GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 51

51

Circle theorems and coordinate circles

Higher-level GCSE material

Use circle geometry to justify angles and tangents, then connect circles centred at the origin to coordinate equations. Every theorem needs its geometric conditions; sketches support reasoning but are not evidence by themselves.

$$x^2 + y^2 = r^2$$

Circle theorems require their stated configurations.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Use isosceles-triangle and parallel-line angle facts, Pythagoras and straight-line equations.

Your route through this notebook

1. Angles at the centre and in a semicircle
2. Same segment and cyclic quadrilaterals
3. Tangent-radius, equal tangents and alternate segments
4. Building circle-theorem proofs
5. Circle equations and tangent lines

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Angles at the centre and in a semicircle

For the same arc, the angle subtended at the centre is twice the angle subtended at the circumference. Choose the arc not containing the circumference vertex; in some configurations the relevant central angle is reflex. Identify the endpoints of the common chord before comparing angles.

An angle in a semicircle is 90°: the diameter subtends 180° at the centre, so its angle at the circumference is half of that. Conversely, a triangle whose vertices lie on a circle and whose one side is a diameter has a right angle opposite the diameter. Radii joined to a circumference point create isosceles triangles useful in explanations and proofs.

A reliable method

1. Identify the chord endpoints and the arc used by each angle.

2. Use centre angle = 2 × circumference angle for that arc.

3. For a diameter, use a 180° central angle to obtain a right angle.

Worked example 1

A central angle for an arc is 124°. Find a circumference angle subtending the same arc.

STEP-BY-STEP SOLUTION

The circumference angle is 124/2 = 62°.

Worked example 2

AB is a diameter and P is another point on the circle. If angle PAB = 37°, find angle PBA.

STEP-BY-STEP SOLUTION

Angle APB = 90° by the semicircle theorem. Thus PBA = 180 − 90 − 37 = 53°.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Two angles sharing chord endpoints may use different arcs. Do not halve an arbitrary central angle without checking where the circumference vertex lies.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 A circumference angle is 48°. Find the central angle subtending the same arc.

1.2 A matching central reflex angle is 240°. Find its circumference angle.

1.3 AB is a diameter. State angle ACB for C on the circle.

1.4 A triangle in a semicircle has an acute angle 26°. Find its other acute angle.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Same segment and cyclic quadrilaterals

Angles subtended by the same chord in the same segment are equal. The circumference vertices must lie in the same segment relative to the chord; angles on opposite sides generally sum to 180° instead. A cyclic quadrilateral has all four vertices on one circle. Its opposite interior angles sum to 180°.

An exterior angle of a cyclic quadrilateral equals the opposite interior angle, since both supplement the same adjacent interior angle. These conclusions depend on cyclicity. A general quadrilateral has angle sum 360° but need not have supplementary opposite angles. State which points lie on the circle when invoking the theorem.

A reliable method

1. Identify a shared chord or a cyclic quadrilateral.

2. Check whether angle vertices lie in the same or opposite segments.

3. Use equality or supplementation with a named reason.

Worked example 1

A,B,P,Q lie on a circle with P and Q in the same segment above chord AB. If APB = 57°, find AQB.

STEP-BY-STEP SOLUTION

Both subtend chord AB in the same segment, so AQB = 57°.

Worked example 2

ABCD is cyclic and angle A = 112°. Find angle C.

STEP-BY-STEP SOLUTION

Opposite angles in a cyclic quadrilateral total 180°, so C = 68°.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A quadrilateral drawn inside a circle is not necessarily cyclic unless every vertex is on the circumference. A vertex strictly inside the circle invalidates the theorem.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Cyclic opposite angles are $3x + 10$ and $2x + 20$. Find x.

2.2 A cyclic quadrilateral has one angle $83°$. Find its opposite.

2.3 An exterior angle of a cyclic quadrilateral is $126°$. Find the opposite interior angle.

2.4 Two circumference angles subtend the same chord from opposite segments. One is $72°$. Find the other.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Tangent-radius, equal tangents and alternate segments

A tangent is perpendicular to the radius at its point of contact. Two tangents drawn from the same external point have equal lengths. The line from that external point to the centre also bisects the angle between the tangents, as shown by congruent right triangles.

The alternate segment theorem states that the angle between a tangent and a chord equals the angle subtended by that chord in the alternate segment. Identify the tangent, contact point, chord and opposite-segment angle explicitly. A diagram often contains both an acute and obtuse tangent-chord angle; use the angle matching the chosen segment. These theorems combine with triangle totals and Pythagoras.

A reliable method

1. Mark a right angle where a radius meets a tangent.

2. Use equal lengths only for tangents from the same external point.

3. Match a tangent-chord angle to its alternate-segment angle.

Worked example 1

P is external, PA tangent at A, OA = 5 cm and OP = 13 cm. Find PA.

STEP-BY-STEP SOLUTION

OA is perpendicular to PA, so PA = $\sqrt{(13^2 - 5^2)} = 12$ cm.

Worked example 2

A tangent-chord angle is 38°. Find the corresponding alternate-segment angle.

STEP-BY-STEP SOLUTION

It is 38°, by the alternate segment theorem, using the specified opposite segment.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The equal-tangent theorem concerns complete segments from a single external point to the two contacts. It does not state that all tangent segments drawn anywhere have the same length.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A radius meets its tangent at angle x. Find x.

3.2 Tangents PA and PB have lengths $3x + 2$ and $5x - 8$. Find x.

3.3 A tangent-chord angle is 64°. Find its matching alternate-segment angle.

3.4 Two tangents from P have angle APB = 70°. Find the smaller central angle AOB.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Building circle-theorem proofs

A circle proof combines general facts in a stated configuration. Join the centre to relevant circumference points to create equal radii and isosceles triangles. Mark angles using variables and express one angle in terms of another. Finish with the specific required conclusion rather than a numerical example.

Some theorem proofs need separate diagram cases. A proof drawn for the centre lying inside an inscribed angle does not by itself cover every possible position. State the configuration you are proving. A short, complete proof of equal tangents can use RHS congruence and works for an external point with two tangent contacts.

A reliable method

1. Add useful radii or a shared line without assuming the conclusion.
2. State every equality with its reason.
3. Use congruence or angle sums to conclude the claimed relationship.

Worked example 1

Prove tangents PA and PB from the same external point P have equal lengths.

STEP-BY-STEP SOLUTION

OA = OB (radii), OP shared, and angles OAP = OBP = 90° (radius-tangent). Right triangles OAP and OBP are congruent by RHS. Corresponding sides PA and PB are equal.

Worked example 2

In cyclic ABCD, prove its exterior angle at A equals interior angle C.

STEP-BY-STEP SOLUTION

Interior A + C = 180° by cyclicity. Exterior A + interior A = 180° by a straight line. Subtract the shared interior A to get exterior A = C.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A proof should not rely on measuring the supplied diagram. Measurements can check a drawing, but exact geometric reasons establish the claim.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Why are OA and OB equal in one circle?

4.2 Which congruence criterion proves the equal-tangent result using OP?

4.3 Why is OP the hypotenuse in triangles OAP and OBP?

4.4 If equal tangent lengths are established, what follows about angles APO and OPB?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 5 / UNDERSTAND

Circle equations and tangent lines

A circle of radius r centred at the origin has equation $x^2 + y^2 = r^2$ by Pythagoras. A point lies on it exactly when its coordinates satisfy the equation. To find missing coordinates, solve the resulting square equation and consider both signs. A circle equation usually describes two y values for many x inputs, so it is not a single-valued function y of x over its full domain.

A tangent at a point on the circle is perpendicular to the radius through that point. Find the radius gradient and use the negative reciprocal when it is finite and non-zero. At a horizontal or vertical radius use the corresponding vertical or horizontal tangent. For an origin-centred circle and point (a,b) on it, the tangent equation can be written $ax + by = r^2$.

A reliable method

1. Read r from the square on the right side.

2. Check or calculate point coordinates using $x^2 + y^2 = r^2$.

3. Use the perpendicular radius or $ax + by = r^2$, checking the contact point.

Worked example 1

Find the tangent at (3,4) on $x^2 + y^2 = 25$.

STEP-BY-STEP SOLUTION

Radius gradient 4/3, tangent gradient −3/4. Equation $y - 4 = -3(x - 3)/4$, or $3x + 4y = 25$.

Worked example 2

Find tangents at (5,0) and (0,5) on that circle.

STEP-BY-STEP SOLUTION

At (5,0), the radius is horizontal, so tangent $x = 5$. At (0,5), radius is vertical, so tangent $y = 5$.

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LESSON 5 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The circle $x^2 + y^2 = 49$ has radius 7, not 49. The right-hand side is the squared distance from the origin.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

5.1 State radius of $x^2 + y^2 = 81$.

5.2 Does (5,12) lie on $x^2 + y^2 = 169$?

5.3 On $x^2 + y^2 = 100$, find y when $x = 6$.

5.4 Find tangent at $(-3,4)$ on $x^2 + y^2 = 25$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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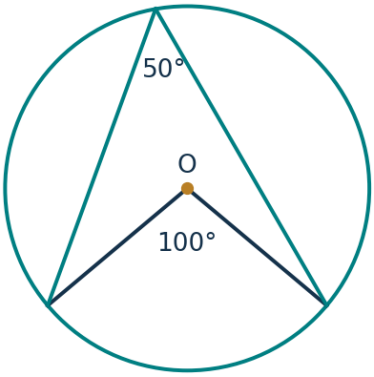
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

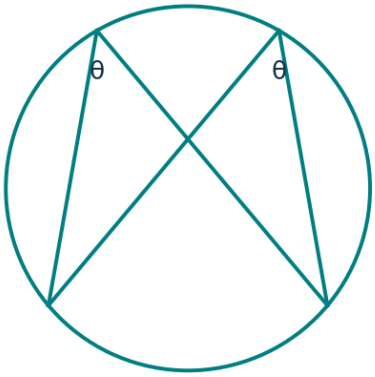
Read the geometry • 1

Circle theorems refer to specific chords and arcs

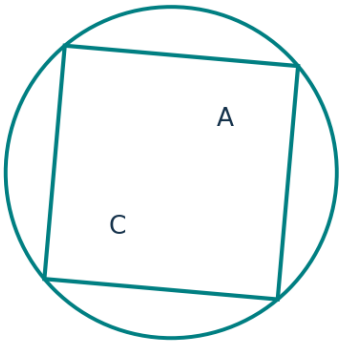
Centre angle is twice circumference angle



Same segment

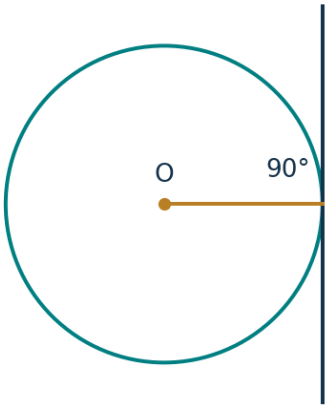


Cyclic opposite angles



A + C = 180°

Radius meets tangent at 90°



WHAT THE FIGURES ESTABLISH

The first angles use the same chord and lower arc: the 100° centre angle corresponds to 50° at the circumference. Same-segment angles share a chord with their vertices on the same side. Opposite angles supplement only when the quadrilateral is cyclic. The radius-tangent right angle supplies the right triangle for tangent-length problems.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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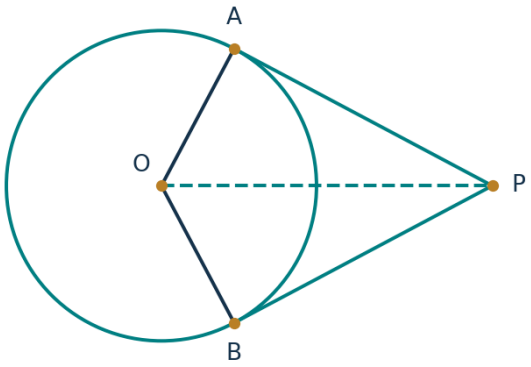
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

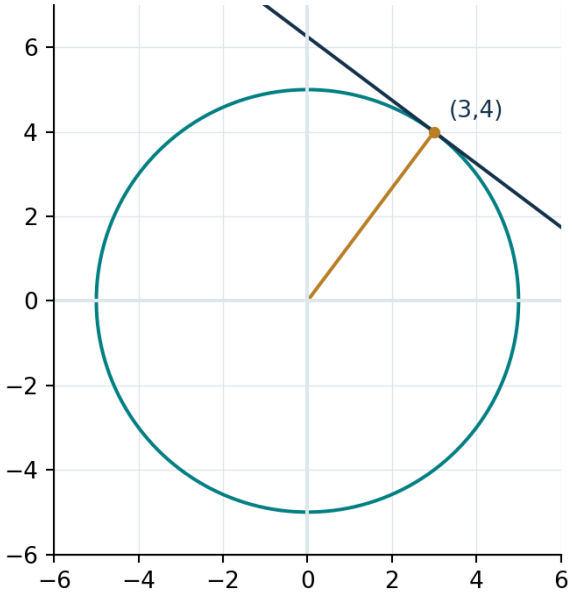
Read the geometry • 2

Tangents: congruence and perpendicular gradients

Equal tangents from P



$x^2 + y^2 = 25$, tangent at (3,4)



WHAT THE FIGURES ESTABLISH

Equal radii, the shared hypotenuse OP and right angles at A and B give RHS congruence, so PA = PB. In coordinates, radius OP to (3,4) has gradient 4/3. The tangent has gradient −3/4 and equation $3x + 4y = 25$. At axial contact points the tangent may be vertical or horizontal instead.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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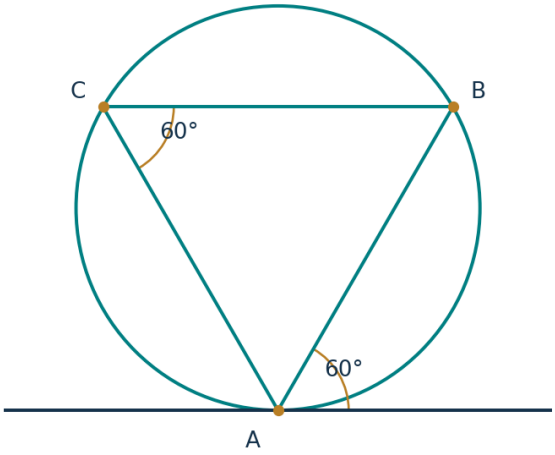
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

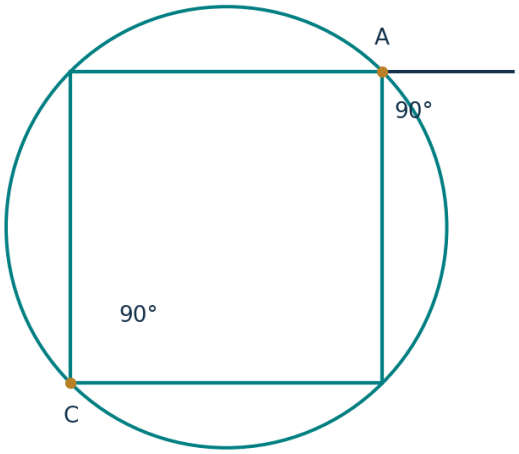
Read the geometry • 3

Match the tangent-chord angle to the alternate segment

Alternate segment theorem



Cyclic exterior angle



WHAT THE FIGURES ESTABLISH

At A, the tangent points right and chord AB makes an angle of 60° with it. The matching angle ACB in the alternate segment is also 60°. In the cyclic exterior-angle example, extending BA through A creates an exterior angle equal to opposite interior C; both are 90° in this square example. The theorem applies to every cyclic quadrilateral, not only squares.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. A central angle is 146°. Find the angle subtending the same arc at the circumference. [2 marks]

R2. AB is a diameter, C on the circle, and angle CAB = 24°. Find CBA. [2 marks]

R3. A cyclic quadrilateral has angle B = 119°. Find D. [2 marks]

R4. A tangent-chord angle is 47°. Find its matching alternate-segment angle. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Tangents from P touch at A,B with angle APB = 82°. Find the smaller AOB. [3 marks]

R6. A tangent from P touches at A, OA = 8, OP = 17. Find PA. [3 marks]

R7. State radius of $x^2 + y^2 = 100$ and find y when x = 8. [3 marks]

R8. Find the tangent at (5,12) on $x^2 + y^2 = 169$. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Angles at the centre and in a semicircle

1.1 A circumference angle is 48°. Find the central angle subtending the same arc.

96°.

1.2 A matching central reflex angle is 240°. Find its circumference angle.

120°.

1.3 AB is a diameter. State angle ACB for C on the circle.

90°.

1.4 A triangle in a semicircle has an acute angle 26°. Find its other acute angle.

64°.

Lesson 2 • Same segment and cyclic quadrilaterals

2.1 Cyclic opposite angles are $3x + 10$ and $2x + 20$. Find x.

$5x + 30 = 180$, so $x = 30$.

2.2 A cyclic quadrilateral has one angle 83°. Find its opposite.

97°.

2.3 An exterior angle of a cyclic quadrilateral is 126°. Find the opposite interior angle.

126°.

2.4 Two circumference angles subtend the same chord from opposite segments. One is 72°. Find the other.

108°.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Tangent-radius, equal tangents and alternate segments

3.1 A radius meets its tangent at angle x. Find x.

90°.

3.2 Tangents PA and PB have lengths 3x + 2 and 5x − 8. Find x.

3x + 2 = 5x − 8 gives x = 5.

3.3 A tangent-chord angle is 64°. Find its matching alternate-segment angle.

64°.

3.4 Two tangents from P have angle APB = 70°. Find the smaller central angle AOB.

Quadrilateral OAPB has right angles at A and B, so AOB = 360 − 90 − 90 − 70 = 110°.

Lesson 4 • Building circle-theorem proofs

4.1 Why are OA and OB equal in one circle?

They are radii of the same circle.

4.2 Which congruence criterion proves the equal-tangent result using OP?

RHS.

4.3 Why is OP the hypotenuse in triangles OAP and OBP?

The right angles are at the contact points A and B, so OP is opposite them.

4.4 If equal tangent lengths are established, what follows about angles APO and OPB?

They are equal by the same triangle congruence, so OP bisects angle APB.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 5 • Circle equations and tangent lines

5.1 State radius of $x^2 + y^2 = 81$.

9.

5.2 Does (5,12) lie on $x^2 + y^2 = 169$?

Yes, 25 + 144 = 169.

5.3 On $x^2 + y^2 = 100$, find y when x = 6.

$y^2 = 64$, so $y = \pm 8$.

5.4 Find tangent at (−3,4) on $x^2 + y^2 = 25$.

$-3x + 4y = 25$, or $y = 3x/4 + 25/4$.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. A central angle is 146°. Find the angle subtending the same arc at the circumference. [2 marks]

73°, half the central angle. [1 for theorem; 1 for result.]

R2. AB is a diameter, C on the circle, and angle CAB = 24°. Find CBA. [2 marks]

ACB = 90°; CBA = 66°. [1 for semicircle; 1 for triangle total.]

R3. A cyclic quadrilateral has angle B = 119°. Find D. [2 marks]

D = 61°, opposite angles supplement. [1 for theorem; 1 for result.]

R4. A tangent-chord angle is 47°. Find its matching alternate-segment angle. [2 marks]

47°, by alternate segment theorem. [1 for result; 1 for reason.]

R5. Tangents from P touch at A,B with angle APB = 82°. Find the smaller AOB. [3 marks]

Right angles at A,B; AOB = 360 − 180 − 82 = 98°. [1 for radius-tangent; 1 for quadrilateral sum; 1 for result.]

R6. A tangent from P touches at A, OA = 8, OP = 17. Find PA. [3 marks]

PA = √(289 − 64) = 15. [1 for right angle; 1 for subtraction; 1 for root.]

R7. State radius of $x^2 + y^2 = 100$ and find y when x = 8. [3 marks]

Radius 10; $y^2 = 36$, so y = ±6. [1 for radius; 1 for substitution; 1 for both values.]

R8. Find the tangent at (5,12) on $x^2 + y^2 = 169$. [4 marks]

Radius gradient 12/5; tangent gradient −5/12. Through the point: 5x + 12y = 169. [1 for radius gradient; 1 for perpendicular; 1 for point equation; 1 for simplified line.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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