GCSE ONLINE TUTORING

PROBABILITY / TOPIC GUIDE 54

54

Listing outcomes and sample spaces

Shared core with Higher counting extensions

Make chance problems visible by listing possibilities in a systematic order. Build sample-space grids, understand why dice sums are not equally likely and use counting rules with clear restrictions. Separate counting outcomes from assigning their probabilities.

Count elementary outcomes systematically.

Equal-looking categories need not be equally likely.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Multiply whole numbers, simplify fractions and understand equally likely outcomes.

Your route through this notebook

1. Systematic lists and ordered outcomes
2. Sample-space grids for two stages
3. Two fair dice and non-uniform totals
4. Product rule for counting — Higher extension
5. Unordered selections and avoiding false equal likelihood

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Systematic lists and ordered outcomes

A systematic list follows a fixed order so that no possibility is missed or duplicated. For two stages, hold the first result fixed while listing every permitted second result, then move to the next first result. Labels help distinguish objects that look similar.

State whether order matters. Choosing red then blue is a different ordered sequence from blue then red, but both may describe the same final unordered pair. Replacement and repetition also change what is allowed. A list of possibilities is not automatically a list of equally likely outcomes: probabilities depend on the random mechanism. Establish that mechanism before dividing favourable entries by all entries.

A reliable method

1. Define the stages, available choices and any restrictions.

2. Hold one choice fixed while cycling through the next choices.

3. State whether the list is ordered and whether its entries are equally likely.

Worked example 1

List outcomes of a fair coin followed by an independent uniform choice of 1, 2, 3.

STEP-BY-STEP SOLUTION

H1, H2, H3, T1, T2, T3. Each has probability (1/2)(1/3) = 1/6.

Worked example 2

List ordered two-letter strings from A, B, C with no repeated letter.

STEP-BY-STEP SOLUTION

AB, AC, BA, BC, CA, CB. AA, BB, CC are excluded by the restriction.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A compact list should still be auditable. Writing “and so on” is not a complete answer when the task asks for every possible outcome.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 List outcomes of two independent coin tosses in order.

1.2 List two-digit codes from digits 1, 2 with repetition allowed.

1.3 List ordered two-letter strings from A, B with no repetition.

1.4 For a uniform choice of one card labelled 1, 2, 3 followed by a fair coin, how many equally likely outcomes?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Sample-space grids for two stages

A sample-space grid uses rows for one stage and columns for another. Each cell represents one ordered pair, even when the recorded total or product repeats. Fill each cell with the requested derived result while retaining the row and column labels. This shows where the same result can arise in different ways.

For two independent uniform selections, every cell in a full rectangular grid is equally likely. Independence and uniformity both matter. A grid for a biased coin still lists all outcomes, but its cells need weights rather than equal counts. For selection without replacement, some cells may be forbidden or their probabilities may depend on the first choice.

A reliable method

1. Label rows and columns with the elementary outcomes.

2. Fill cells with the required pair, sum or product.

3. Count qualifying cells only when the model makes them equally likely.

Worked example 1

Spinner A has equal sectors 1, 2. Independent spinner B has equal sectors 1, 2, 3. List sum rows and find P(sum = 4).

STEP-BY-STEP SOLUTION

Row A=1: 2, 3, 4. Row A=2: 3, 4, 5. Two of six cells total 4, so probability 1/3.

Worked example 2

Using the same grid, find P(sum greater than 3).

STEP-BY-STEP SOLUTION

Qualifying cells are (1, 3), (2, 2), (2, 3), with sums 4, 4, 5. Probability 3/6 = 1/2.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Do not count the different total names and assume equal probability. In the example, total 3 occurs in two cells while total 2 occurs in only one.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Use A = 1, 2 and B = 1, 2, 3 independently and uniformly. Find P(sum = 2).

2.2 For that grid, find P(sum = 3).

2.3 For that grid, find P(product = 2).

2.4 If A is biased toward 1, are the six cells still equally likely?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Two fair dice and non-uniform totals

Two distinguishable independent fair six-sided dice produce 36 equally likely ordered pairs. A result such as (2, 5) is distinct from (5, 2). Totals from 2 to 12 have different numbers of generating pairs, so the eleven possible sum values are not equally likely.

A diagonal in a sum grid contains pairs with a common total. Total 7 has six pairs, while totals 2 and 12 have one each. Events such as a double, at least one six or a product condition are easier to count when the pair structure is preserved. Check that an outcome satisfying two conditions is not counted twice in an “or” event.

A reliable method

1. Use $6 \times 6 = 36$ ordered equally likely pairs.

2. Count pairs satisfying the actual event, not distinct sum labels.

3. Simplify the fraction and verify any overlap.

Worked example 1

Find probability of total 7.

STEP-BY-STEP SOLUTION

Pairs (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) give $6/36 = 1/6$.

Worked example 2

Find probability of at least one six.

STEP-BY-STEP SOLUTION

Six pairs have first die 6 and six have second die 6, but (6, 6) is counted twice. There are 11 distinct pairs, giving 11/36.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The probability of an even total is not found by counting how many of 2, 3, ..., 12 are even. Count ordered die outcomes or use parity of the two independent dice.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find P(total = 2).

3.2 Find P(total = 6).

3.3 Find P(a double).

3.4 Find P(product = 6).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Product rule for counting — Higher extension

If a first choice has m possibilities and each such choice can be followed by n possibilities, there are mn ordered outcomes. More stages multiply in the same way when the listed count at each stage is valid for every preceding route. This is a counting rule; it does not require random choices or imply equal probabilities.

Restrictions change the count at a stage. Without repetition, later choices have fewer available objects. For a conventional multi-digit number, the first digit cannot be zero; for a code, it may be allowed if stated. If different first choices allow different numbers of later choices, split into cases and add their disjoint counts instead of forcing one product.

A reliable method

1. Define stages and whether zero, repetition and order are allowed.

2. Count valid choices at each stage after earlier restrictions.

3. Multiply within a route pattern and add disjoint cases when necessary.

Worked example 1

There are 3 shirts, 4 trousers and 2 pairs of shoes. Count outfits.

STEP-BY-STEP SOLUTION

$3 \times 4 \times 2 = 24$, assuming any shirt can be paired with any trousers and shoes.

Worked example 2

Count three-digit numbers with no repeated digit.

STEP-BY-STEP SOLUTION

First digit: 9 choices (1–9). Second: 9 remaining digits including zero. Third: 8. Total $9 \times 9 \times 8 = 648$.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Counting a code does not need an independence assumption; it needs a correct description of permitted choices. Assigning probabilities to those codes is a separate step.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Count two-letter strings from A, B, C, D with repetition allowed.

4.2 Count them without repetition.

4.3 Count four-digit codes using 0–9 with repetition allowed and leading zero allowed.

4.4 Count two-digit numbers with no repeated digit.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Unordered selections and avoiding false equal likelihood

An unordered selection records which objects were chosen, not their sequence. Two distinct objects can usually be selected in two orders, so counting ordered pairs and dividing by two can count unordered pairs when every valid pair has exactly two orders. This reasoning must be justified rather than applied blindly when repetitions are allowed.

For four distinct counters selected two without replacement, there are $4 \times 3 = 12$ ordered pairs and 6 unordered pairs. If colours repeat, colour-category names may have different probabilities because they contain different numbers of individual-object outcomes. Label repeated-colour objects separately to reveal the weighting. In a probability calculation, use a representation whose equal likelihood has been established.

A reliable method

1. Decide what the final recorded outcome retains or forgets.

2. Count individual objects before collapsing repeated labels.

3. Check how many elementary routes lead to each recorded category.

Worked example 1

From distinct objects A, B, C, D choose two without order. List all pairs.

STEP-BY-STEP SOLUTION

AB, AC, AD, BC, BD, CD: six pairs. Each has two possible orders in a uniform sequential draw without replacement.

Worked example 2

A bag has R_1, R_2, B. Draw two uniformly without replacement. Find P(same colour).

STEP-BY-STEP SOLUTION

Six ordered object pairs are possible. Same colour occurs as R_1R_2 or R_2R_1, so probability $2/6 = 1/3$. The category names RR and mixed are not equally likely.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“Only two possibilities: same or different” does not give a probability of 1/2 to each. These event categories can contain very different numbers of elementary outcomes.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 List unordered pairs from A, B, C.

5.2 From four distinct objects chosen uniformly two without replacement, find P(pair is A and B).

5.3 A bag has R_1, R_2, B. Find P(mixed colours) in two draws without replacement.

5.4 Why can a grid of sum labels contain repeated numbers?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 1

A grid cell represents an ordered elementary outcome

Independent uniform spinners: sums

	B=1	B=2	B=3
A=1	2	3	4
A=2	3	4	5

Six equally likely pairs

Sum 4: (1,3) or (2,2)

P(sum 4) = 2/6 = 1/3

Different sum labels have different weights.

WHAT THE REPRESENTATION SHOWS

Rows label the first spinner and columns the second. The entries are sums, so a repeated number records a different route rather than a duplicate error. Exactly two of six cells contain 4. Equal cell probabilities require the independent uniform spinner model stated here.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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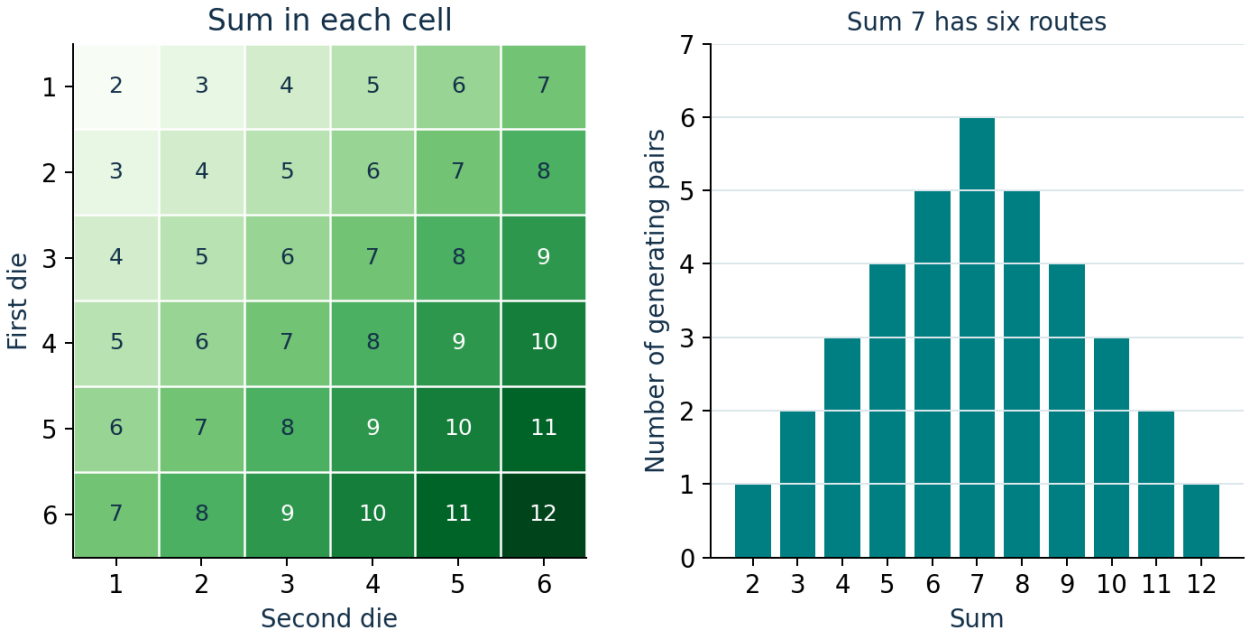
VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 2

Two distinguishable fair dice give 36 ordered pairs



WHAT THE REPRESENTATION SHOWS

The sum grid has 36 equally likely cells under the independent fair-dice model. The bar heights count how many cells produce each total. Total 7 has six routes and probability 1/6; total 2 has one route and probability 1/36. Counting eleven sum names as equally likely would be incorrect.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. List all ordered results of a fair coin and independent equal-sector spinner labelled 1, 2. [2 marks]

R2. Independent uniform spinners have labels 1, 2 and 1, 2, 3. Find P(sum = 5). [2 marks]

R3. Two independent fair dice are rolled. Find P(total = 8). [3 marks]

R4. For those dice, find P(total at least 11). [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. There are 5 starters, 3 mains and 4 desserts. Count complete meals if all combinations are permitted. [2 marks]

R6. Higher: count three-letter codes from five distinct letters with no repetition. [2 marks]

R7. Higher: count three-digit numbers with no repeated digit and first digit fixed as 4. [3 marks]

R8. Four distinct counters A, B, C, D are drawn uniformly two without replacement. Find P(the unordered pair contains A). [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Systematic lists and ordered outcomes

1.1 List outcomes of two independent coin tosses in order.

HH, HT, TH, TT.

1.2 List two-digit codes from digits 1, 2 with repetition allowed.

11, 12, 21, 22.

1.3 List ordered two-letter strings from A, B with no repetition.

AB, BA.

1.4 For a uniform choice of one card labelled 1, 2, 3 followed by a fair coin, how many equally likely outcomes?

3 × 2 = 6: 1H, 1T, 2H, 2T, 3H, 3T.

Lesson 2 • Sample-space grids for two stages

2.1 Use A = 1, 2 and B = 1, 2, 3 independently and uniformly. Find P(sum = 2).

Only (1, 1): 1/6.

2.2 For that grid, find P(sum = 3).

(1, 2), (2, 1): 2/6 = 1/3.

2.3 For that grid, find P(product = 2).

(1, 2), (2, 1): 1/3.

2.4 If A is biased toward 1, are the six cells still equally likely?

No. Their probabilities reflect the unequal first-stage probabilities, even if B remains independent and uniform.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Two fair dice and non-uniform totals

3.1 Find P(total = 2).

Only (1, 1): 1/36.

3.2 Find P(total = 6).

Five pairs (1, 5), (2, 4), (3, 3), (4, 2), (5, 1): 5/36.

3.3 Find P(a double).

Six pairs (1, 1) through (6, 6): 1/6.

3.4 Find P(product = 6).

(1, 6), (2, 3), (3, 2), (6, 1): 4/36 = 1/9.

Lesson 4 • Product rule for counting — Higher extension

4.1 Count two-letter strings from A, B, C, D with repetition allowed.

4 × 4 = 16.

4.2 Count them without repetition.

4 × 3 = 12.

4.3 Count four-digit codes using 0–9 with repetition allowed and leading zero allowed.

10^4 = 10,000.

4.4 Count two-digit numbers with no repeated digit.

9 × 9 = 81.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Unordered selections and avoiding false equal likelihood

5.1 List unordered pairs from A, B, C.

AB, AC, BC.

5.2 From four distinct objects chosen uniformly two without replacement, find P(pair is A and B).

Two of 12 orders, or one of 6 unordered pairs: 1/6.

5.3 A bag has R_1, R_2, B. Find P(mixed colours) in two draws without replacement.

Complement of same colour: $1 - 1/3 = 2/3$.

5.4 Why can a grid of sum labels contain repeated numbers?

Different ordered elementary outcomes can produce the same sum; each cell represents its own route.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. List all ordered results of a fair coin and independent equal-sector spinner labelled 1, 2. [2 marks]

H1, H2, T1, T2. [1 for systematic method; 1 for complete list.]

R2. Independent uniform spinners have labels 1, 2 and 1, 2, 3. Find P(sum = 5). [2 marks]

Only (2, 3) qualifies among six pairs: 1/6. [1 for sample count; 1 for result.]

R3. Two independent fair dice are rolled. Find P(total = 8). [3 marks]

Five pairs (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), giving 5/36. [1 for denominator; 1 for favourable pairs; 1 for probability.]

R4. For those dice, find P(total at least 11). [3 marks]

(5, 6), (6, 5), (6, 6): 3/36 = 1/12. [1 for denominator; 1 for count; 1 for simplification.]

R5. There are 5 starters, 3 mains and 4 desserts. Count complete meals if all combinations are permitted. [2 marks]

5 × 3 × 4 = 60. [1 for product rule; 1 for result.]

R6. Higher: count three-letter codes from five distinct letters with no repetition. [2 marks]

5 × 4 × 3 = 60. [1 for decreasing choices; 1 for result.]

R7. Higher: count three-digit numbers with no repeated digit and first digit fixed as 4. [3 marks]

First fixed; second 9 choices including zero but excluding 4; third 8. Total 72. [1 for first restriction; 1 for remaining counts; 1 for result.]

R8. Four distinct counters A, B, C, D are drawn uniformly two without replacement. Find P(the unordered pair contains A). [3 marks]

Favourable unordered pairs AB, AC, AD among six: 3/6 = 1/2. [1 for six pairs; 1 for three favourable; 1 for result.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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