GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 03

03

Fractions

Shared GCSE foundations

A fraction is both a number and a way to describe a division or a proportion. This guide connects those meanings, then develops reliable methods for comparing, calculating and solving problems. The denominator describes the unit; the numerator counts how many of those units you have.



UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Recall factors and multiples. Be able to multiply and divide whole numbers.

Your route through this notebook

1. Meaning, equivalence and comparison
2. Improper fractions and mixed numbers
3. Adding and subtracting fractions
4. Multiplying, dividing and finding a fraction of an amount

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Meaning, equivalence and comparison

In 3/5, the denominator 5 says that a whole has been split into five equal parts. The numerator 3 says that three of those parts are taken. The word equal matters: three unequal pieces of a cake need not be three fifths. A fraction can also represent division, so 3/5 = 3 ÷ 5.

Equivalent fractions name the same point on a number line. Multiplying numerator and denominator by the same non-zero number changes the size and number of pieces together, preserving the total amount. Simplifying reverses this process. To compare fractions with different denominators, rewrite them using a common unit; for positive fractions with the same denominator, compare numerators.

A reliable method

1. Simplify by dividing numerator and denominator by a common factor, ideally the HCF.

2. For comparison, use a common denominator, often the LCM.

3. Do not compare denominators alone when the numerators differ.

Worked example 1

Simplify 42/56.

STEP-BY-STEP SOLUTION

The HCF of 42 and 56 is 14. Divide both by 14: 42/56 = 3/4. Dividing only the top would change the value.

Worked example 2

Which is larger, 5/6 or 7/9?

STEP-BY-STEP SOLUTION

Use denominator 18: 5/6 = 15/18 and 7/9 = 14/18. Therefore 5/6 is larger by 1/18.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

With the same numerator, a larger denominator gives a smaller positive fraction: 1/8 < 1/5. That rule cannot be applied blindly to 7/8 and 1/5 because both parts of the fraction have changed.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Simplify 18/30.

1.2 Write 3/7 with denominator 42.

1.3 Order 2/3, 3/4 and 5/6, smallest first.

1.4 Explain why 4/10 = 2/5.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Improper fractions and mixed numbers

An improper fraction has a numerator at least as large as its denominator. It is a perfectly valid number, not an error. A mixed number writes a whole-number part and a proper fractional part together: 2 3/5 means 2 + 3/5. For calculations, improper fractions are often easier because the entire number is represented by a single fraction.

To change a mixed number into an improper fraction, count all the fractional units in its whole-number part, then add the remaining units. To change back, divide the numerator by the denominator: the quotient counts whole groups and the remainder counts the fractional units left. Keep the original denominator.

A reliable method

1. Convert a b/c to (a × c + b)/c for positive mixed numbers.

2. For an improper fraction, divide the numerator by the denominator.

3. For a negative mixed number, the minus applies to the whole value: −2 1/3 means −(2 + 1/3).

Worked example 1

Convert 3 2/7 to an improper fraction.

STEP-BY-STEP SOLUTION

Three wholes contain 3 × 7 = 21 sevenths. Add 2 sevenths: 23/7.

Worked example 2

Convert 29/6 to a mixed number.

STEP-BY-STEP SOLUTION

29 ÷ 6 = 4 remainder 5. There are four whole groups of six sixths and five sixths left: 4 5/6.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The notation 2 1/4 is not 2 × 1/4. It conventionally means 2 + 1/4. In algebra, adjacent symbols usually indicate multiplication, so mixed-number notation should be used carefully and unambiguously.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Convert 4 3/8 to an improper fraction.

2.2 Convert 38/9 to a mixed number.

2.3 Write −1 3/4 as a single fraction.

2.4 Which is larger, 2 1/3 or 19/8?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Adding and subtracting fractions

Addition combines quantities measured in the same units. You can add three fifths and one fifth directly, just as you can add three metres and one metre. You cannot directly add thirds and quarters until you rename them in a common unit. Equivalent fractions make this possible without changing either quantity.

Once denominators match, add or subtract only the numerators. The denominator stays fixed because the size of each piece has not changed. Convert mixed numbers to improper fractions when this makes subtraction clearer, and simplify the answer. For signed fractions, use the same sign reasoning as with whole numbers.

A reliable method

1. Choose a common denominator and rewrite every fraction.

2. Add or subtract the numerators while keeping that denominator.

3. Simplify; convert to a mixed number if useful in the context.

Worked example 1

Calculate 3/4 + 5/6.

STEP-BY-STEP SOLUTION

The LCM of 4 and 6 is 12. Rewrite as 9/12 + 10/12 = 19/12 = 1 7/12.

Worked example 2

Calculate 2 1/5 − 3/4.

STEP-BY-STEP SOLUTION

Convert 2 1/5 to 11/5. With denominator 20: 44/20 − 15/20 = 29/20 = 1 9/20.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Adding numerator to numerator and denominator to denominator does not preserve the operation. For example, 1/2 + 1/2 = 1, whereas 2/4 = 1/2. Testing a simple case can expose a false rule.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Calculate 2/7 + 3/7.

3.2 Calculate 5/6 − 1/4.

3.3 Calculate 1 2/3 + 2 3/5.

3.4 Calculate −3/4 + 1/6.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Multiplying, dividing and finding a fraction of an amount

Multiplying by a fraction scales an amount. Taking 2/3 of 3/5 means keeping two thirds of three fifths, which gives 6/15 = 2/5. This explains the rule of multiplying numerators together and denominators together. Common factors can be cancelled before multiplying because numerator and denominator of the overall product are being divided by the same number.

Division asks how many groups fit, or reverses a multiplication. Dividing by 2/3 is the same as multiplying by 3/2: two thirds of the unknown result must recover the starting number. Only invert the divisor. A fraction of a quantity is found by dividing by the denominator and multiplying by the numerator; either order can be used if handled accurately.

A reliable method

1. Convert mixed numbers before multiplying or dividing.

2. For division, keep the first fraction, change ÷ to ×, and invert only the second fraction.

3. Cancel factors across a product, never across a sum.

Worked example 1

Calculate (4/9) × (3/10).

STEP-BY-STEP SOLUTION

Cancel 3 with 9 to leave 1 and 3; cancel 4 with 10 to leave 2 and 5. The product is 2/(3 × 5) = 2/15.

Worked example 2

Calculate 1 1/2 ÷ 3/4.

STEP-BY-STEP SOLUTION

Convert to 3/2, then multiply by the reciprocal: (3/2) × (4/3) = 2. Two groups of three quarters fit into one and a half.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Division need not make a number smaller. Dividing 6 by 1/2 asks how many halves are in 6; there are 12. The size of the divisor determines whether the result grows or shrinks.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Find 3/8 of £64.

4.2 Calculate (5/12) × (9/10).

4.3 Calculate 7/8 ÷ 14/15.

4.4 Three fifths of a class is 18 pupils. How many pupils are in the class?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Simplify 54/72. [2 marks]

R2. Calculate 2 1/4 − 5/6. [3 marks]

R3. Calculate (3/7) × (14/15). [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Calculate 2 2/3 ÷ 4/5. [3 marks]

R5. A tank contains 48 litres and is 3/5 full. Find its capacity. [3 marks]

R6. A walker completes 2/5 of a route in the morning and 1/3 in the afternoon. What fraction remains? [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Meaning, equivalence and comparison

1.1 Simplify 18/30.

Divide both parts by 6: 18/30 = 3/5.

1.2 Write 3/7 with denominator 42.

Multiply both parts by 6: 18/42.

1.3 Order 2/3, 3/4 and 5/6, smallest first.

Use twelfths: 8/12, 9/12, 10/12. Therefore 2/3 < 3/4 < 5/6.

1.4 Explain why 4/10 = 2/5.

Each pair of tenths forms one fifth. Dividing numerator and denominator by 2 gives the same amount in larger units.

Lesson 2 • Improper fractions and mixed numbers

2.1 Convert 4 3/8 to an improper fraction.

(4 × 8 + 3)/8 = 35/8.

2.2 Convert 38/9 to a mixed number.

38 = 4 × 9 + 2, so 4 2/9.

2.3 Write −1 3/4 as a single fraction.

−(1 + 3/4) = −7/4.

2.4 Which is larger, 2 1/3 or 19/8?

2 1/3 = 7/3 = 56/24; 19/8 = 57/24. Therefore 19/8 is larger by 1/24.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Adding and subtracting fractions

3.1 Calculate 2/7 + 3/7.

The units already match: 5/7.

3.2 Calculate 5/6 − 1/4.

10/12 − 3/12 = 7/12.

3.3 Calculate 1 2/3 + 2 3/5.

5/3 + 13/5 = 25/15 + 39/15 = 64/15 = 4 4/15.

3.4 Calculate −3/4 + 1/6.

−9/12 + 2/12 = −7/12.

Lesson 4 • Multiplying, dividing and finding a fraction of an amount

4.1 Find 3/8 of £64.

64 ÷ 8 × 3 = £24.

4.2 Calculate (5/12) × (9/10).

45/120 = 3/8, or cancel before multiplying.

4.3 Calculate 7/8 ÷ 14/15.

(7/8) × (15/14) = 15/16.

4.4 Three fifths of a class is 18 pupils. How many pupils are in the class?

One fifth is 18 ÷ 3 = 6. Five fifths is 6 × 5 = 30 pupils.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Simplify 54/72. [2 marks]

HCF = 18, so 54/72 = 3/4. [1 for a common-factor step; 1 for simplest form.]

R2. Calculate 2 1/4 − 5/6. [3 marks]

9/4 − 5/6 = 27/12 − 10/12 = 17/12 = 1 5/12. [1 for conversion; 1 for common denominator; 1 for answer.]

R3. Calculate (3/7) × (14/15). [2 marks]

42/105 = 2/5. [1 for correct product/cancellation; 1 for simplified answer.]

R4. Calculate 2 2/3 ÷ 4/5. [3 marks]

8/3 × 5/4 = 10/3 = 3 1/3. [1 for mixed conversion; 1 for reciprocal method; 1 for result.]

R5. A tank contains 48 litres and is 3/5 full. Find its capacity. [3 marks]

One fifth = 48 ÷ 3 = 16 litres; capacity = 16 × 5 = 80 litres. [1 for dividing by 3; 1 for multiplying by 5; 1 for 80 litres.]

R6. A walker completes 2/5 of a route in the morning and 1/3 in the afternoon. What fraction remains? [3 marks]

Completed = 6/15 + 5/15 = 11/15. Remaining = 1 − 11/15 = 4/15. [1 for common denominator; 1 for total; 1 for remainder.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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