GCSE ONLINE TUTORING

NUMBER / TOPIC GUIDE 01

01

Number foundations and calculations

Shared GCSE foundations

Numbers describe amounts, positions and changes. This guide builds the meaning behind written calculations so that you can choose a method, explain it and check whether an answer is sensible. Work without a calculator first; then use one to check selected answers.

Equal steps represent equal changes.



UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Read whole numbers and recall basic multiplication facts. If a fact is unfamiliar, build it by repeated addition rather than guessing.

Your route through this notebook

1. Place value, decimals and ordering
2. Negative numbers and directed change
3. Written methods and inverse checks
4. Order of operations and calculator discipline

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Place value, decimals and ordering

Our number system uses powers of ten. Moving one place to the left makes the value of a digit ten times as large; moving right makes it one tenth as large. In 4,507.06, the 5 means five hundreds, while the 6 means six hundredths. Zero holds a place even when that place contributes nothing to the total.

To compare decimals, align the decimal points and compare from the left. Extra zeros at the end of a decimal do not change its value: 0.6 = 0.60. A longer decimal is not automatically larger. Multiplying by ten changes every digit’s place value; it is safer to describe digits moving than to say that a decimal point moves.

A reliable method

1. Write numbers in columns headed thousands, hundreds, tens, units, tenths and hundredths.

2. For ordering, pad decimal places with zeros, then compare the first place that differs.

3. Use < for less than and > for greater than. The open side faces the larger number.

Worked example 1

Order 0.7, 0.07 and 0.707 from smallest to largest.

STEP-BY-STEP SOLUTION

Write 0.700, 0.070 and 0.707. Compare tenths first, then hundredths and thousandths. Therefore 0.07 < 0.7 < 0.707.

Worked example 2

Work out 3.046 × 100.

STEP-BY-STEP SOLUTION

Each digit moves two place-value columns to the left. The 3 becomes 300, the 4 becomes 4, and the 6 becomes 0.6. The answer is 304.6.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A pupil says 0.48 is greater than 0.5 because 48 is greater than 5. The comparison treats hundredths and tenths as the same unit. Rewrite 0.5 as 0.50: 48 hundredths is less than 50 hundredths.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 What is the value of the digit 7 in 27,406.18?

1.2 Order 2.05, 2.5, 2.005 and 2.055, smallest first.

1.3 Calculate 0.063 × 1,000 and 84.2 ÷ 100.

1.4 Explain why 4.090 and 4.09 are equal.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Negative numbers and directed change

A negative number lies below zero on a number line. Further left means smaller, so $-8 < -3$ even though $8 > 3$. The distance of -8 from zero is 8. A minus sign can name a negative number or tell us to subtract; those jobs must be distinguished.

Adding a negative change moves left. Subtracting a negative reverses that change, so it moves right. For multiplication and division, equal signs give a positive result and different signs give a negative result. These sign rules describe products and quotients; they do not mean that every calculation with two negative numbers is positive.

A reliable method

1. For addition and subtraction, start at the first number and describe the change.
2. Rewrite subtracting a negative as adding the corresponding positive.
3. For multiplication or division, decide the sign, then calculate with the sizes.

Worked example 1

The temperature is -4°C and rises by 9°C. What is the new temperature?

STEP-BY-STEP SOLUTION

Start at -4. A rise of 4 reaches zero and the remaining rise of 5 reaches 5. The new temperature is 5°C.

Worked example 2

Calculate $-6 - (-10)$, then $(-6) \times (-10)$.

STEP-BY-STEP SOLUTION

Subtracting -10 means adding 10, giving 4. The product has two negative factors, so it is positive: $6 \times 10 = 60$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Compare $-3 + (-5) = -8$ with $(-3) \times (-5) = 15$. “Two negatives make a positive” is incomplete advice: first identify the operation. For division, remember that division by zero is undefined.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Calculate $-7 + 12$.

2.2 Calculate $5 - (-8)$.

2.3 Calculate $(-4) \times 7$ and $(-36) \div (-6)$.

2.4 A balance is $-£18$. A payment of £25 arrives and £11 is then spent. Find the balance.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Written methods and inverse checks

An efficient calculation method keeps track of the value of every digit. In addition and subtraction, align place values, including decimal points. Exchange one unit of a larger place for ten units of the next smaller place when needed. Multiplication can be broken into partial products, and division can be understood as finding a missing factor.

An inverse operation reverses a calculation. Addition checks subtraction, and multiplication checks division. Estimation supplies a different kind of check: it can reveal a misplaced decimal point even when the written steps appear neat. A calculator is useful for checking, but the answer still needs interpreting in context.

A reliable method

1. Estimate the size of the answer before starting.

2. Keep place-value columns aligned; show partial products or exchanges.

3. Check with an inverse operation and interpret any remainder.

Worked example 1

Calculate 3.6 × 24.

STEP-BY-STEP SOLUTION

Split 24 into 20 + 4. Then 3.6 × 20 = 72 and 3.6 × 4 = 14.4. Add to get 86.4. The estimate 4 × 24 = 96 supports this size.

Worked example 2

Divide 875 by 12 and interpret it as seats in 12-seat vehicles.

STEP-BY-STEP SOLUTION

12 × 72 = 864, leaving 11. Thus 875 ÷ 12 = 72 remainder 11. To seat all 875 people, 73 vehicles are needed; the remainder cannot be ignored.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The same numerical quotient can need different answers in context. A remainder may be reported, converted into a fraction, or require rounding up. Read what the quantity represents before deciding.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Calculate 48.7 + 6.85.

3.2 Calculate 70.02 − 8.76.

3.3 Calculate 2.4 × 35.

3.4 Pack 157 items into boxes holding at most 8. How many boxes are needed?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Order of operations and calculator discipline

A mathematical expression has an agreed reading order. Deal with brackets, then powers and roots, then multiplication and division from left to right, then addition and subtraction from left to right. Multiplication does not always come before division; they have equal priority. The same is true of addition and subtraction.

A fraction bar acts as a grouping symbol: calculate the whole numerator and the whole denominator before dividing. On a calculator, use brackets to reproduce that structure. The expressions -3^2 and $(-3)^2$ differ because a power acts on its base: in the first expression the base is 3, while in the second it is -3.

A reliable method

1. Mark grouped quantities, including the top and bottom of a fraction.

2. Evaluate powers before ordinary multiplication.

3. Write an intermediate line so each operation can be checked.

Worked example 1

Calculate $18 - 3 \times (2 + 1)^2$.

STEP-BY-STEP SOLUTION

Brackets give 3, then $3^2 = 9$. Multiply $3 \times 9 = 27$. Finally $18 - 27 = -9$.

Worked example 2

Calculate $24 \div 6 \times 2$ and compare -4^2 with $(-4)^2$.

STEP-BY-STEP SOLUTION

Division and multiplication go left to right: $24 \div 6 \times 2 = 4 \times 2 = 8$. Also $-4^2 = -16$, while $(-4)^2 = 16$.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A calculator entry such as 8 + 4 ÷ 2 does not represent (8 + 4) ÷ 2. The first equals 10 and the second equals 6. An accurate machine can still answer a different question from the one intended.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Evaluate $7 + 5 \times 3$.

4.2 Evaluate $(7 + 5) \times 3$.

4.3 Evaluate $30 \div 5 \times 2 - 3$.

4.4 Evaluate $(9 + 15) \div (7 - 3)$, and explain the brackets.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Order −0.6, −0.06, 0.006 and −0.606 from smallest to largest. [2 marks]

R2. Work out 6.04 × 100 − 0.8 × 10. [2 marks]

R3. A lift starts on floor −3, rises 8 floors and falls 6. Where does it finish? [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. A trip has 286 passengers. Each coach seats 49. How many coaches are needed? [3 marks]

R5. Calculate $40 - 2 \times (3 + 1)^2$. [3 marks]

R6. A pupil writes $-5^2 = 25$. Explain and correct the error. [2 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Place value, decimals and ordering

1.1 What is the value of the digit 7 in 27,406.18?

The 7 is in the thousands column, so its value is 7,000.

1.2 Order 2.05, 2.5, 2.005 and 2.055, smallest first.

Use 2.050, 2.500, 2.005 and 2.055: $2.005 < 2.05 < 2.055 < 2.5$.

1.3 Calculate $0.063 \times 1{,}000$ and $84.2 \div 100$.

Moving digits three columns left gives 63. Moving digits two columns right gives 0.842.

1.4 Explain why 4.090 and 4.09 are equal.

The final zero adds zero thousandths. Both numbers are 4 units and 9 hundredths.

Lesson 2 • Negative numbers and directed change

2.1 Calculate $-7 + 12$.

A rise of 7 reaches zero; 5 remains. Answer: 5.

2.2 Calculate $5 - (-8)$.

Subtracting -8 is adding 8: $5 + 8 = 13$.

2.3 Calculate $(-4) \times 7$ and $(-36) \div (-6)$.

Different signs give -28. Equal signs give $+6$.

2.4 A balance is $-£18$. A payment of £25 arrives and £11 is then spent. Find the balance.

$-18 + 25 - 11 = 7 - 11 = -4$. The final balance is $-£4$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Written methods and inverse checks

3.1 Calculate 48.7 + 6.85.

Write 48.70 + 6.85 and align the decimal points. Answer: 55.55.

3.2 Calculate 70.02 − 8.76.

Exchange across place-value columns, or use 70.02 − 8 − 0.76. Answer: 61.26. Check: 61.26 + 8.76 = 70.02.

3.3 Calculate 2.4 × 35.

2.4 × (30 + 5) = 72 + 12 = 84.

3.4 Pack 157 items into boxes holding at most 8. How many boxes are needed?

157 ÷ 8 = 19 remainder 5. Nineteen full boxes leave 5 items, so 20 boxes are needed.

Lesson 4 • Order of operations and calculator discipline

4.1 Evaluate 7 + 5 × 3.

Multiply first: 7 + 15 = 22.

4.2 Evaluate (7 + 5) × 3.

Brackets first: 12 × 3 = 36.

4.3 Evaluate 30 ÷ 5 × 2 − 3.

Work left to right at equal priority: 6 × 2 − 3 = 12 − 3 = 9.

4.4 Evaluate (9 + 15) ÷ (7 − 3), and explain the brackets.

The grouped numerator is 24 and denominator is 4, so 24 ÷ 4 = 6. Both groups must be calculated before the division.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Order −0.6, −0.06, 0.006 and −0.606 from smallest to largest. [2 marks]

−0.606 < −0.6 < −0.06 < 0.006. For the negative values, the larger distance below zero means the smaller number. [1 for correct negative order; 1 for complete order.]

R2. Work out 6.04 × 100 − 0.8 × 10. [2 marks]

604 − 8 = 596. [1 for the two products; 1 for the result.]

R3. A lift starts on floor −3, rises 8 floors and falls 6. Where does it finish? [2 marks]

−3 + 8 − 6 = −1. [1 for a correct calculation; 1 for floor −1.]

R4. A trip has 286 passengers. Each coach seats 49. How many coaches are needed? [3 marks]

49 × 5 = 245, while 49 × 6 = 294. Five coaches are insufficient; 6 are needed. [1 for division/comparison; 1 for remainder/capacity; 1 for rounding up.]

R5. Calculate $40 - 2 \times (3 + 1)^2$. [3 marks]

$(3 + 1)^2 = 16$; $2 \times 16 = 32$; $40 - 32 = 8$. [1 for each stage.]

R6. A pupil writes $-5^2 = 25$. Explain and correct the error. [2 marks]

The square applies to 5 before the leading minus: $-5^2 = -25$. To square negative five, write $(-5)^2 = 25$. [1 for explanation; 1 for correction.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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