING

RATIO, PROPORTION & RATES / TOPIC GUIDE 32

32

Proportion in everyday problems

Shared GCSE core

Use multiplicative reasoning to scale recipes, compare prices, interpret scale diagrams and model practical work. Learn to recognise when a constant-rate assumption is justified and when a fixed charge or real constraint changes the calculation.

New amount = old amount × scale factor

Check units, assumptions and whole-package constraints.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Work with ratios, fractions, decimals and metric units.

Your route through this notebook

1. The unitary method and proportional tables
2. Recipes, mixtures and fractional scale factors
3. Best buys and unit pricing
4. Scales, maps and geometric proportionality
5. Work, resources and model assumptions

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

The unitary method and proportional tables

The unitary method finds the value for one unit, then scales to the required number. If five identical notebooks cost £12, each costs £12/5, and any number can be priced at that rate if there are no discounts or extra fees. A proportional table has a constant ratio output/input for non-zero inputs.

You can also use a direct scale factor: moving from 5 units to 15 multiplies both quantity and cost by 3. Doubling both quantities preserves a proportional relationship; adding the same number to both does not. A fixed charge breaks direct proportionality, even when the remaining cost changes at a constant rate. Check the model before calculating.

A reliable method

1. Identify the linked quantities and the constant-rate assumption.

2. Find one unit or a convenient common unit.

3. Scale to the requested amount and check the units.

Worked example 1

Seven identical pens cost £8.40. Find the cost of 12.

STEP-BY-STEP SOLUTION

One costs £8.40/7 = £1.20. Twelve cost £14.40, assuming the same unit price.

Worked example 2

A machine packs 84 items in 6 minutes at a steady rate. How long for 210?

STEP-BY-STEP SOLUTION

Rate = 14 items/minute. Time = 210/14 = 15 minutes. The constant-rate assumption excludes setup delays.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A taxi charge £5 + £2 per mile is not directly proportional to mileage: zero miles still gives £5. Dividing a 5-mile cost by five does not recover the marginal rate.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Four tickets cost £30. Find the cost of 10 at the same rate.

1.2 A tap fills 45 litres in 3 minutes. Find the volume in 8 minutes at the same rate.

1.3 A table pairs inputs 2,4,6 with outputs 7,14,21. Is it proportional?

1.4 A service charges C = 10 + 3n. Explain why C is not directly proportional to n.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Recipes, mixtures and fractional scale factors

A recipe scales proportionally when every ingredient is multiplied by the same factor. The factor is required servings/original servings. It may be fractional: scaling eight servings to five uses 5/8, not a subtraction of three grams from every ingredient. Convert ingredient units before comparing or combining amounts.

For a mixture, keep the named ingredient ratio fixed while changing the batch size. The total number of ratio parts identifies what fraction each ingredient supplies. Practical limits, such as buying whole packets or enough liquid for a minimum number of servings, may require rounding the purchase amount without changing the intended recipe quantities.

A reliable method

1. Find required/original scale factor.

2. Multiply every ingredient by that same factor.

3. Separate recipe quantities from whole-package purchase decisions.

Worked example 1

A recipe for 8 uses 600 g flour and 400 ml milk. Scale it to 5.

STEP-BY-STEP SOLUTION

Scale factor 5/8. Flour = 375 g; milk = 250 ml. Both amounts use the same factor.

Worked example 2

Paint is mixed red:white = 2:7. Make 4.5 litres.

STEP-BY-STEP SOLUTION

9 parts in total, so one is 0.5 litre. Red 1 litre and white 3.5 litres.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

If servings decrease from 8 to 5, the amount falls to 5/8 of the original, not by 5/8. Distinguish the remaining fraction from the fraction removed.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 A recipe for 6 uses 450 g flour. How much for 10?

2.2 A drink uses concentrate:water = 1:5. Make 3 litres.

2.3 A recipe for 12 needs 9 eggs. How many mathematically for 8 servings?

2.4 A recipe needs 750 g flour, sold only in 500 g bags. How many bags must be bought?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Best buys and unit pricing

A fair price comparison uses a common amount. You can calculate cost per gram, per 100 g or per kilogram, or scale both packages to the same quantity. State the unit with the price; 0.42 pence per gram and £0.42 per gram differ by a factor of 100.

The cheapest unit price is not necessarily the lowest total cost for a required purchase. Whole packages, minimum quantities, offers and waste can change the best choice. Distinguish “best value per unit” from “cheapest way to buy at least a specified amount”. Only compare products assumed equivalent for the purpose of the question.

A reliable method

1. Convert both quantities to the same unit.

2. Calculate a common unit price or equivalent cost.

3. If a required amount is given, compare whole-package totals too.

Worked example 1

Compare 750 g for £3.15 with 1.2 kg for £4.80.

STEP-BY-STEP SOLUTION

First: £3.15/0.75 = £4.20/kg. Second: £4.80/1.2 = £4.00/kg. The 1.2 kg pack has the lower unit price.

Worked example 2

You need at least 1.4 kg. Packs are 500 g for £1.80 or 900 g for £3.00. Which single pack type is cheaper to buy?

STEP-BY-STEP SOLUTION

Three small packs give 1.5 kg for £5.40. Two large packs give 1.8 kg for £6.00. Small packs cost less for this requirement, despite the large pack having a lower unit price.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An answer about best value should state the basis of comparison. If mixing pack types is allowed, also test mixed combinations rather than comparing only repeated identical packs.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Which is cheaper per kg: 400 g at £1.52 or 700 g at £2.80?

3.2 Six bottles cost £4.50. A single bottle costs 80p. Find savings per bottle in the pack.

3.3 Compare 3 for £5 with individual items at £1.75.

3.4 You need 1.4 kg; 500 g costs £1.80 and 900 g costs £3.00. Mixed packs are allowed. Find the cheapest combination.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Scales, maps and geometric proportionality

A scale of 1:n means one length unit on a drawing represents n of the same units in reality. Convert after scaling or convert both quantities first. A scale drawing preserves corresponding length ratios; it does not automatically preserve numerical areas using the same factor.

For similar shapes, a length multiplier k produces an area multiplier k^2 and a volume multiplier k^3. This is because areas multiply two dimensions and volumes multiply three. These conclusions require similar shapes, not merely two shapes with one matching side ratio. Maps also show straight-line distances; an actual route may be longer.

A reliable method

1. Interpret the scale using identical units.

2. For lengths apply k; for similar areas k^2; for similar volumes k^3.

3. Convert units at a clearly recorded stage.

Worked example 1

On a 1:25,000 map, two points are 6.4 cm apart. Find real straight-line distance.

STEP-BY-STEP SOLUTION

$6.4 \times 25{,}000 = 160{,}000$ cm = 1,600 m = 1.6 km.

Worked example 2

Similar solids have length ratio 2:3. Find their volume ratio.

STEP-BY-STEP SOLUTION

Cube corresponding factors: $2^3:3^3 = 8:27$. The area ratio would instead be 4:9.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A 1:100 drawing does not make every area 100 times smaller. Each of two lengths is divided by 100, so area is divided by 10,000.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 A model uses 1:50. Its length is 12 cm. Find actual length.

4.2 An actual 18 m wall is drawn at 1:200. Find drawing length.

4.3 Similar shapes have lengths 3:5. Find area ratio.

4.4 A solid is enlarged by length factor 4. Its original volume is 30 cm^3. Find new volume.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Work, resources and model assumptions

If identical workers operate at a constant independent rate, total work can be measured in worker-hours. Four workers for six hours supply 24 worker-hours. For a fixed job, increasing the number of workers reduces time only if the work divides efficiently and there are no bottlenecks.

Similar reasoning applies to machine-hours, food supplies and coverage rates. Identify whether the total job is fixed, whether the rate is per person or for the whole group, and whether all workers or machines are equivalent. A real-world model can be useful without being exact; stating assumptions makes its limits clear. Direct proportion and inverse proportion use different fixed quantities.

A reliable method

1. Identify the fixed job or resource total.

2. Calculate total worker-hours, machine-hours or person-days.

3. Divide by the new resource count and state the constant-rate assumptions.

Worked example 1

Six identical workers complete a job in 8 hours. Find time for 4.

STEP-BY-STEP SOLUTION

The job needs $6 \times 8 = 48$ worker-hours. Four workers need $48/4 = 12$ hours, assuming equal productivity and divisible work.

Worked example 2

Food feeds 12 people for 15 days. How long for 20 people?

STEP-BY-STEP SOLUTION

There are 180 person-days of food. At the same consumption per person, $180/20 = 9$ days.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Doubling workers does not guarantee halving time for every real job. A single narrow workspace or a task that must be done in sequence can invalidate the model.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 Three machines make 540 parts in 6 hours. Find parts per machine-hour.

5.2 At that rate, how many parts can 5 machines make in 4 hours?

5.3 Supplies feed 8 people for 18 days. How many days for 12 people?

5.4 A job takes 10 workers 9 days. Find time for 6 at the same individual rate.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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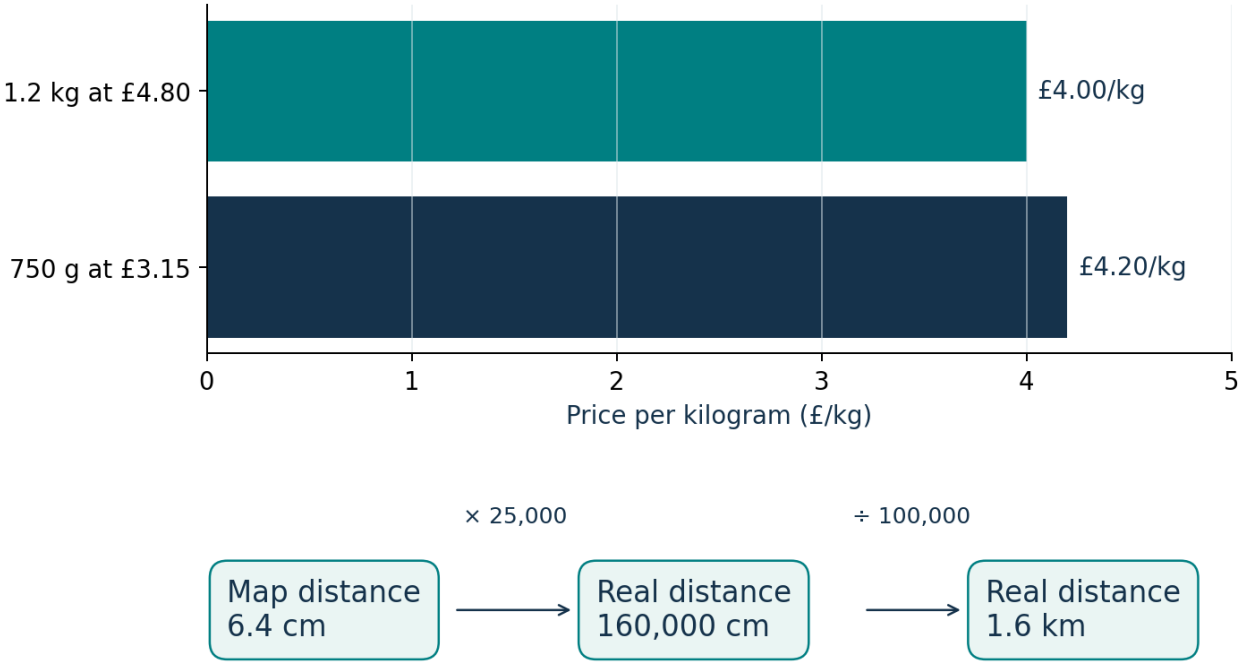
VISUAL EXPLAINER / MAKE THE RELATIONSHIP VISIBLE

Connect the representations

Proportional comparisons need matching units



READ THE FIGURE

The price comparison uses kilograms for both packs, giving £4.20/kg versus £4.00/kg. This ranks unit value, not necessarily the cheapest whole-pack purchase for a particular need. The map diagram scales centimetres first at 1:25,000, then converts real centimetres into kilometres.

Discuss: identify the reference whole, fixed quantity or rate in this picture. Recalculate one labelled result without looking at the explanation. Say which assumptions make the comparison valid.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Nine identical items cost £13.50. Find cost of 14. [3 marks]

R2. A recipe for 8 uses 320 g sugar. Find sugar for 14. [3 marks]

R3. Make 7.2 litres of a 2:7 concentrate:water drink. [3 marks]

R4. Compare 600 g for £2.34 with 1 kg for £4.10. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. On a 1:40,000 map a distance is 3.5 cm. Find real distance in km. [3 marks]

R6. Similar shapes have length ratio 4:7. Find area and volume ratios. [2 marks]

R7. Eight workers take 15 hours for a job. Find time for 12 equivalent workers. [3 marks]

R8. Need at least 2 kg. Packs are 600 g for £2.10 or 1 kg for £3.80; mixing is allowed. Find the cheapest purchase. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • The unitary method and proportional tables

1.1 Four tickets cost £30. Find the cost of 10 at the same rate.

One costs £7.50; ten cost £75.

1.2 A tap fills 45 litres in 3 minutes. Find the volume in 8 minutes at the same rate.

15 litres/minute; 120 litres.

1.3 A table pairs inputs 2,4,6 with outputs 7,14,21. Is it proportional?

Yes, output/input = 3.5 throughout the supplied table.

1.4 A service charges C = 10 + 3n. Explain why C is not directly proportional to n.

The fixed £10 means C/n is not constant; the graph does not pass through the origin.

Lesson 2 • Recipes, mixtures and fractional scale factors

2.1 A recipe for 6 uses 450 g flour. How much for 10?

450 × 10/6 = 750 g.

2.2 A drink uses concentrate:water = 1:5. Make 3 litres.

6 parts; concentrate 0.5 litre, water 2.5 litres.

2.3 A recipe for 12 needs 9 eggs. How many mathematically for 8 servings?

9 × 8/12 = 6 eggs.

2.4 A recipe needs 750 g flour, sold only in 500 g bags. How many bags must be bought?

750/500 = 1.5 bags, so buy 2. The recipe still uses 750 g, leaving 250 g.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Best buys and unit pricing

3.1 Which is cheaper per kg: 400 g at £1.52 or 700 g at £2.80?

£3.80/kg versus £4.00/kg. The 400 g pack is cheaper per kg.

3.2 Six bottles cost £4.50. A single bottle costs 80p. Find savings per bottle in the pack.

Pack price 75p each; saving 5p per bottle.

3.3 Compare 3 for £5 with individual items at £1.75.

Offer unit price £5/3 ≈ £1.6667; three singles cost £5.25, so the offer saves 25p for three.

3.4 You need 1.4 kg; 500 g costs £1.80 and 900 g costs £3.00. Mixed packs are allowed. Find the cheapest combination.

One of each gives exactly 1.4 kg for £4.80. Other qualifying small combinations cost £5.40 or more, so £4.80 is cheapest.

Lesson 4 • Scales, maps and geometric proportionality

4.1 A model uses 1:50. Its length is 12 cm. Find actual length.

600 cm = 6 m.

4.2 An actual 18 m wall is drawn at 1:200. Find drawing length.

18 m = 1800 cm; 1800/200 = 9 cm.

4.3 Similar shapes have lengths 3:5. Find area ratio.

9:25.

4.4 A solid is enlarged by length factor 4. Its original volume is 30 cm³. Find new volume.

Volume factor $4^3 = 64$; new volume 1920 cm³.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Work, resources and model assumptions

5.1 Three machines make 540 parts in 6 hours. Find parts per machine-hour.

540/(3 × 6) = 30 parts per machine-hour.

5.2 At that rate, how many parts can 5 machines make in 4 hours?

5 × 4 × 30 = 600 parts.

5.3 Supplies feed 8 people for 18 days. How many days for 12 people?

144 person-days divided by 12 = 12 days.

5.4 A job takes 10 workers 9 days. Find time for 6 at the same individual rate.

90 worker-days / 6 = 15 days, assuming the job divides effectively.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Nine identical items cost £13.50. Find cost of 14. [3 marks]

One costs £1.50; 14 cost £21. [1 for division; 1 for scaling; 1 for money result.]

R2. A recipe for 8 uses 320 g sugar. Find sugar for 14. [3 marks]

Factor 14/8 = 1.75; sugar = 560 g. [1 for factor; 1 for product; 1 for unit.]

R3. Make 7.2 litres of a 2:7 concentrate:water drink. [3 marks]

9 parts, 0.8 litre each: 1.6 litres concentrate,5.6 litres water. [1 for parts; 1 for unit amount; 1 for both.]

R4. Compare 600 g for £2.34 with 1 kg for £4.10. [3 marks]

First costs £3.90/kg, below £4.10/kg. The 600 g pack is better unit value. [1 for common unit; 1 for prices; 1 for conclusion.]

R5. On a 1:40,000 map a distance is 3.5 cm. Find real distance in km. [3 marks]

3.5 × 40,000 = 140,000 cm = 1.4 km. [1 for scale; 1 for conversion; 1 for answer.]

R6. Similar shapes have length ratio 4:7. Find area and volume ratios. [2 marks]

Area 16:49; volume 64:343. [1 each.]

R7. Eight workers take 15 hours for a job. Find time for 12 equivalent workers. [3 marks]

120 worker-hours / 12 = 10 hours. [1 for fixed work; 1 for division; 1 for result/assumption.]

R8. Need at least 2 kg. Packs are 600 g for £2.10 or 1 kg for £3.80; mixing is allowed. Find the cheapest purchase. [4 marks]

Two 1 kg packs cost £7.60. Four 600 g packs cost £8.40. One 1 kg plus two 600 g packs costs £8.00. One of each is only 1.6 kg. Thus £7.60 is cheapest. [1 for required combinations; 2 for costs; 1 for conclusion.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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