GCSE ONLINE TUTORING

PROBABILITY / TOPIC GUIDE 55

55

Sets, Venn diagrams and probability tables

Shared core with Higher extensions

Organise overlapping information using sets, Venn diagrams and two-way tables. Learn the notation, fill each disjoint region correctly and translate counts into probabilities. Higher work includes three sets and probabilities within a restricted group.

Union: A ∪ B • **Intersection: A ∩ B**

Fill the overlap first and identify the correct whole.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Independent educational resource

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Before you begin

Use complements, fractions and totals, and understand a sample space.

Your route through this notebook

1. Set notation and the universal set
2. Filling two-set Venn diagrams
3. Union, exactly one and complements
4. Three-set Venn diagrams — Higher
5. Two-way tables and restricted groups

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Set notation and the universal set

A set is a collection of distinct elements. The universal set U specifies every element under consideration; a complement only has meaning relative to that universe. In a Venn diagram the outer rectangle represents U, and circles represent named sets. Elements outside a circle may still belong to the universe.

A ∩ B is the intersection: elements in both sets. A ∪ B is the union: elements in A or B or both. A′ is the complement: elements in U not in A. The symbol ∅ denotes the empty set, and n(A) counts elements of A. This series uses P(A) for a probability, which is different from n(A). Name the universe before interpreting “neither” or “not”.

A reliable method

1. Identify U and list distinct elements only once.

2. Translate intersection as both and union as inclusive or.

3. Interpret complements relative to U.

Worked example 1

Let U = {1, 2, 3, 4, 5, 6}, A = {2, 4, 6}, B = {2, 3, 5}. Find A ∩ B and A ∪ B.

STEP-BY-STEP SOLUTION

Intersection {2}. Union {2, 3, 4, 5, 6}; the shared 2 appears only once.

Worked example 2

For the same universe, find A′ and n(B).

STEP-BY-STEP SOLUTION

A′ = {1, 3, 5}. There are three elements in B, so n(B) = 3.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A ∪ B does not normally mean exactly one of the sets. Exactly one excludes the overlap; inclusive union includes it.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 U = {1, 2, 3, 4, 5}, A = {1, 3, 5}. Find A′.

1.2 A = {a, b, c}, B = {b, c, d}. Find A ∩ B.

1.3 Find A ∪ B for those sets.

1.4 What does A ∩ B = ∅ mean?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Filling two-set Venn diagrams

A two-set Venn diagram divides the universe into four disjoint regions: A only, both, B only and neither. A total such as n(A) includes the overlap. Put n(A ∩ B) in first, then subtract it from each set total to find the only regions. Finally subtract the union from the universal total to find neither.

Check that all regions are non-negative and sum to n(U). Negative counts or a union exceeding the universe reveal inconsistent data or double counting. If individual objects are selected uniformly, their region counts can be converted into probabilities by dividing by the relevant selection total. Different circle sizes on a sketch do not encode counts unless an area-proportional diagram is explicitly intended.

A reliable method

1. Fill both before either only region.

2. Subtract the overlap from each named set total.

3. Find neither using the universe minus all inside regions.

Worked example 1

Of 40 students, 22 take A, 18 take B and 7 take both. Fill the four regions.

STEP-BY-STEP SOLUTION

Both 7; A only 22 − 7 = 15; B only 18 − 7 = 11. Union 33; neither 40 − 33 = 7.

Worked example 2

In that group, find probability of neither for a uniform selection.

STEP-BY-STEP SOLUTION

There are 7 students outside both sets among 40: probability 7/40.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Writing 22 in A only would treat all A students as excluding B and then add the overlap again. Total set membership and an only-region count are different quantities.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 A total 30 has n(A)=16, n(B)=12, both=5. Find A only.

2.2 For that group find B only and union.

2.3 Find neither in that group.

2.4 Could a 20-person group have n(A)=15, n(B)=14 and overlap 2? Explain.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Union, exactly one and complements

Adding n(A) and n(B) counts the intersection twice. Therefore n(A ∪ B) = n(A) + n(B) − n(A ∩ B). The probability version has the same form: P(A ∪ B) = P(A) + P(B) − P(A ∩ B). This general addition rule also works when the overlap is zero.

Exactly one means A only or B only, so subtract the overlap twice from n(A)+n(B). Neither is the complement of the union. Not both is the complement of the intersection and includes the two only regions as well as neither. Translating the words before calculating is more reliable than searching for a keyword such as “not”.

A reliable method

1. Translate the phrase into disjoint regions or set notation.

2. Avoid double counting the overlap.

3. Use a complement when it describes the desired region more simply.

Worked example 1

P(A)=0.5, P(B)=0.4, P(A ∩ B)=0.15. Find union and neither.

STEP-BY-STEP SOLUTION

Union = 0.5 + 0.4 − 0.15 = 0.75. Neither = 1 − 0.75 = 0.25.

Worked example 2

Using counts A only 15, both 7, B only 11, neither 7, find exactly one and not both.

STEP-BY-STEP SOLUTION

Exactly one = 15 + 11 = 26. Not both = 40 − 7 = 33. These are different events.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“Neither A nor B” is not the same as “not both”. A person in A only fails the both-condition but does not belong to neither.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 P(A)=0.6, P(B)=0.3, both=0.1. Find union.

3.2 For those probabilities find exactly one.

3.3 For those probabilities find neither and not both.

3.4 If A and B are disjoint, simplify P(A ∪ B).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Three-set Venn diagrams — Higher

Three sets create eight disjoint regions, including the outside region. Start with the triple intersection, then the pairwise-only overlaps, then the single-set-only regions. A statement such as n(A ∩ B)=8 normally includes those who also belong to C. Subtract the triple count before placing the AB-only region.

Each full set total must equal the sum of its four internal regions. When using inclusion-exclusion, add the three set totals, subtract the three pairwise intersection totals and add the triple back once. The diagram method is often clearer than a formula because it shows which regions each statement includes. Use non-negative counts and an overall total check to detect errors.

A reliable method

1. Place the triple intersection first.

2. Subtract it from each inclusive pairwise total.

3. Use set totals for single-only regions, then find outside.

Worked example 1

In a group of 50, n(A)=24, n(B)=20, n(C)=18; pairwise intersections AB=9, AC=7, BC=6 and triple=3. Find all regions.

STEP-BY-STEP SOLUTION

Triple 3. AB only 6, AC only 4, BC only 3. A only 24 − 6 − 4 − 3 = 11; B only 8; C only 8. Union 43, neither 7.

Worked example 2

For that diagram find exactly two sets.

STEP-BY-STEP SOLUTION

Use pairwise-only regions: 6 + 4 + 3 = 13. The triple region is excluded.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A pairwise intersection is not automatically a pairwise-only region. The word “only” changes whether the triple overlap is included.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Triple = 4 and n(A ∩ B)=11. Find AB only.

4.2 Three single-only regions contain 5, 6, 7; three pair-only regions 2, 3, 4; triple 1. Find union.

4.3 If the universe for that diagram has 35 elements, find outside.

4.4 For that diagram find number in at least two sets.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Two-way tables and restricted groups

A two-way table classifies each member by one row category and one column category. Interior cells count joint categories. Row totals, column totals and the grand total provide checks and missing values. Each member belongs in exactly one cell when the chosen categories are disjoint and exhaustive.

For a uniform selection from the entire group, divide the relevant cell or marginal total by the grand total. If the question restricts the selection to one row, the denominator is that row total instead. “Among bus travellers” is not a request for a probability over every person. A Venn diagram and a table can encode the same information; translating between them helps confirm which total is relevant.

A reliable method

1. Complete interior cells and marginal totals.

2. Underline whether selection is from everyone or a restricted group.

3. Use the total of that selection group as denominator.

Worked example 1

A table has boys: bus 12, walk 8; girls: bus 18, walk 12. Find totals and P(bus) for a uniform pupil.

STEP-BY-STEP SOLUTION

Boys 20, girls 30, bus 30, walk 20, grand total 50. P(bus)=30/50=3/5.

Worked example 2

In that table, a bus traveller is selected uniformly. Find probability they are a girl.

STEP-BY-STEP SOLUTION

Restrict to the 30 bus travellers. Girls within that group = 18, so probability 18/30 = 3/5. Dividing by 50 would answer a different question.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A conditional denominator is a group size, not a notation trick. Re-read who can actually be selected before dividing.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 A row contains 9 cyclists and 15 non-cyclists. Find row total.

5.2 In a group of 60, 24 are cyclists. Find P(cyclist) under uniform selection.

5.3 Of those 24 cyclists, 9 are in Year 10. Find P(Year 10 given cyclist).

5.4 A table has row totals 22 and 28 and first-column total 19. Find grand total and second-column total.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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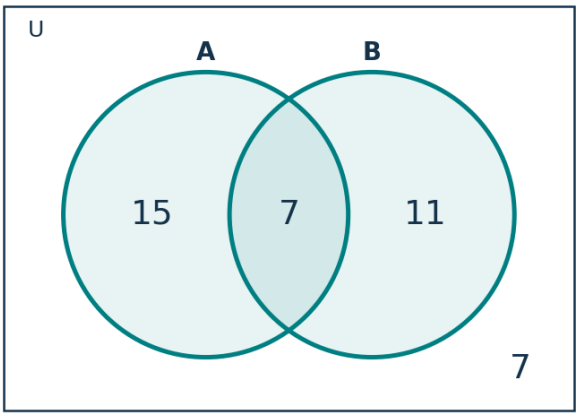
VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 1

Venn regions are disjoint; set totals include overlaps



Universe: 40

n(A) = 15 + 7 = 22

n(B) = 11 + 7 = 18

Union: 15 + 7 + 11 = 33

Neither: 7

Exactly one: 15 + 11 = 26

WHAT THE REPRESENTATION SHOWS

Fill the overlap 7 first. Then A only is 22−7=15 and B only is 18−7=11. The four disjoint regions sum to 40. The same displayed 7 appears in the overlap and the outside by coincidence; they represent different groups. Not both contains 15+11+7=33, whereas neither contains only the outside 7.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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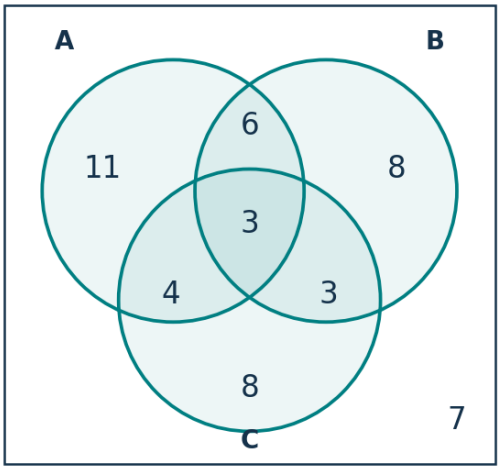
VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 2

Identify inclusive intersections and the selection denominator

Select from bus group: denominator 30

50 people; AB includes triple 3



	Bus	Walk	Total
Boys	12	8	20
Girls	18	12	30
Total	30	20	50

WHAT THE REPRESENTATION SHOWS

Left: triple 3 is included in every pairwise intersection total. AB=6+3=9, AC=4+3=7, BC=3+3=6; the seven inside regions total 43, leaving 7 outside. Right: a uniform pupil selection uses total 50. Given bus, use only the highlighted 30 people; 18/30 are girls. The two panels are different datasets.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. U={1, 2, 3, 4, 5, 6, 7, 8}, A={2, 4, 6, 8}, B={1, 2, 3, 4}. Find intersection and A′. [2 marks]

R2. A group of 60 has A=32, B=25, both=12. Find A only, B only and neither. [3 marks]

R3. For that group find P(exactly one) under uniform selection. [2 marks]

R4. P(A)=0.55, P(B)=0.4, P(both)=0.2. Find P(neither). [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Higher: triple=2, pairwise totals AB=7, AC=6, BC=5. Find exactly two-set count. [3 marks]

R6. Higher: three set totals are 20, 18, 16; pairwise totals 8, 7, 6; triple 3. Find union. [3 marks]

R7. A table has row 1: bus 14, walk 6; row 2: bus 16, walk 14. Find P(bus) for a uniformly selected member. [2 marks]

R8. From the bus travellers in that table, one is selected uniformly. Find probability of row 1. [2 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Set notation and the universal set

1.1 U = {1, 2, 3, 4, 5}, A = {1, 3, 5}. Find A′.

{2, 4}.

1.2 A = {a, b, c}, B = {b, c, d}. Find A ∩ B.

{b, c}.

1.3 Find A ∪ B for those sets.

{a, b, c, d}.

1.4 What does A ∩ B = ∅ mean?

The sets have no shared elements; corresponding events are mutually exclusive.

Lesson 2 • Filling two-set Venn diagrams

2.1 A total 30 has n(A)=16, n(B)=12, both=5. Find A only.

16 − 5 = 11.

2.2 For that group find B only and union.

B only 7; union 11 + 5 + 7 = 23.

2.3 Find neither in that group.

30 − 23 = 7.

2.4 Could a 20-person group have n(A)=15, n(B)=14 and overlap 2? Explain.

No. Union 15 + 14 − 2 = 27 exceeds 20.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Union, exactly one and complements

3.1 P(A)=0.6, P(B)=0.3, both=0.1. Find union.

0.8.

3.2 For those probabilities find exactly one.

0.6 + 0.3 − 2(0.1) = 0.7.

3.3 For those probabilities find neither and not both.

Neither 0.2; not both 0.9.

3.4 If A and B are disjoint, simplify P(A ∪ B).

P(A) + P(B), because intersection probability is zero.

Lesson 4 • Three-set Venn diagrams — Higher

4.1 Triple = 4 and n(A ∩ B)=11. Find AB only.

11 − 4 = 7.

4.2 Three single-only regions contain 5, 6, 7; three pair-only regions 2, 3, 4; triple 1. Find union.

5 + 6 + 7 + 2 + 3 + 4 + 1 = 28.

4.3 If the universe for that diagram has 35 elements, find outside.

35 − 28 = 7.

4.4 For that diagram find number in at least two sets.

2 + 3 + 4 + 1 = 10.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Two-way tables and restricted groups

5.1 A row contains 9 cyclists and 15 non-cyclists. Find row total.

24.

5.2 In a group of 60, 24 are cyclists. Find P(cyclist) under uniform selection.

24/60 = 2/5.

5.3 Of those 24 cyclists, 9 are in Year 10. Find P(Year 10 given cyclist).

9/24 = 3/8.

5.4 A table has row totals 22 and 28 and first-column total 19. Find grand total and second-column total.

Grand total 50; second column 31.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. U={1, 2, 3, 4, 5, 6, 7, 8}, A={2, 4, 6, 8}, B={1, 2, 3, 4}. Find intersection and A′. [2 marks]

A ∩ B={2, 4}; A′={1, 3, 5, 7}. [1 each.]

R2. A group of 60 has A=32, B=25, both=12. Find A only, B only and neither. [3 marks]

A only 20; B only 13; union 45, neither 15. [1 for each requested region.]

R3. For that group find P(exactly one) under uniform selection. [2 marks]

(20 + 13)/60 = 33/60 = 11/20. [1 for count; 1 for probability.]

R4. P(A)=0.55, P(B)=0.4, P(both)=0.2. Find P(neither). [3 marks]

Union 0.75; neither 0.25. [1 for addition rule; 1 for union; 1 for complement.]

R5. Higher: triple=2, pairwise totals AB=7, AC=6, BC=5. Find exactly two-set count. [3 marks]

Pair-only counts 5, 4, 3; total 12. [1 for subtracting triple; 1 for counts; 1 for sum.]

R6. Higher: three set totals are 20, 18, 16; pairwise totals 8, 7, 6; triple 3. Find union. [3 marks]

20 + 18 + 16 − 8 − 7 − 6 + 3 = 36. [1 for totals; 1 for overlap correction; 1 for result.]

R7. A table has row 1: bus 14, walk 6; row 2: bus 16, walk 14. Find P(bus) for a uniformly selected member. [2 marks]

Bus 30 of grand total 50, so 3/5. [1 for totals; 1 for probability.]

R8. From the bus travellers in that table, one is selected uniformly. Find probability of row 1. [2 marks]

14/30 = 7/15. [1 for restricted denominator; 1 for result.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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