GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 02

02

Factors, multiples and primes

Shared GCSE foundations

Factors describe how a whole number can be built by multiplication. Multiples describe repeated groups. Understanding both helps with fractions, repeating events and efficient calculations. This guide explains why prime factorisation works, rather than treating factor trees as a drawing exercise.

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

Prime building blocks: $2^3 \times 3^2$

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Know multiplication and division facts and the meaning of whole numbers.

Your route through this notebook

1. Factors, multiples and divisibility
2. Prime numbers and systematic checking
3. Prime factorisation and index notation
4. Highest common factors and lowest common multiples

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Factors, multiples and divisibility

A factor divides a whole number exactly, leaving no remainder. Factors occur in pairs: because 3 × 8 = 24, both 3 and 8 are factors of 24. A multiple is the result of multiplying a number by a whole number. Positive multiples of 6 include 6, 12, 18 and 24.

A positive integer has finitely many positive factors but infinitely many positive multiples. Divisibility tests help identify factors quickly. A number is divisible by 2 if its last digit is even, by 5 if it ends in 0 or 5, and by 10 if it ends in 0. For 3 or 9, examine the sum of its digits. A test tells you whether division is exact; it does not give the quotient.

A reliable method

1. List factor pairs in increasing order of the first factor.

2. Stop when the pairs would repeat; include a square-root factor only once.

3. For divisibility by 3 or 9, add digits and test that sum.

Worked example 1

List all positive factors of 36.

STEP-BY-STEP SOLUTION

Pairs are 1 × 36, 2 × 18, 3 × 12, 4 × 9 and 6 × 6. The factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Worked example 2

Is 4,725 divisible by 3, 5 and 9?

STEP-BY-STEP SOLUTION

The digit sum is 4 + 7 + 2 + 5 = 18, divisible by both 3 and 9. The final digit is 5, so the number is divisible by 5 as well.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“8 is a multiple of 24” reverses the relationship. Since $24 = 3 \times 8$, 8 is a factor of 24 and 24 is a multiple of 8. Always identify which number divides the other.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 List all positive factors of 28.

1.2 Write the first five positive multiples of 7.

1.3 Is 5,148 divisible by 3 and by 9?

1.4 Explain why 9 is a factor of 63.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Prime numbers and systematic checking

A prime number is a positive integer with exactly two positive factors: 1 and itself. A composite number has more than two. The number 1 is neither prime nor composite because it has only one positive factor. The number 2 is the only even prime: every larger even number has 2 as an additional factor.

To test whether a number is prime, try prime divisors up to its square root. If a composite number had both factors greater than its square root, their product would be greater than the number. Therefore one factor must be at most the square root. This explains why a finite set of checks is sufficient.

A reliable method

1. Reject numbers below 2 when testing for primality.

2. Try the primes 2, 3, 5, 7, 11 and so on, only as far as the square root.

3. Finding one exact divisor proves a number composite; a few failed checks alone may not prove it prime.

Worked example 1

Show that 29 is prime.

STEP-BY-STEP SOLUTION

The square root of 29 is between 5 and 6. Test the primes 2, 3 and 5. It is odd, has digit sum 11 and does not end in 0 or 5. None divides it, so 29 is prime.

Worked example 2

Is 91 prime?

STEP-BY-STEP SOLUTION

Its square root is between 9 and 10. Test 2, 3, 5 and 7. Since $91 = 7 \times 13$, it is composite.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An odd number need not be prime: 9, 15 and 21 are counterexamples. A counterexample disproves a claim that is supposed to hold for every number.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Identify the primes: 1, 2, 17, 21, 27, 31.

2.2 Show that 47 is prime.

2.3 Find a factor proving that 119 is composite.

2.4 Give an example disproving “the sum of two primes is always odd”.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Prime factorisation and index notation

Prime factorisation writes a whole number as a product of primes. You may split a number using different factor pairs, but fully breaking down every composite factor produces the same collection of primes, apart from their order. A factor tree records the splitting; repeated division records the same reasoning in another layout.

Index notation compresses repeated multiplication: $2 \times 2 \times 2 \times 3$ means $2^3 \times 3$. The index counts copies of a factor; it does not multiply the base by the index. Prime factorisation will later let us compare the building blocks of two numbers without listing every factor or multiple.

A reliable method

1. Split the number into any convenient factor pair.

2. Continue until every factor at the ends is prime.

3. Collect equal prime factors using powers, then multiply back to check.

Worked example 1

Express 180 as a product of prime factors.

STEP-BY-STEP SOLUTION

$180 = 18 \times 10 = (2 \times 3 \times 3) \times (2 \times 5)$. Collecting gives $2^2 \times 3^2 \times 5$. Check: $4 \times 9 \times 5 = 180$.

Worked example 2

A number is $2^3 \times 3 \times 5^2$. Find the number.

STEP-BY-STEP SOLUTION

Evaluate the powers first: $8 \times 3 \times 25 = 24 \times 25 = 600$. The notation describes multiplication, not addition.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A factor tree ending in 2, 6 and 7 is unfinished because 6 is composite. Replace 6 by 2 × 3. The final factors must all be prime, and none should be lost or counted twice.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find the prime factorisation of 72.

3.2 Find the prime factorisation of 150.

3.3 Evaluate $2^4 \times 3^2$.

3.4 A pupil writes $84 = 2^2 \times 21$ as a prime factorisation. Complete it.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Highest common factors and lowest common multiples

The highest common factor (HCF) is the largest positive integer that divides both numbers exactly. It helps when splitting quantities into the largest identical groups without leftovers. The lowest common multiple (LCM) is the smallest positive number that is a multiple of both. It helps when repeating events must coincide.

With prime factorisations, the HCF contains only shared prime factors, using the smaller power of each. The LCM contains every required prime factor, using the larger power of each. Think of HCF as the largest common building kit and LCM as the smallest kit capable of containing either original number.

A reliable method

1. Decide whether the situation asks for dividing into groups or matching repeated cycles.

2. Factorise both numbers into primes.

3. For HCF take shared primes with minimum powers; for LCM take all primes with maximum powers.

Worked example 1

Find the HCF and LCM of 24 and 60.

STEP-BY-STEP SOLUTION

$24 = 2^3 \times 3$ and $60 = 2^2 \times 3 \times 5$. HCF $= 2^2 \times 3 = 12$. LCM $= 2^3 \times 3 \times 5 = 120$.

Worked example 2

Lights flash every 18 seconds and 24 seconds. They flash together now. When next?

STEP-BY-STEP SOLUTION

The next coincidence is a common multiple. $18 = 2 \times 3^2$ and $24 = 2^3 \times 3$. LCM $= 2^3 \times 3^2 = 72$ seconds.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A question mentioning “largest” does not always mean HCF, and “smallest” alone does not establish LCM. Identify the mathematical relationship. In a two-number check, HCF × LCM equals the product of the original positive integers.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Find HCF(18, 30).

4.2 Find LCM(12, 20).

4.3 Make the greatest number of identical packs from 48 pencils and 60 pens, using everything. Find the number and contents of each pack.

4.4 Two alarms sound every 15 and 20 minutes. They sound together at 09:00. When next?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. List all positive factors of 45. [2 marks]

R2. Is 143 prime? Justify. [2 marks]

R3. Write 252 as a product of prime factors. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Find the HCF and LCM of 36 and 84. [4 marks]

R5. Ribbons are 96 cm and 144 cm long. Cut equal pieces of the greatest possible length without waste. Find the length and total number of pieces. [3 marks]

R6. Buses leave every 16 and 24 minutes. Both leave at 08:10. Find the next common departure. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Factors, multiples and divisibility

1.1 List all positive factors of 28.

Factor pairs 1 × 28, 2 × 14 and 4 × 7 give 1, 2, 4, 7, 14, 28.

1.2 Write the first five positive multiples of 7.

7, 14, 21, 28, 35.

1.3 Is 5,148 divisible by 3 and by 9?

Digit sum = 18. Since 18 is divisible by both 3 and 9, so is 5,148.

1.4 Explain why 9 is a factor of 63.

63 ÷ 9 = 7 exactly, or equivalently 9 × 7 = 63.

Lesson 2 • Prime numbers and systematic checking

2.1 Identify the primes: 1, 2, 17, 21, 27, 31.

2, 17 and 31 are prime. 1 has one factor; 21 = 3 × 7 and 27 = 3 × 9.

2.2 Show that 47 is prime.

Its square root is below 7. Test 2, 3 and 5: it is odd, digit sum 11, and final digit 7. All fail, so 47 is prime.

2.3 Find a factor proving that 119 is composite.

119 = 7 × 17. The factor 7 is neither 1 nor 119.

2.4 Give an example disproving “the sum of two primes is always odd”.

3 + 5 = 8, which is even. Any valid pair of odd primes is a counterexample.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Prime factorisation and index notation

3.1 Find the prime factorisation of 72.

$72 = 8 \times 9 = 2^3 \times 3^2$.

3.2 Find the prime factorisation of 150.

$150 = 15 \times 10 = 2 \times 3 \times 5^2$.

3.3 Evaluate $2^4 \times 3^2$.

$16 \times 9 = 144$.

3.4 A pupil writes $84 = 2^2 \times 21$ as a prime factorisation. Complete it.

Since $21 = 3 \times 7$, the complete result is $2^2 \times 3 \times 7$.

Lesson 4 • Highest common factors and lowest common multiples

4.1 Find HCF(18, 30).

$18 = 2 \times 3^2$ and $30 = 2 \times 3 \times 5$. Shared minimum powers give $2 \times 3 = 6$.

4.2 Find LCM(12, 20).

$12 = 2^2 \times 3$ and $20 = 2^2 \times 5$. Maximum powers give $2^2 \times 3 \times 5 = 60$.

4.3 Make the greatest number of identical packs from 48 pencils and 60 pens, using everything. Find the number and contents of each pack.

HCF(48, 60) = 12 packs. Each contains $48 \div 12 = 4$ pencils and $60 \div 12 = 5$ pens.

4.4 Two alarms sound every 15 and 20 minutes. They sound together at 09:00. When next?

LCM(15, 20) = 60 minutes. They next coincide at 10:00.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. List all positive factors of 45. [2 marks]

1, 3, 5, 9, 15, 45. [1 for correct factor pairs; 1 for a complete list.]

R2. Is 143 prime? Justify. [2 marks]

No: 143 = 11 × 13. [1 for a non-trivial divisor; 1 for the justified conclusion.]

R3. Write 252 as a product of prime factors. [3 marks]

252 = 4 × 63 = $2^2 \times 3^2 \times 7$. [1 for a valid split; 1 for complete prime factors; 1 for correct powers.]

R4. Find the HCF and LCM of 36 and 84. [4 marks]

36 = $2^2 \times 3^2$; 84 = $2^2 \times 3 \times 7$. HCF = 12 and LCM = 252. [1 for each factorisation; 1 for each answer.]

R5. Ribbons are 96 cm and 144 cm long. Cut equal pieces of the greatest possible length without waste. Find the length and total number of pieces. [3 marks]

HCF = 48 cm. The ribbons make 2 and 3 pieces, so 5 altogether. [1 for HCF method; 1 for 48 cm; 1 for 5 pieces.]

R6. Buses leave every 16 and 24 minutes. Both leave at 08:10. Find the next common departure. [3 marks]

LCM = 48 minutes, so 08:58. [1 for a common-multiple method; 1 for 48; 1 for the time.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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