GCSE ONLINE TUTORING

PROBABILITY / TOPIC GUIDE 56

56

Combined and conditional probability

Shared combined events with Higher extensions

Combine events through probability trees, distinguish independence from disjointness and adjust the model when objects are not replaced. Higher work conditions on observed information, reverses viewpoints and uses complements for repeated trials.

P(A | B) = P(A ∩ B) / P(B)

Given B,restrict the possible outcomes to B; P(B) > 0.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Use sample spaces, fractions, complements, Venn diagrams and two-way tables.

Your route through this notebook

1. Independent events and the multiplication rule

2. Probability trees with replacement

3. Without replacement and changing branches — Higher

4. Conditioning on information — Higher

5. Combining routes, reversing conditions and repeated trials — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Independent events and the multiplication rule

Events A and B are independent when knowing whether one occurred does not change the probability of the other. Then $P(A \cap B)=P(A)P(B)$. In a sequential model this means the second-stage probability stays the same after either first-stage outcome. Independence is an assumption to justify, not a consequence of the word “and”.

Mutually exclusive events cannot happen together, so their intersection has probability zero. Two non-zero-probability mutually exclusive events are therefore not independent. For events that may be dependent, the general rule uses a conditional probability: $P(A \cap B)=P(A)P(B \text{ given } A)$. This guide writes the latter as $P(B|A)$, with $P(A)>0$. A tree makes those conditional branches visible.

A reliable method

1. Decide whether the second event’s chance changes after the first.
2. Use the product of independent probabilities only when justified.
3. For dependent stages, multiply by the appropriate conditional branch.

Worked example 1

A fair coin and an independent fair die are used. Find P(head and six).

STEP-BY-STEP SOLUTION

$(1/2)(1/6)=1/12$, using independence.

Worked example 2

$P(A)=0.4$, $P(B)=0.3$ and they are independent. Find P(both) and P(neither).

STEP-BY-STEP SOLUTION

Both $0.4\times0.3=0.12$. Their complements are also independent, so neither $0.6\times0.7=0.42$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“And multiply, or add” is incomplete. Multiplication needs the correct second-stage probability, and addition needs disjoint routes or an overlap correction.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Independent events have probabilities 0.2 and 0.6. Find both.

1.2 Two independent fair coins are tossed. Find P(two tails).

1.3 On a single die roll, are score 1 and score 6 independent?

1.4 If $P(A)=1/3$ and $P(B|A)=3/5$, find $P(A \cap B)$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Probability trees with replacement

A probability tree separates stages and records conditional probabilities on branches. Branches leaving the same node sum to 1. Multiply along a route to find the probability of that complete outcome sequence. Add mutually exclusive routes when several sequences satisfy the requested event.

With replacement means the selected object is returned before the next draw. If the bag is remixed and each draw is uniform, the composition is restored and draws are independent. Same-colour probabilities still require adding red-red and blue-blue routes, while exactly one red uses red-blue and blue-red. Do not add the branch labels along a route.

A reliable method

1. Label branches from every node with probabilities summing to 1.

2. Multiply probabilities along each desired complete route.

3. Add only the disjoint routes that belong to the event.

Worked example 1

A bag has 3 red, 2 blue. Draw twice with replacement and remixing. Find P(two red).

STEP-BY-STEP SOLUTION

Each red probability is 3/5. Route RR has probability $(3/5)^2=9/25$.

Worked example 2

For that bag find P(exactly one red).

STEP-BY-STEP SOLUTION

Routes RB and BR each have probability $(3/5)(2/5)=6/25$. Total 12/25.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

RB and BR are different ordered routes but belong to the same exactly-one-red event. Missing either route halves the correct answer in this example.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 With the same 3-red, 2-blue replacement model, find P(BB).

2.2 Find P(same colour) in that model.

2.3 Find P(at least one red) in that model.

2.4 A tree node has one branch 0.37 and one other exhaustive branch. Find the other.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Without replacement and changing branches — Higher

Without replacement changes the bag after a selection. The total number of remaining objects decreases and the selected colour's count decreases. Therefore the second-stage probabilities depend on the first-stage outcome. Write a separate second-stage branch set for each first result.

For two selections from 3 red and 2 blue counters, after red there are 2 red and 2 blue among 4; after blue there are 3 red and 1 blue among 4. Multiply the correct conditional fractions along each route and add disjoint routes as usual. The method stays the same while the branch values change. Check that all complete route probabilities sum to 1.

A reliable method

1. Update the remaining total and relevant colour count after each draw.

2. Write second-stage probabilities separately under each first branch.

3. Multiply route probabilities and add matching event routes.

Worked example 1

A bag has 3 red, 2 blue. Draw twice without replacement. Find P(two red).

STEP-BY-STEP SOLUTION

P(RR)=(3/5)(2/4)=3/10.

Worked example 2

For that model find P(mixed colours).

STEP-BY-STEP SOLUTION

RB=(3/5)(2/4)=3/10. BR=(2/5)(3/4)=3/10. Mixed probability = 3/5.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Reducing the denominator but forgetting to reduce the selected colour count creates an invalid branch model. After a red is removed from 3 red, only 2 red remain.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A bag has 4 red, 3 blue. Draw two without replacement. Find P(RR).

3.2 For that bag find P(BB).

3.3 For that bag find P(mixed colours).

3.4 A bag has 5 green, 1 yellow. After yellow is drawn without replacement, find P(green next).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Conditioning on information — Higher

Conditional probability restricts the sample space to outcomes where the given condition holds. P(A|B)=P(A ∩ B)/P(B), provided P(B)>0. In a count table with uniform selection, this becomes the count in both divided by the count in B. The denominator is the group known to contain the selected member.

P(A|B) need not equal P(B|A). Reversing the wording changes the denominator. A tree may similarly require adding several leaf routes to find the condition before taking a ratio. Do not condition on an event of probability zero using this formula. For a fair die known to be even, the permitted outcomes are 2, 4, 6, each equally likely under the restricted model.

A reliable method

1. Identify the information after given and restrict to that group.

2. Find the favourable intersection within the group.

3. Divide by the condition’s probability or count and interpret the direction.

Worked example 1

A fair die is rolled and is known to be even. Find P(score greater than 3).

STEP-BY-STEP SOLUTION

Restricted outcomes {2, 4, 6}; favourable {4, 6}. Probability 2/3.

Worked example 2

Among 50 pupils, 20 play music, 15 play sport and 8 do both. Find P(sport|music) and P(music|sport).

STEP-BY-STEP SOLUTION

First denominator 20: 8/20=2/5. Reversed denominator 15: 8/15. The two conditions give different probabilities.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Given information does not mean repeat the experiment. It changes what is known about the same selected outcome and hence which outcomes remain possible.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 $P(A \cap B)=0.12$, $P(B)=0.3$. Find $P(A|B)$.

4.2 A fair die is known to exceed 2. Find P(even).

4.3 Of 30 cyclists, 12 are Year 10. Of all 20 Year 10 pupils, 12 cycle. Find both conditional probabilities.

4.4 Can the usual formula $P(A|B)=P(A \cap B)/P(B)$ be used when $P(B)=0$?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Combining routes, reversing conditions and repeated trials — Higher

A condition can collect several complete routes in a tree. Add those route probabilities to find the condition's total, then divide the favourable subset by it. This is especially useful when a result could have come from different groups. A natural-frequency model can make the denominator visible by following a hypothetical large group through the stated probabilities.

For repeated independent trials with success probability p, the probability of no successes in n trials is $(1-p)^n$. Therefore at least one success has probability $1-(1-p)^n$. This complement is often simpler than listing every success pattern. "Exactly one" is a different event: count routes with one success and no others. State independence and constant p before using a power.

A reliable method

1. Add all disjoint routes leading to the conditioning result.

2. Divide the desired route subtotal by that result's total.

3. For at least one, consider the no-success complement under a justified model.

Worked example 1

Half the items come from group A with P(red|A)=0.8; half from B with P(red|B)=0.2. An item is red. Find P(A|red).

STEP-BY-STEP SOLUTION

A-and-red=0.5×0.8=0.4; B-and-red=0.5×0.2=0.1. Total red=0.5. Thus P(A|red)=0.4/0.5=0.8.

Worked example 2

A trial succeeds with probability 0.2, independently each time. Find P(at least one success in 3 trials).

STEP-BY-STEP SOLUTION

No successes has probability $0.8^3=0.512$. At least one = $1-0.512=0.488$.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A high P(red|A) does not alone determine P(A|red). The frequency of group A and alternative routes to red also affect the reversed conditional probability.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 Independent trials have p=0.3. Find P(at least one success in 2 trials).

5.2 Three independent fair coins are tossed. Find P(exactly one head).

5.3 Group A supplies 25% of items with red probability 0.6; B supplies 75% with red probability 0.2. Find P(red).

5.4 In that model, find P(A|red).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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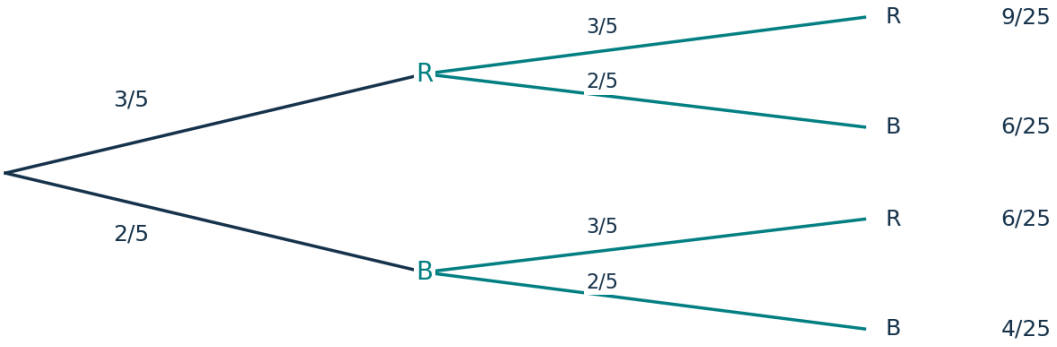
VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

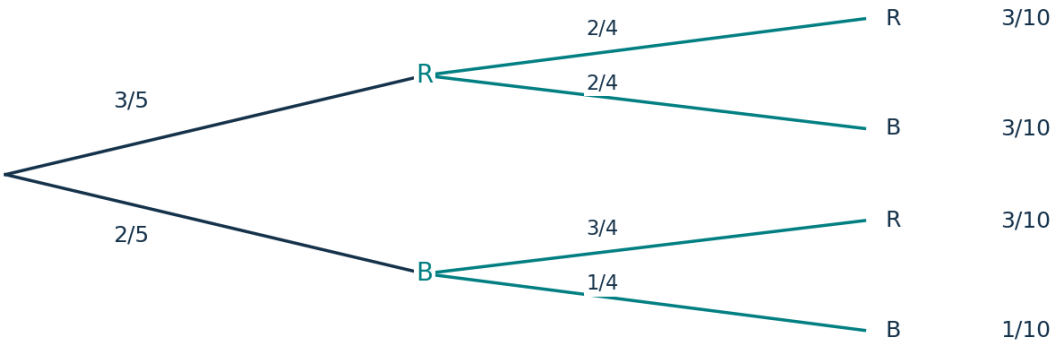
Read the model • 1

Multiply along routes; add disjoint qualifying routes

3 red,2 blue: replacement and remixing



Same bag: without replacement



WHAT THE REPRESENTATION SHOWS

R and B mean red and blue. With replacement the composition resets, so the second branch probabilities stay 3/5 and 2/5. Without replacement, branches update separately after red and blue. The leaf probabilities in each tree sum to 1. Mixed colours combine the RB and BR routes: 12/25 with replacement, 3/5 without.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 2

Given red,restrict to the red items before dividing

Natural frequencies for a hypothetical group of 1000

	Red	Not red	Total
Group A	200	200	400
Group B	120	480	600
Total	320	680	1000

40% from A; P(red | A) = 0.5

60% from B; P(red | B) = 0.2

Given red: 200 of the 320 are from A

P(A | red) = 200/320 = 5/8

WHAT THE REPRESENTATION SHOWS

This frequency representation encodes the supplied model, not observed survey data. Of 1000 hypothetical items,400 come from A and 200 of those are red; 600 come from B and 120 are red. Given red, only the 320 red items remain possible, so P(A|red)=200/320=5/8. P(red|A)=200/400=1/2 is a different conditional probability.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Independent events have P(A)=0.25, P(B)=0.4. Find both and neither. [3 marks]

R2. A bag has 4 red, 1 blue. Draw twice with replacement and remixing. Find P(exactly one blue). [3 marks]

R3. Higher: from 4 red, 1 blue draw twice without replacement. Find P(two red). [2 marks]

R4. Higher: from 5 red, 3 blue draw twice without replacement. Find P(mixed colours). [4 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Higher: a fair die is known to be odd. Find P(score at least 3). [2 marks]

R6. Higher: P(A ∩ B)=0.18, P(A)=0.45, P(B)=0.3. Find P(A|B) and P(B|A). [3 marks]

R7. Higher: five independent trials each succeed with probability 0.2. Find P(at least one success). [3 marks]

R8. Higher: 40% of items come from A with red probability 0.5; 60% from B with red probability 0.2. Given red, find P(A). [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Independent events and the multiplication rule

1.1 Independent events have probabilities 0.2 and 0.6. Find both.

0.12.

1.2 Two independent fair coins are tossed. Find P(two tails).

1/4.

1.3 On a single die roll, are score 1 and score 6 independent?

No. They are mutually exclusive with positive probabilities; knowing score 1 rules out score 6.

1.4 If P(A)=1/3 and P(B|A)=3/5, find P(A ∩ B).

(1/3)(3/5)=1/5.

Lesson 2 • Probability trees with replacement

2.1 With the same 3-red, 2-blue replacement model, find P(BB).

4/25.

2.2 Find P(same colour) in that model.

9/25+4/25=13/25.

2.3 Find P(at least one red) in that model.

1−P(BB)=1−4/25=21/25.

2.4 A tree node has one branch 0.37 and one other exhaustive branch. Find the other.

0.63.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Without replacement and changing branches — Higher

3.1 A bag has 4 red, 3 blue. Draw two without replacement. Find P(RR).

(4/7)(3/6)=2/7.

3.2 For that bag find P(BB).

(3/7)(2/6)=1/7.

3.3 For that bag find P(mixed colours).

1−2/7−1/7=4/7, or add RB and BR.

3.4 A bag has 5 green, 1 yellow. After yellow is drawn without replacement, find P(green next).

1, because all five remaining counters are green.

Lesson 4 • Conditioning on information — Higher

4.1 P(A ∩ B)=0.12, P(B)=0.3. Find P(A|B).

0.12/0.3=0.4.

4.2 A fair die is known to exceed 2. Find P(even).

Restricted {3, 4, 5, 6}; two even out of four, so 1/2.

4.3 Of 30 cyclists, 12 are Year 10. Of all 20 Year 10 pupils, 12 cycle. Find both conditional probabilities.

P(Year 10|cyclist)=12/30=2/5; P(cyclist|Year 10)=12/20=3/5.

4.4 Can the usual formula P(A|B)=P(A ∩ B)/P(B) be used when P(B)=0?

No. Its denominator is zero; the formula is undefined.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Combining routes, reversing conditions and repeated trials — Higher

5.1 Independent trials have p=0.3. Find P(at least one success in 2 trials).

$1-0.7^2=0.51$.

5.2 Three independent fair coins are tossed. Find P(exactly one head).

HTT, THT, TTH: 3 routes each 1/8, so 3/8.

5.3 Group A supplies 25% of items with red probability 0.6; B supplies 75% with red probability 0.2. Find P(red).

$0.25\times0.6+0.75\times0.2=0.3$.

5.4 In that model, find P(A|red).

A-and-red=0.15; total red=0.3. Ratio 0.15/0.3=1/2.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Independent events have P(A)=0.25, P(B)=0.4. Find both and neither. [3 marks]

Both 0.1; neither 0.75×0.6=0.45. [1 for product rule; 1 for both; 1 for neither.]

R2. A bag has 4 red, 1 blue. Draw twice with replacement and remixing. Find P(exactly one blue). [3 marks]

RB and BR each 4/25, total 8/25. [1 for route probabilities; 1 for both routes; 1 for sum.]

R3. Higher: from 4 red, 1 blue draw twice without replacement. Find P(two red). [2 marks]

(4/5)(3/4)=3/5. [1 for updated branch; 1 for product.]

R4. Higher: from 5 red, 3 blue draw twice without replacement. Find P(mixed colours). [4 marks]

RB=(5/8)(3/7)=15/56; BR=(3/8)(5/7)=15/56. Sum 15/28. [1 for each route; 1 for addition; 1 for simplification.]

R5. Higher: a fair die is known to be odd. Find P(score at least 3). [2 marks]

Restricted {1, 3, 5}; favourable {3, 5}, so 2/3. [1 for restricted space; 1 for result.]

R6. Higher: P(A ∩ B)=0.18, P(A)=0.45, P(B)=0.3. Find P(A|B) and P(B|A). [3 marks]

0.18/0.3=0.6; 0.18/0.45=0.4. [1 for correct denominators; 1 for each value.]

R7. Higher: five independent trials each succeed with probability 0.2. Find P(at least one success). [3 marks]

$1-0.8^5=1-0.32768=0.67232$. [1 for no-success complement; 1 for power; 1 for result.]

R8. Higher: 40% of items come from A with red probability 0.5; 60% from B with red probability 0.2. Given red, find P(A). [4 marks]

A-and-red=0.2; B-and-red=0.12; total red=0.32. P(A|red)=0.2/0.32=5/8. [1 for each route; 1 for total; 1 for conditional ratio.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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