GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 49

49

Right-angled trigonometry

Shared core with exact-value extensions

Use sine, cosine and tangent to connect angles with side ratios in right triangles. Choose a ratio from labelled sides, recover angles with inverse functions and solve height and distance models. Include exact values for familiar angles.

sin θ = O/H cos θ = A/H tan θ = O/A

Choose sides relative to the selected acute angle.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

INDEPENDENT GCSE MATHS • WEBSITE EDITION • NOVEMBER 2026

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Your learning guide

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Independent educational resource

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Before you begin

Identify a hypotenuse, use ratios and rearrange equations. Set a calculator to degrees.

Your route through this notebook

1. Opposite, adjacent and hypotenuse
2. Finding missing sides
3. Finding angles and modelling heights
4. Exact values and multi-step right triangles

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Opposite, adjacent and hypotenuse

For a chosen acute angle in a right triangle, the hypotenuse is opposite the right angle, the opposite side faces the chosen angle, and the adjacent side is the remaining shorter side next to it. Opposite and adjacent switch when you switch the chosen acute angle; the hypotenuse does not.

Sine, cosine and tangent are ratios: $\sin\theta$ = opposite/hypotenuse, $\cos\theta$ = adjacent/hypotenuse, $\tan\theta$ = opposite/adjacent. Similar right triangles with the same acute angle have the same ratios, regardless of size. SOHCAHTOA helps recall them, but correct labelling is the crucial first step. These side-ratio definitions here apply to right triangles with acute θ.

A reliable method

1. Mark the right angle and chosen acute angle.

2. Label O,A,H relative to that chosen angle.

3. Select the ratio involving the two relevant sides.

Worked example 1

For a 3–4–5 triangle, θ is opposite the side of length 3. Find $\sin\theta$,$\cos\theta$,$\tan\theta$.

STEP-BY-STEP SOLUTION

O = 3,A = 4,H = 5. Thus $\sin\theta = 3/5$, $\cos\theta = 4/5$ and $\tan\theta = 3/4$.

Worked example 2

If opposite and hypotenuse are the known sides, which ratio is useful?

STEP-BY-STEP SOLUTION

Sine, because $\sin\theta$ = O/H. Cosine and tangent require the adjacent side.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The adjacent side is not the hypotenuse even though both meet the acute angle. Exclude the hypotenuse before naming the adjacent leg.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 θ is opposite 5 in a 5–12–13 right triangle. State sinθ.

1.2 For the same θ, state cosθ.

1.3 For the same θ, state tanθ.

1.4 What changes when the other acute angle is selected?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Finding missing sides

Select the trigonometric ratio containing the known side, the unknown side and the given angle. Write an equation before calculating. If the unknown is in a numerator, multiply by the denominator; if it is in the denominator, rearrange carefully instead of multiplying automatically.

Use degree mode for these GCSE angle problems. A hypotenuse must exceed either leg, and sine or cosine of an acute angle lies between 0 and 1, giving a quick size check. Keep full precision until the final answer. A sketch should show which side the answer describes and whether the triangle model is justified.

A reliable method

1. Choose the ratio from O,A,H.

2. Write and rearrange the equation for the missing side.

3. Calculate in degrees and check relative lengths.

Worked example 1

A right triangle has hypotenuse 10 cm and angle 30°. Find the opposite side.

STEP-BY-STEP SOLUTION

$\sin 30° = x/10$, so $x = 10 \times 1/2 = 5$ cm.

Worked example 2

The opposite side is 6 cm and $\theta = 30°$. Find the hypotenuse.

STEP-BY-STEP SOLUTION

$\sin 30° = 6/h$. Rearranging $h = 6/\sin 30° = 12$ cm. Multiplying 6 by $\sin 30°$ would find the wrong quantity.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The phrase “use sine” is not a complete calculation. The written ratio shows whether the unknown belongs above or below the fraction bar.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find opposite side for hypotenuse 18 cm and angle 30°.

2.2 Find adjacent side for hypotenuse 14 cm and angle 60°.

2.3 Find opposite side for adjacent 8 cm and angle 45°.

2.4 Find hypotenuse for adjacent 5 cm and angle 60°.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Finding angles and modelling heights

To recover an angle from a side ratio, use the inverse function: $\theta = \sin^{-1}(O/H)$, $\cos^{-1}(A/H)$ or $\tan^{-1}(O/A)$. On a calculator these inverse keys are different from taking a reciprocal. The side ratios identify an acute angle in the right-triangle setting.

An angle of elevation is measured upward from a horizontal line; depression is measured downward from a horizontal line. If an observer's eye is above ground, a trig height calculated from eye level must be adjusted to get the full object height. Horizontal and vertical directions justify the right angle. Model assumptions such as level ground should be stated.

A reliable method

1. Choose the ratio using known sides.
2. Apply the matching inverse function in degree mode.
3. Interpret the angle or add any stated eye-height offset.

Worked example 1

A right triangle has opposite 6 and hypotenuse 10. Find θ to 1 decimal place.

STEP-BY-STEP SOLUTION

$\theta = \sin^{-1}(0.6) \approx 36.9°$.

Worked example 2

From a point 20 m horizontally from a tower, its top has elevation 45°. The observer's eye is 1.6 m high. Find tower height.

STEP-BY-STEP SOLUTION

Height above eye = $20\tan45° = 20$ m. Total tower height = 21.6 m.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Using the sloping line-of-sight distance as the horizontal adjacent side changes the model. Label the ground distance and the hypotenuse separately.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Opposite and adjacent are both 7 cm. Find θ.

3.2 Adjacent is 4 cm and hypotenuse 8 cm. Find θ.

3.3 A tower top has elevation 30° from $10\sqrt{3}$ m horizontally away at ground level. Find height.

3.4 Why is $\sin^{-1}(0.5)$ not $1/\sin(0.5)$?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Exact values and multi-step right triangles

Exact trigonometric values come from familiar triangles. An isosceles right triangle gives sin45° = cos45° = √2/2 and tan45° = 1. Splitting an equilateral triangle gives sin30° = 1/2, cos30° = √3/2, tan30° = √3/3; sin60° = √3/2, cos60° = 1/2, tan60° = √3.

At endpoints, sin0° = 0, cos0° = 1, sin90° = 1 and cos90° = 0; tan90° is undefined. For a multi-step figure, first solve one right triangle, then transfer a shared length into the next. Pythagoras and trigonometry can be combined, but only apply each formula where its geometric conditions hold.

A reliable method

1. Use a standard-triangle value for an exact-angle problem.

2. Keep surds and fractions exact through intermediate steps.

3. For a compound figure, identify each right triangle separately.

Worked example 1

Find exact opposite length for hypotenuse 12 cm and θ = 60°.

STEP-BY-STEP SOLUTION

12sin60° = 12(√3/2) = 6√3 cm.

Worked example 2

A rectangle has width 6 cm and a diagonal making 30° with its longer horizontal side. Find that horizontal side.

STEP-BY-STEP SOLUTION

tan30° = 6/l, so l = 6/(√3/3) = 6√3 cm. The diagonal would be 12 cm.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Exact means keep the mathematical value, such as $6\sqrt{3}$, rather than a rounded decimal. If a question asks for exact form, a calculator approximation alone is insufficient.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 State $\sin 30°$, $\cos 60°$ and $\tan 45°$.

4.2 State $\sin 45°$ exactly.

4.3 Find exact adjacent length for hypotenuse 10 and angle 30°.

4.4 Find exact hypotenuse for opposite 5 and angle 45°.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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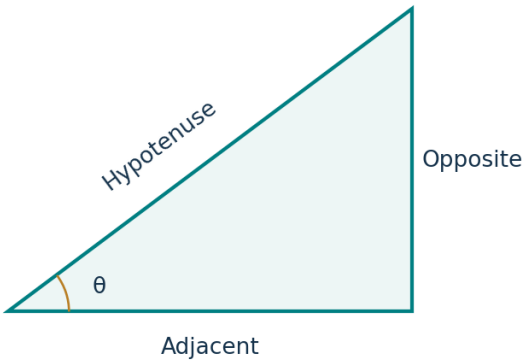
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

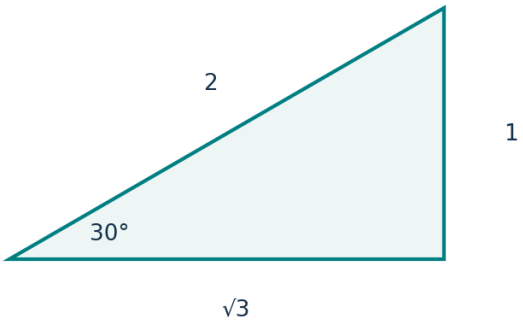
Read the geometry

Label sides relative to the chosen angle first

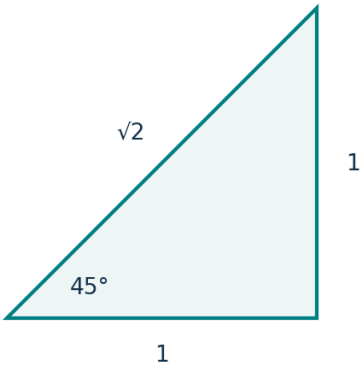
The three ratios



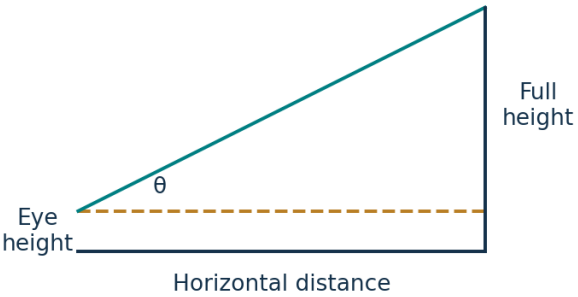
30°–60°–90° triangle



45°–45°–90° triangle



Elevation from eye level



WHAT THE FIGURES ESTABLISH

Opposite and adjacent refer to the selected angle; the hypotenuse is always opposite the right angle. The standard-triangle panels show ratios 1:√3:2 and 1:1:√2, not centimetre measurements. In a height problem, the trig calculation may find only height above the observer's eye, so add the stated eye height.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. For θ opposite 8 in an 8–15–17 triangle, state sinθ and tanθ. [2 marks]

R2. Find opposite side for hypotenuse 16 cm and angle 30°. [2 marks]

R3. Find adjacent side for hypotenuse 20 cm and angle 60°. [2 marks]

R4. Find hypotenuse for opposite 9 cm and angle 30°. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. A right triangle has opposite 5 and adjacent 5. Find θ. [2 marks]

R6. A 12 m horizontal distance gives elevation 45° from eye level 1.5 m. Find full height. [3 marks]

R7. Give exact opposite side for hypotenuse 8 and angle 45°. [2 marks]

R8. A rectangle is 7 cm high and its diagonal makes 30° with the horizontal base. Find base exactly. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Opposite, adjacent and hypotenuse

1.1 θ is opposite 5 in a 5–12–13 right triangle. State sinθ.

5/13.

1.2 For the same θ, state cosθ.

12/13.

1.3 For the same θ, state tanθ.

5/12.

1.4 What changes when the other acute angle is selected?

Opposite and adjacent interchange; hypotenuse stays the same.

Lesson 2 • Finding missing sides

2.1 Find opposite side for hypotenuse 18 cm and angle 30°.

18sin30° = 9 cm.

2.2 Find adjacent side for hypotenuse 14 cm and angle 60°.

14cos60° = 7 cm.

2.3 Find opposite side for adjacent 8 cm and angle 45°.

8tan45° = 8 cm.

2.4 Find hypotenuse for adjacent 5 cm and angle 60°.

5/cos60° = 10 cm.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Finding angles and modelling heights

3.1 Opposite and adjacent are both 7 cm. Find θ.

$\tan^{-1}(1) = 45°$.

3.2 Adjacent is 4 cm and hypotenuse 8 cm. Find θ.

$\cos^{-1}(1/2) = 60°$.

3.3 A tower top has elevation 30° from $10\sqrt{3}$ m horizontally away at ground level. Find height.

$10\sqrt{3}\tan30° = 10$ m.

3.4 Why is $\sin^{-1}(0.5)$ not $1/\sin(0.5)$?

The inverse function finds an angle from a sine value; a reciprocal is a different operation. In degrees $\sin^{-1}(0.5) = 30°$.

Lesson 4 • Exact values and multi-step right triangles

4.1 State $\sin30°$,$\cos60°$ and $\tan45°$.

1/2,1/2,1.

4.2 State $\sin45°$ exactly.

$\sqrt{2}/2$.

4.3 Find exact adjacent length for hypotenuse 10 and angle 30°.

$5\sqrt{3}$ units.

4.4 Find exact hypotenuse for opposite 5 and angle 45°.

$5/(\sqrt{2}/2) = 5\sqrt{2}$ units.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. For θ opposite 8 in an 8–15–17 triangle, state sinθ and tanθ. [2 marks]

8/17 and 8/15. [1 each.]

R2. Find opposite side for hypotenuse 16 cm and angle 30°. [2 marks]

16sin30° = 8 cm. [1 for ratio; 1 for result.]

R3. Find adjacent side for hypotenuse 20 cm and angle 60°. [2 marks]

20cos60° = 10 cm. [1 for ratio; 1 for result.]

R4. Find hypotenuse for opposite 9 cm and angle 30°. [2 marks]

9/sin30° = 18 cm. [1 for rearrangement; 1 for result.]

R5. A right triangle has opposite 5 and adjacent 5. Find θ. [2 marks]

$\tan^{-1}(1) = 45°$. [1 for ratio; 1 for inverse.]

R6. A 12 m horizontal distance gives elevation 45° from eye level 1.5 m. Find full height. [3 marks]

12tan45° + 1.5 = 13.5 m. [1 for ratio; 1 for height above eye; 1 for offset.]

R7. Give exact opposite side for hypotenuse 8 and angle 45°. [2 marks]

8√2/2 = 4√2. [1 for value; 1 for simplification.]

R8. A rectangle is 7 cm high and its diagonal makes 30° with the horizontal base. Find base exactly. [3 marks]

tan30° = 7/b; b = 7√3 cm. [1 for diagram/ratio; 1 for rearrangement; 1 for exact value.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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