GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 05

05

Powers, roots and index laws

Shared core with clearly marked Higher extensions

Powers compress repeated multiplication. Roots reverse powers. Index laws become easier to remember when you understand the factors they count. Start with positive whole-number indices, then move to the Higher extensions on negative and fractional indices.

$2^1 = 2$ $2^2 = 4$ $2^3 = 8$

Each increase of 1 in the index multiplies by 2.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Know multiplication, division, prime factors and the order of operations.

Your route through this notebook

1. Powers, roots and exact values
2. Multiplying, dividing and powers of powers
3. Zero and negative indices — Higher extension
4. Fractional indices — Higher extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Powers, roots and exact values

In 5^3, the base is 5 and the index is 3: multiply three copies of 5. A square number is a number multiplied by itself, and a cube number uses three equal factors. A square root reverses squaring; a cube root reverses cubing. The symbol $\sqrt{}$ gives the non-negative square root, so $\sqrt{49} = 7$. However, the equation $x^2 = 49$ has two solutions, 7 and -7.

Roots need not be whole numbers. Since $6^2 = 36$ and $7^2 = 49$, $\sqrt{40}$ lies between 6 and 7. Keep an exact root when a decimal is not requested. For real numbers, a square cannot be negative, but a cube can: the cube root of -8 is -2.

A reliable method

1. Distinguish a power from multiplication by the index.

2. Use known squares and cubes to recognise or estimate roots.

3. Distinguish evaluating $\sqrt{a}$ from solving $x^2 = a$.

Worked example 1

Evaluate 4^3 and the cube root of -27.

STEP-BY-STEP SOLUTION

$4^3 = 4 \times 4 \times 4 = 64$. Since $(-3)^3 = -27$, the cube root of -27 is -3.

Worked example 2

Find consecutive integers between which $\sqrt{70}$ lies.

STEP-BY-STEP SOLUTION

$8^2 = 64$ and $9^2 = 81$. Since $64 < 70 < 81$, we have $8 < \sqrt{70} < 9$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A pupil writes $\sqrt{36} = \pm 6$. The root symbol itself means the principal, non-negative root, so $\sqrt{36} = 6$. The $\pm$ notation belongs when solving an equation such as $x^2 = 36$.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Evaluate 7^2 and 3^4.

1.2 Find $\sqrt{121}$ and the cube root of 125.

1.3 Between which consecutive integers does $\sqrt{50}$ lie?

1.4 Solve $x^2 = 81$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Multiplying, dividing and powers of powers

When powers have the same base, their indices count copies of the same factor. Multiplying joins the lists of factors, so add indices. Dividing cancels equal factors, so subtract indices. Raising a power to another power repeats a group of factors, so multiply the indices. These rules apply to matching bases; $2^3 \times 5^2$ cannot be combined by simply adding its indices.

Brackets decide the scope of a power. In $(3x)^2$, both 3 and x are squared, giving $9x^2$. In $3x^2$, only x is squared. A power does not normally distribute across addition: $(a + b)^2$ contains a middle term and is not simply $a^2 + b^2$.

A reliable method

1. For the same base: $a^m \times a^n = a^{m+n}$.

2. For non-zero a: $a^m \div a^n = a^{m-n}$.

3. For a power of a power: $(a^m)^n = a^{mn}$.

Worked example 1

Simplify $2^3 \times 2^5 \div 2^4$.

STEP-BY-STEP SOLUTION

The base stays 2. Add then subtract indices: $2^{3+5-4} = 2^4 = 16$.

Worked example 2

Simplify $(x^3)^4$ and $(2x^2)^3$.

STEP-BY-STEP SOLUTION

$(x^3)^4 = x^{12}$. In the second expression, cube the coefficient and the power: $2^3 \times (x^2)^3 = 8x^6$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Check a supposed law with small numbers. For instance $(2 + 3)^2 = 25$ but $2^2 + 3^2 = 13$, so a power cannot be distributed over addition in that way.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Simplify $5^2 \times 5^4$.

2.2 Simplify $y^9 \div y^3$, for $y \neq 0$.

2.3 Simplify $(a^4)^3$.

2.4 Simplify $(3b^2)^2$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Zero and negative indices — Higher extension

For a non-zero base, a power with index zero equals 1. One explanation is $a^3 \div a^3$: direct division gives 1, while the division law gives a^0. A negative index indicates a reciprocal. Continuing the sequence 2^3, 2^2, 2^1, 2^0, 2^{-1} divides by 2 each time, producing 8, 4, 2, 1, 1/2.

Therefore $a^{-n} = 1/a^n$ for non-zero a. A negative index does not make the answer negative. The sign of the base is a different issue: $(-2)^{-3} = 1/(-2)^3 = -1/8$, while $2^{-3} = 1/8$. Zero bases need separate care: division by zero is not defined, so these reciprocal rules cannot be used with a = 0.

A reliable method

1. Check that a base used in a reciprocal is non-zero.

2. Replace a negative power by a reciprocal with a positive power.

3. Keep brackets around negative or fractional bases.

Worked example 1

Evaluate 6^0 and 3^{-2}.

STEP-BY-STEP SOLUTION

$6^0 = 1$. Also $3^{-2} = 1/3^2 = 1/9$.

Worked example 2

Evaluate $(2/5)^{-2}$.

STEP-BY-STEP SOLUTION

Invert the whole fraction, then square: $(5/2)^2 = 25/4$. This is the reciprocal of $(2/5)^2 = 4/25$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Compare -2^3, $(-2)^3$ and 2^{-3}. The first two happen to equal -8, but the third equals 1/8. For an even index the first two would differ, so brackets are essential.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Evaluate 11^0.

3.2 Evaluate 4^{-3}.

3.3 Evaluate $(3/4)^{-1}$.

3.4 Simplify $a^3 \times a^{-5}$, for $a \neq 0$, using a positive index.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Fractional indices — Higher extension

For positive bases, an index of 1/n means the nth root. This definition fits the power-of-a-power law: $(a^{1/n})^n = a$. An index of m/n combines a root and a power: $a^{m/n}$ = (the nth root of a)^m. Taking the root first is usually easier for exact arithmetic.

A negative fractional index combines both ideas: find the root and power, then take the reciprocal. This lesson uses positive bases so that the expressions have unambiguous real values. When simplifying more general expressions, domain restrictions matter; not every square root of a squared expression can be replaced by the original signed expression.

A reliable method

1. Read the denominator of the index as the root.

2. Read the numerator as the power.

3. If the index is negative, take the reciprocal of the positive-index result.

Worked example 1

Evaluate $64^{2/3}$.

STEP-BY-STEP SOLUTION

The cube root of 64 is 4. Square it to get $4^2 = 16$.

Worked example 2

Evaluate $16^{-3/4}$.

STEP-BY-STEP SOLUTION

The fourth root of 16 is 2. Cube it to get 8, then take the reciprocal: 1/8.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The denominator of a fractional index does not mean ordinary division of the base. For example, $81^{1/2}$ is $\sqrt{81} = 9$, not $81 \div 2$. Read the index as an instruction about roots.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Evaluate $25^{1/2}$.

4.2 Evaluate $27^{2/3}$.

4.3 Evaluate $32^{3/5}$.

4.4 Evaluate $81^{-3/4}$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Evaluate $5^3 - \sqrt{144}$. [2 marks]

R2. Simplify $(p^3)^2 \times p^4$. [2 marks]

R3. Find consecutive integers enclosing $\sqrt{110}$. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Higher: evaluate $5^{-2} + 2^{0}$. [3 marks]

R5. Higher: evaluate $125^{-2/3}$. [3 marks]

R6. Explain why $(2 + 5)^2$ does not equal $2^2 + 5^2$. [2 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Powers, roots and exact values

1.1 Evaluate 7^2 and 3^4.

$7^2 = 49$; $3^4 = 3 \times 3 \times 3 \times 3 = 81$.

1.2 Find $\sqrt{121}$ and the cube root of 125.

$\sqrt{121} = 11$; the cube root of 125 is 5.

1.3 Between which consecutive integers does $\sqrt{50}$ lie?

$7^2 = 49$ and $8^2 = 64$, so between 7 and 8.

1.4 Solve $x^2 = 81$.

$x = 9$ or $x = -9$ because both square to 81.

Lesson 2 • Multiplying, dividing and powers of powers

2.1 Simplify $5^2 \times 5^4$.

$5^{2+4} = 5^6$.

2.2 Simplify $y^9 \div y^3$, for $y \neq 0$.

$y^{9-3} = y^6$.

2.3 Simplify $(a^4)^3$.

$a^{4 \times 3} = a^{12}$.

2.4 Simplify $(3b^2)^2$.

Square each factor: $3^2 \times b^4 = 9b^4$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Zero and negative indices — Higher extension

3.1 Evaluate 11^0.

The base is non-zero, so the value is 1.

3.2 Evaluate 4^{-3}.

$1/4^3 = 1/64$.

3.3 Evaluate $(3/4)^{-1}$.

The reciprocal is 4/3.

3.4 Simplify $a^3 \times a^{-5}$, for $a \neq 0$, using a positive index.

$a^{3-5} = a^{-2} = 1/a^2$.

Lesson 4 • Fractional indices — Higher extension

4.1 Evaluate $25^{1/2}$.

$\sqrt{25} = 5$.

4.2 Evaluate $27^{2/3}$.

The cube root is 3; square it to get 9.

4.3 Evaluate $32^{3/5}$.

The fifth root of 32 is 2; cube it to get 8.

4.4 Evaluate $81^{-3/4}$.

The fourth root is 3; cube it to get 27; take the reciprocal to obtain 1/27.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Evaluate $5^3 - \sqrt{144}$. [2 marks]

$125 - 12 = 113$. [1 for power/root; 1 for subtraction.]

R2. Simplify $(p^3)^2 \times p^4$. [2 marks]

$p^6 \times p^4 = p^{10}$. [1 for p^6; 1 for combining.]

R3. Find consecutive integers enclosing $\sqrt{110}$. [2 marks]

$10^2 = 100$ and $11^2 = 121$, so $10 < \sqrt{110} < 11$. [1 for bounding squares; 1 for conclusion.]

R4. Higher: evaluate $5^{-2} + 2^0$. [3 marks]

$1/25 + 1 = 26/25$. [1 for reciprocal; 1 for zero power; 1 for addition.]

R5. Higher: evaluate $125^{-2/3}$. [3 marks]

Cube root 125 = 5; $5^2 = 25$; reciprocal = 1/25. [1 for each step.]

R6. Explain why $(2 + 5)^2$ does not equal $2^2 + 5^2$. [2 marks]

The left is 49; the right is 29. Squaring a sum is multiplying the whole sum by itself, which includes cross-products. [1 for values; 1 for explanation.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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