GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 10

10

Recurring decimals

Shared notation with Higher algebraic conversions

Some decimal expansions repeat forever. The repeating pattern can be represented exactly, without writing infinitely many digits. This guide develops recurring notation, fraction-to-decimal division and the algebra used to turn recurring decimals back into fractions.

0.(27) = 0.27272727…

The block 27 repeats without ending.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

INDEPENDENT GCSE MATHS • WEBSITE EDITION • NOVEMBER 2026

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Your learning guide

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Before you begin

Understand decimal place value, fraction simplification, long division and solving simple linear equations.

Your route through this notebook

1. Terminating, recurring and irrational decimals
2. From fractions to decimals and recognising termination
3. Recurring decimals to fractions — Higher extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Terminating, recurring and irrational decimals

A terminating decimal ends after a finite number of places. A recurring decimal repeats a digit or a fixed block forever. In this guide, parentheses show the repeating block: 0.(3) means 0.333… and 0.1(6) means 0.1666… . The brackets are notation for repetition, not multiplication. Dots over a digit, or over the first and last digits of a block, are another common convention.

Every fraction of integers with a non-zero denominator has a decimal expansion that terminates or eventually repeats. In long division, only finitely many remainders are possible. A remainder of zero ends the calculation; a repeated remainder starts the same cycle again. Irrational numbers have decimal expansions that neither terminate nor eventually repeat. A calculator display shows only finitely many digits, so a display alone cannot prove recurrence.

A reliable method

1. Identify exactly which digits repeat and where the repetition begins.

2. Use an ellipsis only when the pattern is clear.

3. Treat a rounded finite display as an approximation unless exact equality has been established.

Worked example 1

Write the first eight decimal digits of 0.(27).

STEP-BY-STEP SOLUTION

Repeat the two-digit block: 0.27272727… . The block is 27, not just 7.

Worked example 2

Explain the difference between 0.1(23) and 0.(123).

STEP-BY-STEP SOLUTION

0.1(23) = 0.123232323… , with non-recurring first digit 1. By contrast, 0.(123) = 0.123123123… . The initial digits agree but the patterns differ.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A rational number does not mean a whole number or a tidy decimal. Both 7/12 and −5/7 are rational because each is a ratio of integers. Their recurring decimals are exact representations.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Write the first six decimal digits of 0.(4).

1.2 Write the first eight decimal digits of 0.2(15).

1.3 Classify 0.375, 0.(18) and √2 as rational or irrational.

1.4 Is 0.3333 exactly equal to 1/3? Explain.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

From fractions to decimals and recognising termination

A fraction becomes a decimal by dividing numerator by denominator. Track the remainder at every stage. When a remainder repeats, all subsequent digits repeat as well. The repeating block may begin immediately or after some non-recurring digits.

A fraction in lowest terms terminates in base ten exactly when its denominator has no prime factors other than 2 and 5. This is because a terminating decimal can be written over a power of ten, and powers of ten contain only those primes. Simplify first: 3/6 terminates because it reduces to 1/2, despite the original denominator containing 3. A denominator with any other prime factor after simplification gives a recurring expansion.

A reliable method

1. Simplify the fraction before using the denominator test.

2. For decimal conversion, divide and record remainders.

3. Identify the whole cycle rather than stopping at the first repeated digit.

Worked example 1

Convert 5/12 to a decimal.

STEP-BY-STEP SOLUTION

50 ÷ 12 gives 4, remainder 2. Bring down zero: 20 ÷ 12 gives 1, remainder 8. Then 80 ÷ 12 gives 6, remainder 8. The remainder 8 repeats, so 5/12 = 0.41(6).

Worked example 2

Will 7/40 terminate? What about 7/30?

STEP-BY-STEP SOLUTION

$40 = 2^3 \times 5$, so 7/40 terminates: 0.175. The simplified denominator 30 has a factor 3, so 7/30 recurs: 0.2(3).

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A repeated digit is not enough to prove that the repeating cycle has begun. In a division calculation, the repeated remainder is the reliable signal. Different remainders can temporarily produce the same next digit.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Convert 3/8 to a decimal.

2.2 Convert 2/11 to a recurring decimal.

2.3 Does 9/24 terminate? Explain.

2.4 Which terminate: 7/25, 5/18 and 13/80?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Recurring decimals to fractions — Higher extension

To convert a recurring decimal to a fraction, give it a variable name and create two expressions whose infinite repeating tails line up. Subtracting those expressions cancels the tails exactly. This is not rounding: the repeated part is identical in both expressions. Solve the resulting linear equation and simplify the fraction.

If repetition begins immediately and its block has k digits, compare x with 10^k x. If there are m non-recurring digits followed by a k-digit block, compare 10^m x with 10^{m+k} x. Both shifted decimals then have the same tail. Writing the first few digits on separate lines helps avoid shifting the wrong number of places.

A reliable method

1. Let x equal the exact recurring decimal.

2. Multiply by powers of ten so the repeating tails match.

3. Subtract, solve for x and simplify using common factors.

Worked example 1

Convert 0.(36) to a fraction.

STEP-BY-STEP SOLUTION

Let x = 0.363636… . Then 100x = 36.363636… . Subtract x: 99x = 36. Thus x = 36/99 = 4/11.

Worked example 2

Convert 0.2(7) to a fraction.

STEP-BY-STEP SOLUTION

Let x = 0.27777… . Then 10x = 2.7777… and 100x = 27.7777… . Subtract: 90x = 25, so x = 25/90 = 5/18.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Why does 0.(9) equal 1? If x = 0.999… , then 10x = 9.999… . Subtraction gives 9x = 9 and x = 1. The recurring representation and the terminating representation name the same number; there is no positive gap between them.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Convert 0.(7) to a fraction.

3.2 Convert 0.(24) to a fraction.

3.3 Convert 0.1(6) to a fraction.

3.4 Convert 0.12(34) to a fraction.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Write the first eight decimal digits of 0.3(24). [2 marks]

R2. Explain whether 11/60 terminates or recurs. [2 marks]

R3. Convert 7/12 to recurring decimal notation. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Higher: convert 0.(54) to a fraction in simplest form. [3 marks]

R5. Higher: convert 0.4(2) to a fraction. [4 marks]

R6. Higher: which is larger, 0.(27) or 0.27? Give the exact difference. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Terminating, recurring and irrational decimals

1.1 Write the first six decimal digits of 0.(4).

0.444444… .

1.2 Write the first eight decimal digits of 0.2(15).

0.21515151… .

1.3 Classify 0.375, 0.(18) and √2 as rational or irrational.

0.375 = 3/8 is rational; 0.(18) is recurring and rational; √2 is irrational.

1.4 Is 0.3333 exactly equal to 1/3? Explain.

No. 0.3333 terminates and is 3333/10000; 1/3 = 0.3333… with infinitely many 3s.

Lesson 2 • From fractions to decimals and recognising termination

2.1 Convert 3/8 to a decimal.

3 ÷ 8 = 0.375.

2.2 Convert 2/11 to a recurring decimal.

2 ÷ 11 gives 0.181818… = 0.(18). Remainders cycle between 2 and 9.

2.3 Does 9/24 terminate? Explain.

9/24 = 3/8. The denominator is 2^3, so it terminates: 0.375.

2.4 Which terminate: 7/25, 5/18 and 13/80?

7/25 and 13/80 terminate because their simplified denominators contain only 2 and/or 5. The denominator 18 contains 3, so 5/18 recurs.

Lesson 3 • Recurring decimals to fractions — Higher extension

3.1 Convert 0.(7) to a fraction.

Let x = 0.777… . Then 10x − x = 7, so 9x = 7 and x = 7/9.

3.2 Convert 0.(24) to a fraction.

100x − x = 24 gives 99x = 24. Thus x = 24/99 = 8/33.

3.3 Convert 0.1(6) to a fraction.

100x − 10x = 16.666… − 1.666… = 15. Hence 90x = 15 and x = 1/6.

3.4 Convert 0.12(34) to a fraction.

100x = 12.3434… and 10000x = 1234.3434… . Subtract to get 9900x = 1222, so x = 1222/9900 = 611/4950.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Write the first eight decimal digits of 0.3(24). [2 marks]

0.32424242… . [1 for non-recurring 3; 1 for repeated block 24.]

R2. Explain whether 11/60 terminates or recurs. [2 marks]

It is in lowest terms and $60 = 2^2 \times 3 \times 5$ includes a factor 3, so it recurs. [1 for denominator reasoning; 1 for conclusion.]

R3. Convert 7/12 to recurring decimal notation. [3 marks]

$7 \div 12 = 0.583333\ldots = 0.58(3)$. [1 for division; 1 for initial digits; 1 for recurring block.]

R4. Higher: convert 0.(54) to a fraction in simplest form. [3 marks]

$100x - x = 54$, so $x = 54/99 = 6/11$. [1 for aligned equations; 1 for subtraction; 1 for simplified answer.]

R5. Higher: convert 0.4(2) to a fraction. [4 marks]

$100x = 42.222\ldots$ and $10x = 4.222\ldots$. Subtract to get $90x = 38$, so $x = 19/45$. [1 for each shift; 1 for subtraction; 1 for fraction.]

R6. Higher: which is larger, 0.(27) or 0.27? Give the exact difference. [3 marks]

$0.(27) = 3/11$; $0.27 = 27/100$. Difference $= (300 - 297)/1100 = 3/1100$, so 0.(27) is larger. [1 for exact conversion; 1 for subtraction; 1 for difference and conclusion.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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