GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 08

08

# Accuracy, error intervals and bounds

Shared accuracy skills with Higher calculations

A rounded measurement represents a range of possible actual values. Bounds describe that range and let us investigate the smallest and largest values that a calculation could approach. This guide separates simple error intervals from the more demanding Higher work with calculations.

$7.5 \leq x < 8.5$: included left endpoint, excluded right endpoint.




**UNDERSTAND THE IDEA. EXPLAIN THE METHOD.**

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

**Name:** _____

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GCSE ONLINE TUTORING / ABOUT THIS BOOK

# Your learning guide

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## Independent educational resource

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## Before you begin

Understand rounding, inequalities, fractions and formula substitution. Use a calculator for the longer calculations.

## Your route through this notebook

1. Intervals for rounded and truncated values

2. Bounds for sums, differences and products — Higher

3. Quotient bounds and justified precision — Higher

## How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

**YOUR PROGRESS**

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.


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LESSON 1 / UNDERSTAND

# Intervals for rounded and truncated values

If a positive length is 8 cm to the nearest centimetre, the actual length could be as low as 7.5 cm and still round to 8. Values just below 8.5 cm also round to 8, but 8.5 rounds to 9 under the usual positive-number school convention. Therefore $7.5 \leq x < 8.5$. The lower endpoint is included; the upper endpoint is excluded.

For nearest-unit rounding, the interval extends half a rounding unit either side of the stated value. Truncation is different: digits are removed rather than rounded. A positive value truncated to 2 decimal places as 3.47 lies from 3.47 inclusive to 3.48 exclusive. With negative values, reason carefully from the rounding or truncation process instead of copying a positive-number template.

## A reliable method

1. Identify the rounding unit, such as 1, 0.1, 0.01 or 100.
2. For nearest rounding, add and subtract half that unit.
3. Use $\leq$ at an included boundary and $<$ at an excluded boundary.

## Worked example 1

A mass is 2.4 kg to the nearest 0.1 kg. Write its interval.

**STEP-BY-STEP SOLUTION**

Half of 0.1 is 0.05. Thus $2.35 \leq m < 2.45$, with m in kilograms.

## Worked example 2

A positive number is truncated to 1 decimal place as 6.2. Write its interval.

**STEP-BY-STEP SOLUTION**

Values from 6.2 up to but not including 6.3 lose their later digits to become 6.2. Thus $6.2 \leq x < 6.3$.


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LESSON 1 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

A value rounded to 2 significant figures has a rounding unit that depends on its size. For 340 to 2 significant figures the last retained place is tens, so the interval is $335 \leq x < 345$. For 3.4 it is tenths.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Write the interval for 56 cm rounded to the nearest centimetre.

1.2 Write the interval for 0.83 rounded to 2 decimal places.

1.3 A positive number truncates to 12.08 at 2 decimal places. Give its interval.

1.4 A positive distance is 4,700 m to 2 significant figures. Give its interval.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 2 / UNDERSTAND

# Bounds for sums, differences and products — Higher

To bound a calculation, first write the possible range for each measured input. Then reason about how the output changes when each input changes. For a sum, the smallest result uses both lower bounds and the upper bound uses both upper bounds. For a difference a − b, the smallest result uses the smallest a and largest b; the largest uses the largest a and smallest b.

For products of positive measurements, using both lower bounds gives the smallest product and both upper bounds gives the upper bound. These rules rely on positivity and independent variation. If values can be negative, consider endpoint combinations and any internal behaviour rather than assuming the same shortcut. An excluded upper bound is a limiting value, not necessarily a value the actual measurement can attain.

## A reliable method

1. Write input intervals with units before calculating.

2. Ask which change makes the required expression increase.

3. Report a bound as a bound; do not imply an excluded endpoint is an attainable maximum.

## Worked example 1

A rectangle is 8 cm by 5 cm, each to the nearest cm. Bound its area.

**STEP-BY-STEP SOLUTION**

$7.5 \le l < 8.5$ and $4.5 \le w < 5.5$. Lower area $= 7.5 \times 4.5 = 33.75$ cm². Upper bound $= 8.5 \times 5.5 = 46.75$ cm². Thus $33.75 \le A < 46.75$.

## Worked example 2

a = 12 and b = 7, each rounded to the nearest integer. Bound a − b.

**STEP-BY-STEP SOLUTION**

$11.5 \le a < 12.5$ and $6.5 \le b < 7.5$. Lower bound $= 11.5 - 7.5 = 4$; upper bound $= 12.5 - 6.5 = 6$. Neither extreme is attained, so $4 < a - b < 6$.


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LESSON 2 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

Adding the same “error amount” after a multiplication is unreliable. An area’s uncertainty depends on both dimensions and their interaction. Work from the input intervals, not a guessed error on the final answer.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Lengths 4.2 m and 3.1 m are each rounded to 0.1 m. Bound their sum.

2.2 A square side is 6 cm to the nearest cm. Give lower and upper bounds for its area.

2.3 $x = 9.4$ and $y = 2.6$, each to 1 decimal place. Find the upper bound for $x - y$.

2.4 A rectangle has lengths 12 m and 7 m, each to the nearest metre. Bound its perimeter.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 3 / UNDERSTAND

# Quotient bounds and justified precision — Higher

For a positive quotient a/b, a larger numerator increases the result and a larger denominator decreases it. Therefore a lower bound is lower(a)/upper(b), and an upper bound is upper(a)/lower(b), provided the denominator stays positive. Using two lower bounds does not generally give the lower quotient.

Bounds can also test whether a rounded answer is guaranteed. Calculate the output interval, then check whether every possible output rounds to the same stated value. If the interval crosses a rounding boundary, the available measurements do not justify that precision. This is stronger reasoning than simply reporting all digits on a calculator display.

## A reliable method

1. Confirm that the denominator cannot be zero or negative in the interval.

2. For a lower quotient use a low numerator and a high denominator; reverse them for the upper quotient.

3. Keep full precision when comparing the result with rounding boundaries.

## Worked example 1

A distance is 100 m to the nearest metre, and time is 20 s to the nearest second. Bound average speed.

**STEP-BY-STEP SOLUTION**

$99.5 \le d < 100.5$ and $19.5 \le t < 20.5$. Lower bound = 99.5/20.5 = 4.853658… m/s. Upper bound = 100.5/19.5 = 5.153846… m/s. Both are limiting, excluded endpoints here.

## Worked example 2

An output is known to satisfy $7.242 \le v < 7.248$. Is 7.2 correct to 1 decimal place for every possible value?

**STEP-BY-STEP SOLUTION**

Yes. Every value lies between 7.15 and 7.25, so every possible value rounds to 7.2. To 2 decimal places the interval crosses 7.245, so a single hundredths answer is not guaranteed.


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LESSON 3 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

Writing rounded decimal bounds as if exact can accidentally narrow an interval. Retain exact fractions for rigorous endpoints or label decimal endpoints as approximations. When requested to give a bound to a stated accuracy, follow the question's wording carefully.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 d = 60 m and t = 12 s, each to the nearest unit. Find the upper bound for d/t.

3.2 A positive mass is 8.0 kg to 1 decimal place and volume is 2.0 litres to 1 decimal place. Find the lower density bound.

3.3 All possible values satisfy $3.26 \leq x < 3.34$. Is 3.3 guaranteed to 1 decimal place?

3.4 All possible values satisfy $3.24 \leq x < 3.34$. Is 3.3 guaranteed to 1 decimal place?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

# Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. A length is 18.6 cm to the nearest 0.1 cm. Write its error interval. [2 marks]

R2. A positive number truncates to 0.047 at 3 decimal places. Write its interval. [2 marks]

R3. Higher: sides of a rectangle are 9 cm and 4 cm, each to the nearest cm. Find its area bounds. [3 marks]


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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

# Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Higher: $x = 15.2$ and $y = 8.1$ to 1 decimal place. Find the lower bound for $x - y$. [3 marks]

R5. Higher: distance is 80 m to the nearest metre and time is 16 s to the nearest second. Find the upper speed bound. [3 marks]

R6. Higher: $12.346 \leq a < 12.354$. Is 12.35 guaranteed to 2 decimal places? Explain. [2 marks]


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SOLUTIONS / LESSON PRACTICE

# Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

## Lesson 1 • Intervals for rounded and truncated values

1.1 Write the interval for 56 cm rounded to the nearest centimetre.

$55.5 \leq x < 56.5$, in cm.

1.2 Write the interval for 0.83 rounded to 2 decimal places.

$0.825 \leq x < 0.835$.

1.3 A positive number truncates to 12.08 at 2 decimal places. Give its interval.

$12.08 \leq x < 12.09$.

1.4 A positive distance is 4,700 m to 2 significant figures. Give its interval.

The rounding unit is 100 m; half is 50 m. Therefore $4{,}650 \leq d < 4{,}750$.

## Lesson 2 • Bounds for sums, differences and products — Higher

2.1 Lengths 4.2 m and 3.1 m are each rounded to 0.1 m. Bound their sum.

Intervals: [4.15, 4.25) and [3.05, 3.15). Sum: $7.20 \leq s < 7.40$ m.

2.2 A square side is 6 cm to the nearest cm. Give lower and upper bounds for its area.

$5.5 \leq s < 6.5$. Squaring positive endpoints gives $30.25 \leq A < 42.25$ cm².

2.3 $x = 9.4$ and $y = 2.6$, each to 1 decimal place. Find the upper bound for $x - y$.

Upper $x = 9.45$; lower $y = 2.55$. Upper bound = 6.90, which is not attained.

2.4 A rectangle has lengths 12 m and 7 m, each to the nearest metre. Bound its perimeter.

$P = 2(l + w)$. Lower $= 2(11.5 + 6.5) = 36$ m; upper $= 2(12.5 + 7.5) = 40$ m. Thus $36 \leq P < 40$.

## Lesson 3 • Quotient bounds and justified precision — Higher

3.1 $d = 60$ m and $t = 12$ s, each to the nearest unit. Find the upper bound for $d/t$.

$60.5/11.5 = 121/23 \approx 5.26087$ m/s.

3.2 A positive mass is 8.0 kg to 1 decimal place and volume is 2.0 litres to 1 decimal place. Find the lower density bound.

Lower mass = 7.95, upper volume = 2.05. Bound $= 7.95/2.05 = 159/41 \approx 3.87805$ kg/litre.

3.3 All possible values satisfy $3.26 \leq x < 3.34$. Is 3.3 guaranteed to 1 decimal place?

Yes: the whole interval lies within $3.25 \leq x < 3.35$.

3.4 All possible values satisfy $3.24 \leq x < 3.34$. Is 3.3 guaranteed to 1 decimal place?

No. Values below 3.25 round to 3.2, while values from 3.25 round to 3.3.


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SOLUTIONS / MIXED REVIEW

# Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. A length is 18.6 cm to the nearest 0.1 cm. Write its error interval. [2 marks]

$18.55 \leq l < 18.65$ cm. [1 for endpoints; 1 for inequality signs.]

R2. A positive number truncates to 0.047 at 3 decimal places. Write its interval. [2 marks]

$0.047 \leq x < 0.048$. [1 for endpoints; 1 for inclusion/exclusion.]

R3. Higher: sides of a rectangle are 9 cm and 4 cm, each to the nearest cm. Find its area bounds. [3 marks]

Lower = $8.5 \times 3.5 = 29.75$ cm²; upper = $9.5 \times 4.5 = 42.75$ cm². [1 for input bounds; 1 for each area bound.]

R4. Higher: x = 15.2 and y = 8.1 to 1 decimal place. Find the lower bound for x − y. [3 marks]

$15.15 - 8.15 = 7.00$. This is a lower bound, not an attained minimum. [1 for each selected bound; 1 for subtraction.]

R5. Higher: distance is 80 m to the nearest metre and time is 16 s to the nearest second. Find the upper speed bound. [3 marks]

$80.5/15.5 = 161/31 \approx 5.19355$ m/s. [1 for input bounds; 1 for quotient choice; 1 for result/unit.]

R6. Higher: $12.346 \leq a < 12.354$. Is 12.35 guaranteed to 2 decimal places? Explain. [2 marks]

Yes. Every value lies in [12.345, 12.355), so every value rounds to 12.35. [1 for rounding interval; 1 for conclusion.]

## Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.


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