GCSE ONLINE TUTORING

RATIO, PROPORTION & RATES / TOPIC GUIDE 31

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Ratio

Shared GCSE core with extension problems

Ratio compares quantities multiplicatively. Build a secure understanding of parts and wholes, then solve sharing, reverse, linked and changing-ratio problems. Bar models and checks help you see what each number represents instead of relying on keywords.

A : B = 3 : 5

A is 3/8 of the whole, but 3/5 of B.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Simplify fractions, multiply and divide decimals, and solve simple linear equations.

Your route through this notebook

1. Meaning, order and equivalent ratios
2. Part-to-part, part-to-whole and sharing a total
3. Finding a total from one share or a difference
4. Combining linked ratios
5. Changing ratios and fixed quantities

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Meaning, order and equivalent ratios

A ratio compares quantities in a stated order. Red:blue = 2:3 means that for every two equal-sized red parts there are three equal-sized blue parts. It does not mean there are only five objects: 20 red and 30 blue give the same relationship. Multiplying or dividing all parts by the same positive factor preserves the ratio.

A ratio normally compares quantities using a common unit. Convert before simplifying a length ratio such as 2 m:50 cm. Ratios may contain decimals or fractions; clear these using a common multiplier before dividing by a common factor. The order must remain unchanged throughout. Unlike adding the same amount to both parts, multiplicative scaling preserves the relationship. A ratio without context does not determine the actual total.

A reliable method

1. Label the quantities in the required order and convert to matching units.

2. Multiply all entries to remove fractions or decimals if needed.

3. Divide all entries by their highest common factor and check equivalent scaling.

Worked example 1

Simplify 1.2 m:45 cm.

STEP-BY-STEP SOLUTION

Convert 1.2 m to 120 cm, giving 120:45. Divide both entries by 15: 8:3. Starting with 1.2:45 would compare different units.

Worked example 2

Simplify 3/4:5/6.

STEP-BY-STEP SOLUTION

Multiply both entries by 12, the LCM of the denominators: 9:10. This is already in whole-number simplest form.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Adding 2 to both entries of 2:3 gives 4:5, which is not equivalent: 2/3 differs from 4/5. Multiplying both entries by 2 gives 4:6, which is equivalent. Test a claim by comparing quotients.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Simplify 18:30.

1.2 Simplify 0.8:1.2:2.

1.3 Write 2 hours:45 minutes in simplest form.

1.4 Simplify 2/3:1/2.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Part-to-part, part-to-whole and sharing a total

A part-to-part ratio compares categories, while a part-to-whole fraction compares one category with the total. If A:B = 3:5, the total contains eight equal parts, so A is 3/8 of the total and A/B = 3/5. These fractions describe different comparisons.

To share a total, add all ratio entries, find the size of one part and multiply by the entries. The shares must sum back to the original total and retain the specified ratio. Three-part sharing follows exactly the same reasoning. If a count of indivisible objects produces fractional shares, the proposed total or exact-ratio condition may be impossible; a money or mass share can legitimately be fractional.

A reliable method

1. Add the ratio parts to identify the whole.

2. Divide the total by that number to find one part.

3. Multiply for each share, then check total and ratio.

Worked example 1

Share £96 in the ratio 3:5.

STEP-BY-STEP SOLUTION

There are 8 parts. One part is £96/8 = £12. Shares are £36 and £60; they sum to £96 and simplify to 3:5.

Worked example 2

A:B:C = 2:3:7 and the total is 180 kg. Find B.

STEP-BY-STEP SOLUTION

There are 12 parts, so one is 15 kg. B is 3 × 15 = 45 kg. Equivalently B is 3/12 = 1/4 of the total.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

In red:blue = 2:7, red is 2/9 of the total, not 2/7. The denominator 7 compares red with blue only. A labelled bar model makes the missing total explicit.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Share £84 in the ratio 2:5.

2.2 Share 240 g in the ratio 1:3:4.

2.3 Boys:girls = 4:7. What fraction of the group are girls?

2.4 Can 25 whole counters be shared exactly in the ratio 2:3:4? Explain.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Finding a total from one share or a difference

A ratio problem need not give the whole. If one named share is known, divide that amount by its own number of parts to find one part. If the difference is known, compare the difference between ratio entries with the difference between actual quantities.

For A:B = 7:4, A contains three more parts than B. A difference of 24 therefore corresponds to 3 parts, not all 11 parts. Once one part is known, recover any share or the total. A labelled diagram or equation helps distinguish a known share, a known total and a known difference. Check the final result against the original condition rather than only the ratio.

A reliable method

1. Identify exactly what the known amount measures.

2. Match it to the corresponding number of ratio parts.

3. Recover one part, then calculate and verify the requested amount.

Worked example 1

A:B = 5:8 and A = 35. Find B and the total.

STEP-BY-STEP SOLUTION

5 parts = 35, so one part is 7. B = 56 and total = 91.

Worked example 2

Two lengths are in the ratio 9:5 and differ by 28 cm. Find them.

STEP-BY-STEP SOLUTION

The difference is 4 parts. One part is 7 cm, so lengths are 63 cm and 35 cm. Their difference checks: 28 cm.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Dividing the known difference by the sum of the ratio entries would treat a difference as a total. Write “difference = ___ parts” before calculating.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A:B = 3:7 and B = 42. Find A.

3.2 A:B = 8:3 and A exceeds B by 45. Find both.

3.3 A:B:C = 2:5:6 and B = 40. Find the total.

3.4 Lengths have ratio 11:7 and differ by 18 m. Find the shorter length.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Combining linked ratios

Linked ratios share a common quantity. To combine A:B and B:C, scale both ratios so that B has the same number of parts in each. Then the A,B,C entries refer to the same underlying unit. The LCM of the two B entries often provides the smallest convenient shared value.

The shared letter must represent the same actual quantity in both relationships. You cannot join ratios merely because their numerical entries look compatible. Once combined, simplify the three-part ratio and use a known share or total if supplied. Keep labels visible to avoid accidentally joining the wrong entries.

A reliable method

1. Write the labels above each ratio.

2. Scale the shared quantity to matching entries.

3. Combine in the required order and check both original ratios.

Worked example 1

A:B = 2:3 and B:C = 4:5. Find A:B:C.

STEP-BY-STEP SOLUTION

Make B = 12: A:B = 8:12 and B:C = 12:15. Therefore A:B:C = 8:12:15.

Worked example 2

Flour:sugar = 5:2 and sugar:butter = 3:4. Find flour:sugar:butter.

STEP-BY-STEP SOLUTION

Make sugar = 6. The ratios become 15:6 and 6:8, so the combined ratio is 15:6:8.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

From A:B = 2:3 and B:C = 4:5, writing 2:3:5 loses the second relationship. Its B:C would be 3:5 rather than 4:5.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Combine A:B = 3:4 and B:C = 2:5.

4.2 Combine X:Y = 5:6 and Y:Z = 9:2.

4.3 A:B:C = 3:4:10 and total is 85. Find C.

4.4 A:B = 4:7 and A:C = 2:3. Find B:C.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Changing ratios and fixed quantities

When quantities are added or removed, the ratio can change because the amounts may not scale together. Represent the original shares as multiples of a common unknown k, then adjust the appropriate share. Use the new ratio to form an equation. Identify what stays fixed: the total, one category, or neither.

If water is added to a mixture, the amount of concentrate remains fixed but its fraction of the total falls. If equal amounts are added to both categories, their difference stays fixed, not their ratio. A valid solution must give non-negative quantities and satisfy both the original and changed situations.

A reliable method

1. Represent original quantities as ak and bk.

2. Apply each stated change to the correct quantity.

3. Use the new ratio, solve for k and verify both states.

Worked example 1

Red:blue = 3:5. Adding 18 red counters makes the ratio 3:2. Find the original counts.

STEP-BY-STEP SOLUTION

Original counts 3k and 5k. Then (3k + 18)/(5k) = 3/2. Thus 6k + 36 = 15k, so k = 4. Original counts are 12 red and 20 blue. After adding 18 red, 30:20 = 3:2.

Worked example 2

Concentrate:water = 2:3 in 25 litres. Add 15 litres of water. Find the new ratio.

STEP-BY-STEP SOLUTION

Original concentrate 10 litres and water 15 litres. New water 30 litres. Ratio 10:30 = 1:3.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An algebraic answer can expose an impossible discrete scenario. In the first example, adding 12 rather than 18 would give k = 8/3 and a non-integer blue count, so no exact whole-counter solution. Do not round such counts and claim the ratio still holds.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 Red:blue = 2:3. Adding 6 red makes the ratio 4:3. Find original counts.

5.2 A mixture is 1:4 concentrate:water and totals 30 litres. Add 6 litres concentrate. Find the new ratio.

5.3 A:B = 5:3. Remove 8 from A to make A:B = 1:1. Find originals.

5.4 A:B = 3:7. Add the same positive amount to both. Does the ratio remain 3:7?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / MAKE THE RELATIONSHIP VISIBLE

Connect the representations

One ratio unit links every share



Whole: 8 equal parts = £96

A : B = 2 : 3 → A : B = 8 : 12

B : C = 4 : 5 → B : C = 12 : 15

Match B first: A : B : C = 8 : 12 : 15

READ THE FIGURE

Top: a 3:5 share of £96 has eight parts worth £12 each. A is 3/8 of the whole but 3/5 of B. Bottom: both linked ratios must use the same B quantity; scaling B to 12 allows the combined ratio 8:12:15. Check the two original ratios by simplifying the relevant pair.

Discuss: identify the reference whole, fixed quantity or rate in this picture. Recalculate one labelled result without looking at the explanation. Say which assumptions make the comparison valid.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Simplify 2.4 kg:600 g. [2 marks]

R2. Share £132 in the ratio 4:7. [3 marks]

R3. A:B = 3:8. What fraction of the total is A? [1 mark]

R4. A:B = 7:4 and A − B = 36. Find A and B. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Combine A:B = 3:5 and B:C = 2:7. [3 marks]

R6. A:B:C = 6:10:35 and B = 80. Find total. [3 marks]

R7. Juice:water = 3:5 in 40 litres. Add 8 litres juice. Find new ratio. [4 marks]

R8. Red:blue = 2:5. Adding 9 red makes 1:1. Find original counts. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Meaning, order and equivalent ratios

1.1 Simplify 18:30.

Divide both by 6: 3:5.

1.2 Simplify 0.8:1.2:2.

Multiply by 10 to get 8:12:20, then divide by 4: 2:3:5.

1.3 Write 2 hours:45 minutes in simplest form.

120:45 = 8:3.

1.4 Simplify 2/3:1/2.

Multiply both by 6: 4:3.

Lesson 2 • Part-to-part, part-to-whole and sharing a total

2.1 Share £84 in the ratio 2:5.

7 parts; one part £12. Shares £24 and £60.

2.2 Share 240 g in the ratio 1:3:4.

8 parts, 30 g each: 30 g,90 g,120 g.

2.3 Boys:girls = 4:7. What fraction of the group are girls?

7/(4 + 7) = 7/11.

2.4 Can 25 whole counters be shared exactly in the ratio 2:3:4? Explain.

No. The 9-part total would require one part 25/9, producing non-integer shares. The 4-part share alone is 100/9, not a whole counter.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Finding a total from one share or a difference

3.1 A:B = 3:7 and B = 42. Find A.

7 parts = 42; one part 6; A = 18.

3.2 A:B = 8:3 and A exceeds B by 45. Find both.

5 parts = 45; one part 9. A = 72, B = 27.

3.3 A:B:C = 2:5:6 and B = 40. Find the total.

One part = 8; total 13 × 8 = 104.

3.4 Lengths have ratio 11:7 and differ by 18 m. Find the shorter length.

4 parts = 18 m; one part 4.5 m; shorter = 7 × 4.5 = 31.5 m.

Lesson 4 • Combining linked ratios

4.1 Combine A:B = 3:4 and B:C = 2:5.

Scale B:C to 4:10. Combined ratio 3:4:10.

4.2 Combine X:Y = 5:6 and Y:Z = 9:2.

Make Y = 18: X:Y = 15:18, Y:Z = 18:4. Ratio 15:18:4.

4.3 A:B:C = 3:4:10 and total is 85. Find C.

17 parts; one part 5; C = 50.

4.4 A:B = 4:7 and A:C = 2:3. Find B:C.

Make A = 4: A:B:C = 4:7:6, so B:C = 7:6.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Changing ratios and fixed quantities

5.1 Red:blue = 2:3. Adding 6 red makes the ratio 4:3. Find original counts.

(2k + 6)/(3k) = 4/3 gives 6k + 18 = 12k, so k = 3. Counts 6 red,9 blue.

5.2 A mixture is 1:4 concentrate:water and totals 30 litres. Add 6 litres concentrate. Find the new ratio.

Original 6 and 24 litres. New 12:24 = 1:2.

5.3 A:B = 5:3. Remove 8 from A to make A:B = 1:1. Find originals.

5k − 8 = 3k, so k = 4. A = 20,B = 12.

5.4 A:B = 3:7. Add the same positive amount to both. Does the ratio remain 3:7?

No. If (3k + t)/(7k + t) = 3/7, cross-multiplication gives 7t = 3t, so t = 0. A positive t changes the ratio.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Simplify 2.4 kg:600 g. [2 marks]

2400:600 = 4:1. [1 for conversion; 1 for simplification.]

R2. Share £132 in the ratio 4:7. [3 marks]

11 parts, £12 each: £48 and £84. [1 for total parts; 1 for part size; 1 for shares.]

R3. A:B = 3:8. What fraction of the total is A? [1 mark]

3/11. [1 for whole of 11 parts.]

R4. A:B = 7:4 and A − B = 36. Find A and B. [3 marks]

3 parts = 36, one part 12. A = 84,B = 48. [1 for difference parts; 1 for unit; 1 for both.]

R5. Combine A:B = 3:5 and B:C = 2:7. [3 marks]

Make B = 10: 6:10:35. [1 for shared value; 1 for scaling; 1 for combined ratio.]

R6. A:B:C = 6:10:35 and B = 80. Find total. [3 marks]

One part 8; total 51 × 8 = 408. [1 for unit; 1 for total parts; 1 for result.]

R7. Juice:water = 3:5 in 40 litres. Add 8 litres juice. Find new ratio. [4 marks]

Original 15 litres juice and 25 water. New 23:25, already simplest. [1 for parts; 1 for originals; 1 for change; 1 for ratio.]

R8. Red:blue = 2:5. Adding 9 red makes 1:1. Find original counts. [4 marks]

2k + 9 = 5k gives k = 3; originally 6 red and 15 blue. [1 for model; 1 for equation; 1 for k; 1 for counts/check.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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