GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 09

09

Surds and exact answers

Higher-level GCSE material

An exact answer preserves a value without decimal rounding. Surds let us work exactly with irrational roots. This guide explains how to simplify and calculate with them, then shows how rationalising a denominator changes the representation without changing the value.

$$\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$$

A square factor comes outside the root.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Independent educational resource

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Before you begin

Know square numbers, factors, fraction arithmetic, expanding brackets and Pythagoras' theorem for the final application.

Your route through this notebook

1. Recognising and simplifying surds
2. Combining and multiplying surds
3. Rationalising denominators and using exact values

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Recognising and simplifying surds

An irrational number cannot be written as a ratio of two integers. Its decimal expansion does not terminate or settle into a repeating block. A surd is an irrational root expression, such as √2 or √7. The expression √49 is not a surd after evaluation because it equals the integer 7. An exact answer such as 3√5 is often more useful than an early decimal approximation.

For non-negative a and b, √(ab) = √a × √b. This lets us take square factors outside a square root. Choose the largest square factor to finish efficiently, or remove smaller square factors in stages. You cannot split addition under a root in the same way: √(a + b) is generally different from √a + √b.

A reliable method

1. Find a square-number factor of the number under the root.

2. Split the root into a product and evaluate the square root of the square factor.

3. Check that the remaining radicand has no square factor greater than 1.

Worked example 1

Simplify √72.

STEP-BY-STEP SOLUTION

72 = 36 × 2. Therefore √72 = √36 × √2 = 6√2. The 2 has no further square factor.

Worked example 2

Simplify 3√50.

STEP-BY-STEP SOLUTION

50 = 25 × 2, so √50 = 5√2. Multiply the coefficient: 3 × 5√2 = 15√2.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

√(9 + 16) = √25 = 5, whereas √9 + √16 = 3 + 4 = 7. This counterexample shows why the product rule cannot be turned into an addition rule.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Simplify √48.

1.2 Simplify √180.

1.3 Simplify 2√75.

1.4 Which are rational: √81, √12, √0.25? Explain.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Combining and multiplying surds

Like surds have the same irrational part after simplification. They combine like algebraic terms: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. Unlike surds generally cannot be combined into a single term by addition. Simplify each root before deciding which terms match. Subtraction works by subtracting coefficients of matching surds.

To multiply, multiply ordinary coefficients and multiply the radicands, then simplify. A surd multiplied by itself becomes the radicand: $\sqrt{a} \times \sqrt{a} = a$ for non-negative a. In brackets, multiply every term in one bracket by every term in the other. Exact cancellation can make the final answer rational even when intermediate terms contain surds.

A reliable method

1. Simplify roots before adding or subtracting.
2. Collect only terms with the same remaining root.
3. For products and brackets, distribute first and simplify fully.

Worked example 1

Simplify $\sqrt{50} + 3\sqrt{8} - \sqrt{18}$.

STEP-BY-STEP SOLUTION

Rewrite as $5\sqrt{2} + 6\sqrt{2} - 3\sqrt{2} = 8\sqrt{2}$.

Worked example 2

Expand and simplify $(2 + \sqrt{3})(5 - \sqrt{3})$.

STEP-BY-STEP SOLUTION

Products are $10 - 2\sqrt{3} + 5\sqrt{3} - 3$. Collect terms to get $7 + 3\sqrt{3}$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

√2 + √3 is already a valid exact expression. It is not √5. Squaring both expressions gives different results: $(\sqrt{2} + \sqrt{3})^2 = 5 + 2\sqrt{6}$, while $(\sqrt{5})^2 = 5$.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Simplify $4\sqrt{3} + 2\sqrt{3} - \sqrt{3}$.

2.2 Simplify $\sqrt{20} + \sqrt{45}$.

2.3 Simplify $(3\sqrt{2})(2\sqrt{6})$.

2.4 Expand $(\sqrt{5} + 2)^2$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Rationalising denominators and using exact values

Rationalising means rewriting a fraction so its denominator is rational. Multiply numerator and denominator by the same non-zero expression, so the fraction's value is preserved. For a single-root denominator, multiply by that root: $\sqrt{a} \times \sqrt{a} = a$. Simplify the resulting fraction where possible.

For a denominator $a + \sqrt{b}$, multiply by its conjugate $a - \sqrt{b}$. The middle terms cancel in $(a + \sqrt{b})(a - \sqrt{b})$, leaving $a^2 - b$. The method requires a non-zero denominator and a suitable non-zero conjugate. Exact values can be carried through geometry calculations, with a decimal approximation taken only at the end if requested.

A reliable method

1. Single root below the fraction bar: multiply top and bottom by that root.

2. Two-term denominator: use the conjugate and expand carefully.

3. Simplify common numerical factors and keep exact form until the question asks otherwise.

Worked example 1

Rationalise $6/\sqrt{3}$.

STEP-BY-STEP SOLUTION

Multiply by $\sqrt{3}/\sqrt{3}$: $6\sqrt{3}/3 = 2\sqrt{3}$. The multiplier equals 1, so the value is unchanged.

Worked example 2

Rationalise $3/(2 + \sqrt{3})$.

STEP-BY-STEP SOLUTION

Multiply top and bottom by $2 - \sqrt{3}$. The denominator is $4 - 3 = 1$; numerator is $6 - 3\sqrt{3}$. Answer: $6 - 3\sqrt{3}$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Rationalising does not remove an irrational value; it moves the root out of the denominator. Both $1/\sqrt{2}$ and $\sqrt{2}/2$ are exact and equal. Which representation is required depends on the question.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Rationalise $5/\sqrt{2}$.

3.2 Rationalise $4/\sqrt{8}$ and simplify.

3.3 Rationalise $1/(3 - \sqrt{2})$.

3.4 A right triangle has shorter sides $\sqrt{6}$ cm and $\sqrt{10}$ cm. Find the exact hypotenuse.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Simplify $\sqrt{200}$. [2 marks]

R2. Simplify $2\sqrt{27} + \sqrt{12}$. [3 marks]

R3. Expand $(3 + \sqrt{2})(4 - \sqrt{2})$. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Rationalise $8/\sqrt{6}$. [3 marks]

R5. Rationalise $2/(\sqrt{5} - 1)$. [3 marks]

R6. A square has area 50 cm^2. Find its exact perimeter. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Recognising and simplifying surds

1.1 Simplify $\sqrt{48}$.

$48 = 16 \times 3$, so $4\sqrt{3}$.

1.2 Simplify $\sqrt{180}$.

$180 = 36 \times 5$, so $6\sqrt{5}$.

1.3 Simplify $2\sqrt{75}$.

$\sqrt{75} = \sqrt{(25 \times 3)} = 5\sqrt{3}$; multiply by 2 to get $10\sqrt{3}$.

1.4 Which are rational: $\sqrt{81}$, $\sqrt{12}$, $\sqrt{0.25}$? Explain.

$\sqrt{81} = 9$ and $\sqrt{0.25} = 0.5$ are rational. $\sqrt{12} = 2\sqrt{3}$ is irrational.

Lesson 2 • Combining and multiplying surds

2.1 Simplify $4\sqrt{3} + 2\sqrt{3} - \sqrt{3}$.

$(4 + 2 - 1)\sqrt{3} = 5\sqrt{3}$.

2.2 Simplify $\sqrt{20} + \sqrt{45}$.

$2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$.

2.3 Simplify $(3\sqrt{2})(2\sqrt{6})$.

$6\sqrt{12} = 6 \times 2\sqrt{3} = 12\sqrt{3}$.

2.4 Expand $(\sqrt{5} + 2)^2$.

$5 + 4\sqrt{5} + 4 = 9 + 4\sqrt{5}$. The middle terms must be included.

Lesson 3 • Rationalising denominators and using exact values

3.1 Rationalise $5/\sqrt{2}$.

Multiply by $\sqrt{2}/\sqrt{2}$ to obtain $5\sqrt{2}/2$.

3.2 Rationalise $4/\sqrt{8}$ and simplify.

$\sqrt{8} = 2\sqrt{2}$, so $4/\sqrt{8} = 2/\sqrt{2} = \sqrt{2}$.

3.3 Rationalise $1/(3 - \sqrt{2})$.

Multiply by $(3 + \sqrt{2})/(3 + \sqrt{2})$: $(3 + \sqrt{2})/(9 - 2) = (3 + \sqrt{2})/7$.

3.4 A right triangle has shorter sides $\sqrt{6}$ cm and $\sqrt{10}$ cm. Find the exact hypotenuse.

By Pythagoras, $c^2 = 6 + 10 = 16$. The positive length is $c = 4$ cm.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Simplify √200. [2 marks]

√(100 × 2) = 10√2. [1 for square factor; 1 for result.]

R2. Simplify 2√27 + √12. [3 marks]

2 × 3√3 + 2√3 = 8√3. [1 for each simplified term; 1 for collection.]

R3. Expand (3 + √2)(4 − √2). [3 marks]

12 − 3√2 + 4√2 − 2 = 10 + √2. [1 for distribution; 1 for √2 × √2 = 2; 1 for collection.]

R4. Rationalise 8/√6. [3 marks]

8√6/6 = 4√6/3. [1 for multiplier; 1 for rational denominator; 1 for simplification.]

R5. Rationalise 2/(√5 − 1). [3 marks]

2(√5 + 1)/(5 − 1) = (√5 + 1)/2. [1 for conjugate; 1 for denominator 4; 1 for simplified result.]

R6. A square has area 50 cm². Find its exact perimeter. [3 marks]

Side = √50 = 5√2 cm. Perimeter = 4 × 5√2 = 20√2 cm. [1 for root; 1 for simplification; 1 for perimeter and unit.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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