GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 52

52

Vectors

Shared coordinate vectors with Higher proofs

Represent directed movement, combine routes and use position vectors to describe points. Higher lessons develop internal division, parallelism and collinearity proofs, keeping the direction and scalar factor explicit.

AB = position of B − position of A

Direction matters; a vector is not just a length.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

INDEPENDENT GCSE MATHS • WEBSITE EDITION • NOVEMBER 2026

© 2026 GCSE Online Tutoring. All rights reserved.

GCSE ONLINE TUTORING / ABOUT THIS BOOK

Your learning guide

© 2026 GCSE Online Tutoring. All rights reserved.

Free for personal and household educational use. Tutors and schools must obtain permission before copying or distributing these books to learners. Resale, paid redistribution, removal of branding and uploading to third-party websites are prohibited without written permission. For permissions, visit www.gcseonlinetutoring.uk.

Independent educational resource

All explanations, examples, diagrams, questions and marking guidance are independently written. No exam-board questions or mark schemes are reproduced. This book is not affiliated with, approved by or endorsed by any awarding organisation.

Before you begin

Work with coordinates, signed numbers, fractions and algebraic expressions.

Your route through this notebook

1. Directed vectors and components
2. Adding vectors and scalar multiples
3. Position vectors, midpoints and ratios — Higher
4. Parallelism, collinearity and geometric proof — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 1 / UNDERSTAND

Directed vectors and components

A vector has magnitude and direction. A displacement vector with components (a,b), also written as a column vector, moves a horizontally and b vertically. Equal vectors have identical components even if drawn in different places. Reversing the arrow changes the vector's sign.

For points $A(x_1,y_1)$ and $B(x_2,y_2)$, vector $AB = (x_2 - x_1, y_2 - y_1)$. This is an endpoint difference, not a midpoint or a point's own coordinates. The magnitude is $\sqrt{(a^2 + b^2)}$ by Pythagoras. Negative components indicate direction, while magnitude is non-negative.

A reliable method

1. Subtract start coordinates from finish coordinates.
2. Keep horizontal and vertical components in order.
3. Use a positive square root for magnitude.

Worked example 1

$A = (-2,3), B = (4,-5)$. Find AB and its magnitude.

STEP-BY-STEP SOLUTION

$AB = (6,-8)$. Magnitude $= \sqrt{(36 + 64)} = 10$.

Worked example 2

If $AB = (3,7)$, find BA.

STEP-BY-STEP SOLUTION

Reverse direction: $BA = (-3,-7)$.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Two equal-length arrows are not necessarily equal vectors: they must also point in the same direction. Opposite arrows have equal magnitude but negative vectors.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find AB for A(1,−4),B(6,8).

1.2 Find magnitude of (5,12).

1.3 Reverse vector (−4,9).

1.4 Are (2,3) and (−2,−3) equal vectors?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 2 / UNDERSTAND

Adding vectors and scalar multiples

Vector addition describes consecutive movement. Adding components gives the overall displacement, and reversing a route contributes a negative vector. For any points A,B,C, AB + BC = AC. The order of component addition does not change the resultant, although a route sketch may look different.

Multiplying by a scalar k changes magnitude by |k|. A positive k preserves direction; a negative k reverses it. Non-zero scalar multiples are parallel. A zero vector has no unique direction, so do not use it as evidence of a particular parallel line. Expressions in vectors a,b can be simplified by collecting coefficients, just as with algebraic terms.

A reliable method

1. Join routes head-to-tail or add corresponding components.

2. Use negatives for reversed segments.

3. For a scalar multiple, multiply every component.

Worked example 1

Let a = (2,−1),b = (−3,4). Find 3a + 2b.

STEP-BY-STEP SOLUTION

3a = (6,−3),2b = (−6,8), so sum (0,5).

Worked example 2

If AB = a and AC = b, express BC.

STEP-BY-STEP SOLUTION

Travel B to A to C: BC = BA + AC = −a + b = b − a.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The displacement for a closed route is zero. AB + BC + CA = 0 because the endpoint is the starting point.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Add (4,−2) and (−1,7).

2.2 Find −3(2,−5).

2.3 Simplify 4a − 2b + 3b − a.

2.4 If AB = 2a and BC = 3b, express AC.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 3 / UNDERSTAND

Position vectors, midpoints and ratios — Higher

A position vector gives a point’s displacement from a fixed origin O. If OA = a and OB = b, then AB = b − a. The midpoint M has position (a + b)/2 because it is reached from A by half of AB.

For an internal division AM:MB = m:n, the fraction of the journey from A to B is m/(m + n). Thus OM = a + [m/(m + n)](b − a). Check the starting point and ratio order: a larger AM fraction places M closer to B. This lesson uses positive internal ratios; external division requires a different interpretation.

A reliable method

1. Use endpoint difference b − a for AB.

2. Take the specified fraction along that directed segment.

3. Add the starting position vector to recover the point’s position.

Worked example 1

OA = a,OB = b. Find midpoint position OM.

STEP-BY-STEP SOLUTION

OM = a + (b − a)/2 = (a + b)/2.

Worked example 2

AP:PB = 2:3. Find OP in terms of a,b.

STEP-BY-STEP SOLUTION

AP = (2/5)(b − a). OP = a + AP = (3/5)a + (2/5)b.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A division vector AP is not the same as the position OP. Add OA when the answer is measured from the origin.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A(2,4),B(8,10). Find midpoint.

3.2 OA = a,OB = b and AP:PB = 1:2. Find OP.

3.3 A(0,0),B(10,15). AP:PB = 2:3. Find P.

3.4 A(1,2),B(7,5). Find point P one third from A towards B.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 4 / UNDERSTAND

Parallelism, collinearity and geometric proof — Higher

Non-zero vectors that are scalar multiples are parallel. To prove three points collinear, show vectors along segments sharing a point are scalar multiples; the shared point ensures they lie on the same line, not merely parallel distinct lines. The scalar can be negative or greater than one when points lie on extensions.

Use position vectors to prove properties independent of a particular coordinate picture. In a parallelogram with adjacent position vectors a,b from O, the opposite vertex has position a + b. Both diagonals then have midpoint (a + b)/2. State what the algebra establishes and finish with the geometric conclusion.

A reliable method

1. Express the relevant directed segments from positions.

2. Simplify to reveal a scalar multiple or equal midpoint.

3. State the shared-point and non-zero conditions needed for the conclusion.

Worked example 1

Show A(1,2),B(3,5),C(7,11) are collinear.

STEP-BY-STEP SOLUTION

AB = (2,3),AC = (6,9) = 3AB. The vectors share A and are non-zero, so all points lie on one line.

Worked example 2

Prove diagonals of parallelogram OACB bisect each other, with OA = a,OB = b,OC = a + b.

STEP-BY-STEP SOLUTION

Midpoint of OC has position (a + b)/2. Midpoint of AB also has (a + b)/2. Equal midpoint proves the diagonals bisect each other.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Parallel vectors alone do not identify the same line. The segment vectors need a shared endpoint or an additional point-on-line argument for collinearity.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Are (2,5) and (6,15) parallel?

4.2 Are (2,5) and (4,12) parallel?

4.3 AB = 2a + 4b and AC = 5a + 10b, with AB ≠ 0. Prove collinearity.

4.4 If AB = v and AC = −2v with v ≠ 0, describe point positions.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

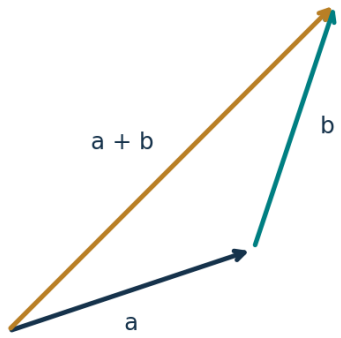
© 2026 GCSE Online Tutoring. All rights reserved.

VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

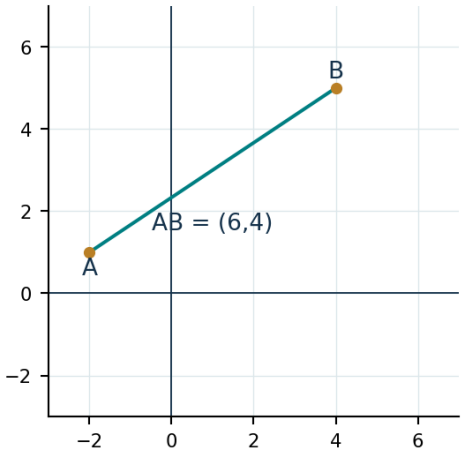
Read the geometry

Vectors describe directed routes and positions

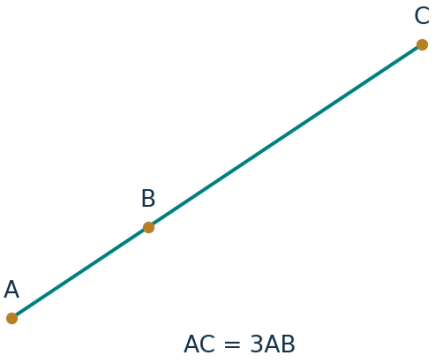
Add routes head-to-tail



Endpoint difference



Shared endpoint plus scalar multiple



Internal ratio AP:PB = 2:3



OP = (3/5)a + (2/5)b

WHAT THE FIGURES ESTABLISH

The overall route is a + b. Subtract start coordinates from finish coordinates for a displacement. For AP:PB = 2:3, P is two fifths of the journey from A to B, so add that fraction of b − a to a. A scalar multiple proves collinearity here because the non-zero segment vectors share A.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

© 2026 GCSE Online Tutoring. All rights reserved.

MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find AB from A(−3,1) to B(5,7). [2 marks]

R2. Find magnitude of (8,6). [2 marks]

R3. Let a = (1,3),b = (4,−2). Find 2a − b. [2 marks]

R4. If OA = a,OB = b, express BA. [1 mark]

© 2026 GCSE Online Tutoring. All rights reserved.

MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Higher: OA = a,OB = b,AM:MB = 3:2. Find OM. [3 marks]

R6. Higher: find midpoint of A(−2,5),B(8,−1). [2 marks]

R7. Higher: prove A(0,1),B(2,4),C(6,10) collinear. [3 marks]

R8. Higher: AB = 3p − 6q,AC = −p + 2q,AB ≠ 0. Describe the relationship of A,B,C. [3 marks]

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Directed vectors and components

1.1 Find AB for A(1,−4),B(6,8).

(5,12).

1.2 Find magnitude of (5,12).

13.

1.3 Reverse vector (−4,9).

(4,−9).

1.4 Are (2,3) and (−2,−3) equal vectors?

No. They are negatives, with equal magnitude and opposite direction.

Lesson 2 • Adding vectors and scalar multiples

2.1 Add (4,−2) and (−1,7).

(3,5).

2.2 Find −3(2,−5).

(−6,15).

2.3 Simplify 4a − 2b + 3b − a.

3a + b.

2.4 If AB = 2a and BC = 3b, express AC.

2a + 3b.

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Position vectors, midpoints and ratios — Higher

3.1 A(2,4),B(8,10). Find midpoint.

(5,7).

3.2 OA = a,OB = b and AP:PB = 1:2. Find OP.

(2/3)a + (1/3)b.

3.3 A(0,0),B(10,15). AP:PB = 2:3. Find P.

P = (2/5)B = (4,6).

3.4 A(1,2),B(7,5). Find point P one third from A towards B.

P = (1,2) + (1/3)(6,3) = (3,3).

Lesson 4 • Parallelism, collinearity and geometric proof — Higher

4.1 Are (2,5) and (6,15) parallel?

Yes, the second is 3 times the first.

4.2 Are (2,5) and (4,12) parallel?

No. Component ratios 2 and 12/5 differ.

4.3 AB = 2a + 4b and AC = 5a + 10b, with AB ≠ 0. Prove collinearity.

AC = (5/2)AB; they share A, so A,B,C are collinear.

4.4 If AB = v and AC = −2v with v ≠ 0, describe point positions.

They are collinear with B and C on opposite sides of A; AC has twice the length of AB.

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find AB from A(−3,1) to B(5,7). [2 marks]

(8,6). [1 each component.]

R2. Find magnitude of (8,6). [2 marks]

√(64 + 36) = 10. [1 for squares; 1 for root.]

R3. Let a = (1,3),b = (4,−2). Find 2a − b. [2 marks]

(2,6) − (4,−2) = (−2,8). [1 for scaling; 1 for result.]

R4. If OA = a,OB = b, express BA. [1 mark]

a − b. [1 for direction.]

R5. Higher: OA = a,OB = b,AM:MB = 3:2. Find OM. [3 marks]

a + (3/5)(b − a) = (2/5)a + (3/5)b. [1 for fraction; 1 for start; 1 for simplification.]

R6. Higher: find midpoint of A(−2,5),B(8,−1). [2 marks]

(3,2). [1 each coordinate.]

R7. Higher: prove A(0,1),B(2,4),C(6,10) collinear. [3 marks]

AB = (2,3),AC = (6,9) = 3AB; shared A gives collinearity. [1 for differences; 1 for multiple; 1 for conclusion.]

R8. Higher: AB = 3p − 6q,AC = −p + 2q,AB ≠ 0. Describe the relationship of A,B,C. [3 marks]

AC = −AB/3. They are collinear, with B and C on opposite sides of A, and AB = 3AC in length. [1 for multiple; 1 for collinearity; 1 for order/length.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

© 2026 GCSE Online Tutoring. All rights reserved.