GCSE ONLINE TUTORING

PROBABILITY / TOPIC GUIDE 53

53

Probability foundations

Shared GCSE foundations and core

Probability measures uncertainty, not a promise about the next result. Build the language of chance, calculate from equally likely outcomes and use experiments to estimate probabilities. Connect complements, expected frequencies and fair models while checking the assumptions behind every calculation.

$$0 \leq P(A) \leq 1 \quad \bullet \quad P(A') = 1 - P(A)$$

A probability is a model of uncertainty,not a guarantee.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Simplify fractions, convert decimals and percentages, and read simple frequency tables.

Your route through this notebook

1. Events, outcomes and the probability scale
2. Equally likely outcomes and weighted categories
3. Complements, exhaustive events and mutually exclusive outcomes
4. Experimental probability and relative frequency
5. Expected frequency, fairness and simulation

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Events, outcomes and the probability scale

An outcome is one possible result of a random process. An event is a set of outcomes, such as rolling an even number on a die. The sample space contains every possible outcome under the model. State the process carefully: rolling a standard die has sample space {1, 2, 3, 4, 5, 6}, while recording only whether its score is even has two recorded categories. Those categories need not be equally likely in every other experiment.

Probability lies between 0 and 1 inclusive. Impossible has probability 0, certain has probability 1, and an even chance has probability 1/2. A probability may be written as a fraction, decimal or percentage, but it has no measurement unit. A probability of 0.8 means a high chance under the model; it does not guarantee success on any particular trial.

A reliable method

1. Define the random process and the event being asked about.

2. Check that the proposed probability lies in [0, 1].

3. Use equivalent fraction, decimal or percentage forms clearly.

Worked example 1

Classify probabilities 0, 0.5, 0.9 and 1.

STEP-BY-STEP SOLUTION

0 is impossible; 0.5 is an even chance; 0.9 is likely; 1 is certain under the model.

Worked example 2

Write 3/8 as a decimal and percentage probability.

STEP-BY-STEP SOLUTION

3/8 = 0.375 = 37.5%. These are three descriptions of the same chance.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“Random” means the outcome is governed by a chance process. It does not mean that every named category is equally likely. A bag with 9 red and 1 blue counter is random to draw from but favours red.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Is 1.2 a valid probability? Explain.

1.2 Write 0.24 as a fraction in simplest form and a percentage.

1.3 For a standard die, list the event score greater than 4.

1.4 Explain why a 75% chance is not a guarantee.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Equally likely outcomes and weighted categories

When elementary outcomes are equally likely, probability = number of favourable outcomes divided by total outcomes. Uniform selection from individual objects makes each object equally likely, not necessarily each colour equally likely. Count the objects or outcomes at the level where equal likelihood is justified.

A fair standard die has six equally likely faces. A fair spinner with equal-angle sectors has equally likely sectors; repeated labels make some recorded scores more likely than others. If sectors have different angles, their probabilities are proportional to angle under an ideal uniform-direction spinner model. Counting colour names alone is then invalid.

A reliable method

1. Identify what is actually equally likely: objects, faces, sectors or directions.

2. Count favourable and total equally likely outcomes.

3. Simplify and check the answer is between 0 and 1.

Worked example 1

A bag contains 5 red, 3 blue and 2 green counters. One counter is selected uniformly. Find P(blue).

STEP-BY-STEP SOLUTION

There are 10 equally likely individual counters; 3 are blue. P(blue) = 3/10.

Worked example 2

A fair equal-sector spinner is labelled 1, 1, 2, 3, 3, 3. Find P(3).

STEP-BY-STEP SOLUTION

Three of six equally likely sectors show 3, so P(3) = 3/6 = 1/2. The three different score names are not equally likely.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The formula favourable/total requires equally likely elementary outcomes. If outcomes carry different probabilities, add the relevant probabilities instead.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find P(even) for a fair standard die.

2.2 A bag has 4 black and 9 white counters. Find P(black) for a uniform draw.

2.3 An ideal spinner has a red sector of 90° out of a full turn. Find P(red).

2.4 A uniform spinner has equal sectors labelled A, A, B, C. Find P(A or B).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Complements, exhaustive events and mutually exclusive outcomes

The complement of event A is the event that A does not happen. Together they exhaust the sample space and cannot occur together, so P(not A) = 1 − P(A). In this series A′ denotes the complement. Make sure “not A” includes every other permitted category, not just one selected alternative.

Mutually exclusive events cannot happen in the same trial. Add their probabilities for an either/or event. A complete list of disjoint, exhaustive outcomes has probabilities summing to 1. Events can be exhaustive without being disjoint: on a die, even and at most 5 cover everything but overlap. Adding overlapping event probabilities directly counts their intersection twice; later guides show how to correct that.

A reliable method

1. Identify all outcomes in the complement or union.

2. For disjoint alternatives, add their probabilities.

3. For a complete disjoint list, use a total of 1 to find any missing value.

Worked example 1

P(rain tomorrow) = 0.35 in a stated model. Find P(no rain tomorrow).

STEP-BY-STEP SOLUTION

The complement is 1 − 0.35 = 0.65.

Worked example 2

A bag has red, blue, green probabilities 0.2, 0.5, p. Find p and P(not blue).

STEP-BY-STEP SOLUTION

The three colour outcomes are disjoint and exhaustive, so p = 1 − 0.7 = 0.3. P(not blue) = 1 − 0.5 = 0.5.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For one die roll, “even” and “prime” are not mutually exclusive because score 2 belongs to both. The word “or” in probability normally includes outcomes satisfying both conditions.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 P(A) = 7/20. Find P(A′).

3.2 Disjoint outcomes have probabilities 0.18, 0.27 and p, and exhaust all possibilities. Find p.

3.3 One die is rolled. Are score 1 and score 6 mutually exclusive?

3.4 A fair die is rolled. Find P(score 1 or score 6).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Experimental probability and relative frequency

An experiment can estimate a probability when the theoretical model is unknown or being tested. Relative frequency = number of occurrences divided by number of trials. It is an estimate, not automatically the exact probability of the underlying process. Record the trial count so that readers can assess the amount of evidence.

For independent trials with a stable underlying probability, larger samples generally give more reliable estimates, though an estimate need not improve after every extra trial. Relative frequencies fluctuate. Past results do not change the next-trial probability for an independent process. Combining results from different experiments is justified only when they concern the same event under comparable conditions; combine counts before dividing.

A reliable method

1. Count successes and all relevant trials.
2. Divide successes by trials and label the result an estimate.
3. Check whether combining or predicting uses a stable, comparable process.

Worked example 1

A spinner lands red 47 times in 200 spins. Estimate P(red).

STEP-BY-STEP SOLUTION

Relative frequency = 47/200 = 0.235.

Worked example 2

In comparable tests the same event occurs 18 times in 60 trials and 56 times in 140 trials. Estimate overall probability.

STEP-BY-STEP SOLUTION

Combine counts: (18 + 56)/(60 + 140) = 74/200 = 0.37. An unweighted mean of separate estimates is not the general method.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Five heads in a row do not make tails “due” on an independent fair coin. The next toss still has probability 1/2 of tails; the earlier sequence is already known.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 An event occurs 36 times in 120 trials. Find relative frequency.

4.2 The same stable process has 12 successes in 40 trials and 28 in 60. Find combined estimate.

4.3 Why is 600 comparable trials usually more informative than 6?

4.4 A fair coin has just shown tails four times. Find P(head on next independent toss).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Expected frequency, fairness and simulation

Expected frequency = number of trials × event probability. This is a long-run average prediction, not a guaranteed count in the next set of trials. It can be fractional even when actual counts must be whole numbers. If a question asks for an estimated whole count, round only as instructed and retain the distinction between expectation and observation.

A fair model should be assessed against how the mechanism works and the evidence. An unequal short-run result does not by itself prove bias. A simulation should represent probabilities correctly, for example using equally likely digits and allocating a suitable number to each event. Use independent random selections if the original trials are independent. If using experimental probability to predict a future count, state the assumption that conditions remain comparable.

A reliable method

1. Find a justified theoretical or estimated probability.

2. Multiply by the number of trials for an expected count.

3. Explain why the observed count can differ and how the model is simulated.

Worked example 1

An event has probability 0.3. Find its expected frequency in 250 independent trials.

STEP-BY-STEP SOLUTION

250 × 0.3 = 75. This is an expectation, not a promise of exactly 75.

Worked example 2

Simulate an event with probability 0.3 using uniformly random digits 0–9.

STEP-BY-STEP SOLUTION

Allocate three digits, say 0, 1, 2, to success and the other seven to failure. Draw digits independently for independent trials.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A probability of 1/6 does not guarantee exactly one success in every six trials. An expected frequency describes an average across repeated sets, while a particular set can vary.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 A fair die is rolled 180 times. Find expected number of sixes.

5.2 P(A) = 2/7. Find expected frequency in 50 trials exactly.

5.3 A game succeeds in 24 of 80 trials. Predict successes in 500 comparable trials.

5.4 Describe a random-digit simulation for probability 0.6.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 1

Chance depends on the model and its elementary outcomes



Six equal-angle sectors: P(score 3) = 3/6



WHAT THE REPRESENTATION SHOWS

Every probability lies from 0 to 1. The spinner has six equally likely sectors but only three score names. Score 3 occupies three sectors, score 1 two and score 2 one. Therefore the score probabilities are 1/2, 1/3, 1/6 rather than one third each. Uniform directions and a fair ideal spinner are assumed.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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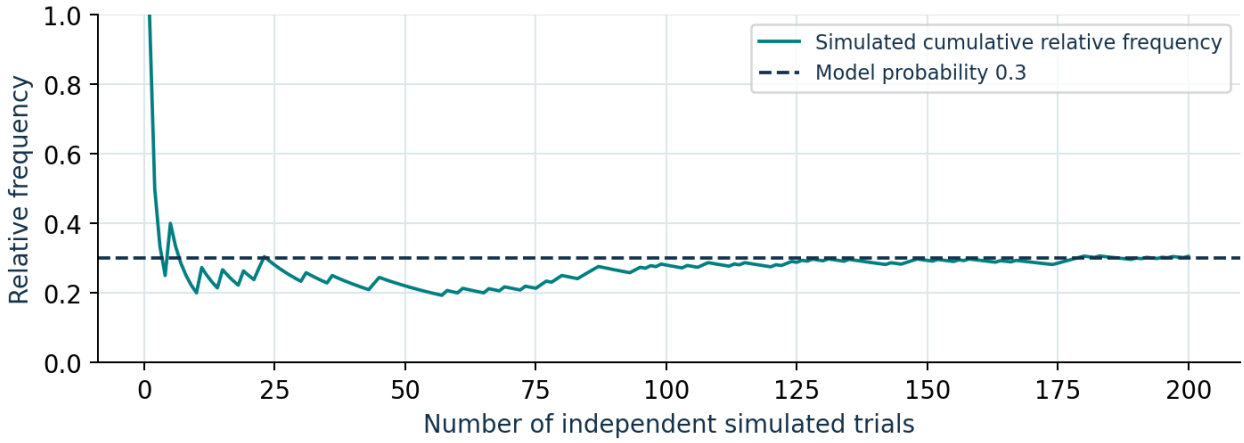
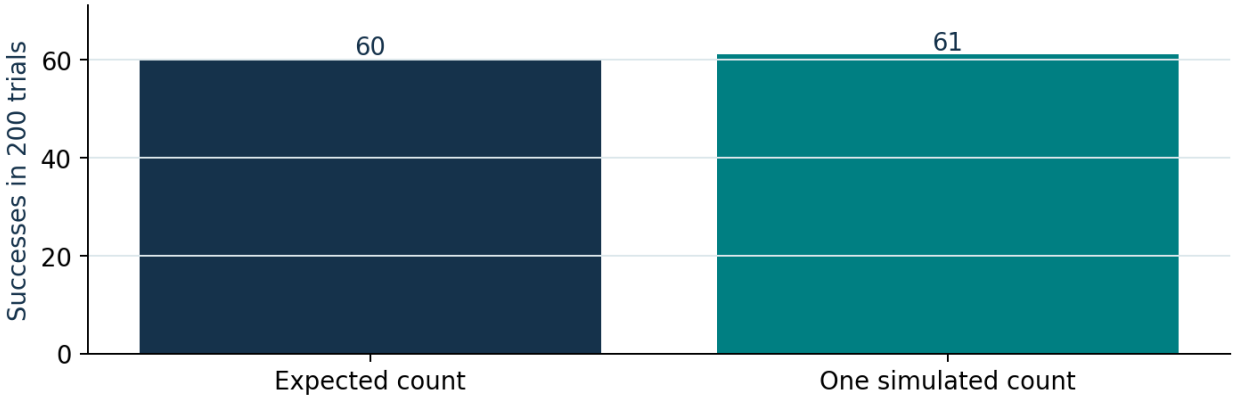
VISUAL EXPLAINER / MAKE THE SAMPLE SPACE VISIBLE

Read the model • 2

Experimental estimates fluctuate; expectation is not a guarantee





WHAT THE REPRESENTATION SHOWS

This reproducible simulation uses 200 independent trials with success probability 0.3. It produces 61 successes, a final relative frequency 61/200 = 0.305, while the expected frequency is 200×0.3 = 60. Fluctuation does not prove a changing probability; one simulated sequence is an illustration, not evidence about a real device.

Discuss: what makes the elementary outcomes equally likely,or why do they need different weights? Identify the event,any given condition and the correct denominator. Recalculate one displayed probability without looking at the explanation.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Write 0.375 as a fraction and percentage probability. [2 marks]

R2. A uniform draw is made from 6 red, 4 blue, 5 green counters. Find P(blue). [2 marks]

R3. P(A) = 0.28. Find P(A′). [1 mark]

R4. Disjoint exhaustive outcomes have probabilities 0.32, 0.41 and p. Find p. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. An event occurs 54 times in 150 trials. Estimate its probability. [2 marks]

R6. Using that estimate, predict successes in 400 comparable trials. [2 marks]

R7. An ideal spinner has red sector angle 144°. Find red probability and expected reds in 250 spins. [3 marks]

R8. A fair coin gives 8 heads in 10 tosses. Explain why this does not prove bias or make tails more likely next. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Events, outcomes and the probability scale

1.1 Is 1.2 a valid probability? Explain.

No. A probability cannot exceed 1.

1.2 Write 0.24 as a fraction in simplest form and a percentage.

24/100 = 6/25; 24%.

1.3 For a standard die, list the event score greater than 4.

{5, 6}.

1.4 Explain why a 75% chance is not a guarantee.

Some permitted outcomes are failures; the model allows failure on the next trial despite success being more likely.

Lesson 2 • Equally likely outcomes and weighted categories

2.1 Find P(even) for a fair standard die.

3/6 = 1/2.

2.2 A bag has 4 black and 9 white counters. Find P(black) for a uniform draw.

4/13.

2.3 An ideal spinner has a red sector of 90° out of a full turn. Find P(red).

90/360 = 1/4.

2.4 A uniform spinner has equal sectors labelled A, A, B, C. Find P(A or B).

Three of four sectors qualify: 3/4.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Complements, exhaustive events and mutually exclusive outcomes

3.1 P(A) = 7/20. Find P(A′).

13/20.

3.2 Disjoint outcomes have probabilities 0.18, 0.27 and p, and exhaust all possibilities. Find p.

1 − 0.18 − 0.27 = 0.55.

3.3 One die is rolled. Are score 1 and score 6 mutually exclusive?

Yes. A single roll cannot show both scores.

3.4 A fair die is rolled. Find P(score 1 or score 6).

1/6 + 1/6 = 1/3, since the events are disjoint.

Lesson 4 • Experimental probability and relative frequency

4.1 An event occurs 36 times in 120 trials. Find relative frequency.

0.3, or 3/10.

4.2 The same stable process has 12 successes in 40 trials and 28 in 60. Find combined estimate.

40/100 = 0.4.

4.3 Why is 600 comparable trials usually more informative than 6?

Sampling variation generally has less impact on the estimated proportion when more independent trials are observed.

4.4 A fair coin has just shown tails four times. Find P(head on next independent toss).

1/2.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Expected frequency, fairness and simulation

5.1 A fair die is rolled 180 times. Find expected number of sixes.

180/6 = 30.

5.2 P(A) = 2/7. Find expected frequency in 50 trials exactly.

100/7 ≈ 14.2857. The expectation may be non-integer.

5.3 A game succeeds in 24 of 80 trials. Predict successes in 500 comparable trials.

Estimated p = 0.3; expected future successes 150, assuming stable conditions.

5.4 Describe a random-digit simulation for probability 0.6.

Choose six of the ten equally likely digits as success, for example 0–5, and the remaining four as failure.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Write 0.375 as a fraction and percentage probability. [2 marks]

3/8 and 37.5%. [1 each.]

R2. A uniform draw is made from 6 red, 4 blue, 5 green counters. Find P(blue). [2 marks]

4/15. [1 for total 15; 1 for favourable count/fraction.]

R3. P(A) = 0.28. Find P(A′). [1 mark]

0.72. [1 for complement.]

R4. Disjoint exhaustive outcomes have probabilities 0.32, 0.41 and p. Find p. [2 marks]

p = 1 − 0.73 = 0.27. [1 for total rule; 1 for result.]

R5. An event occurs 54 times in 150 trials. Estimate its probability. [2 marks]

54/150 = 0.36. [1 for ratio; 1 for simplification.]

R6. Using that estimate, predict successes in 400 comparable trials. [2 marks]

400 × 0.36 = 144, assuming stable conditions. [1 for calculation; 1 for result/assumption.]

R7. An ideal spinner has red sector angle 144°. Find red probability and expected reds in 250 spins. [3 marks]

144/360 = 0.4; expected 100. [1 for angle fraction; 1 for probability; 1 for expected count.]

R8. A fair coin gives 8 heads in 10 tosses. Explain why this does not prove bias or make tails more likely next. [3 marks]

Short-run outcomes fluctuate even under a fair model. More evidence is needed to assess bias, and independence keeps next-toss P(tails) = 1/2. [1 for variation; 1 for evidence limitation; 1 for independence/probability.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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