GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 48

48

Pythagoras’ theorem

Shared core with Higher 3D extensions

Use the right-angle condition to connect side lengths, recognise which side is the hypotenuse and choose addition or subtraction correctly. Extend to coordinates, three-dimensional diagonals and contextual distances.

$$a^2 + b^2 = c^2$$

Identify the right angle and the hypotenuse first.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Square numbers, take square roots, simplify expressions and use length units.

Your route through this notebook

1. The theorem and the hypotenuse
2. Finding a shorter side and checking triangles
3. Coordinates and 3D diagonals — extension
4. Ladders, heights and exact reasoning

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

The theorem and the hypotenuse

In a right-angled triangle, $a^2 + b^2 = c^2$ where c is the hypotenuse, the side opposite the right angle. The theorem relates the areas of squares built on the sides. The hypotenuse is the longest side and must be identified from the right-angle position, not from a letter used in the question.

To find a missing hypotenuse, square both shorter sides, add and take the positive square root. Adding the side lengths themselves is not the theorem. The positive root is selected because the answer is a length. Keep an exact square root or simplified surd if a decimal is not requested.

A reliable method

1. Confirm the triangle is right-angled.

2. Label the hypotenuse and substitute into $a^2 + b^2 = c^2$.

3. Take a positive root and check it exceeds each shorter side.

Worked example 1

Find hypotenuse for shorter sides 6 cm and 8 cm.

STEP-BY-STEP SOLUTION

$c^2 = 36 + 64 = 100$, so c = 10 cm.

Worked example 2

Find hypotenuse for shorter sides 3 m and 5 m exactly.

STEP-BY-STEP SOLUTION

$c^2 = 9 + 25 = 34$, so $c = \sqrt{34}$ m, approximately 5.83 m.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A triangle’s appearance is not enough to apply Pythagoras. A right-angle marker, a stated fact or a justified perpendicular relationship supplies the condition.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find hypotenuse for sides 5 cm and 12 cm.

1.2 Find hypotenuse for sides 8 m and 15 m.

1.3 Find hypotenuse for sides 4 and 7 exactly.

1.4 Where is the hypotenuse relative to the right angle?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Finding a shorter side and checking triangles

If the hypotenuse and one shorter side are known, rearrange to subtract squares: $b^2 = c^2 - a^2$. Subtract in the correct order so that the result is non-negative for valid data. Take the positive root and check that the recovered side is shorter than the hypotenuse.

The converse can test a triangle: if the square of its longest side equals the sum of the squares of the other two, the triangle is right-angled. Use exact given lengths or their stated accuracy; tiny differences from rounded measurements do not necessarily establish a precise classification. First ensure the side lengths can form a triangle at all.

A reliable method

1. Identify the longest side or known hypotenuse.

2. Subtract squares for a missing leg; compare squares for a right-angle test.

3. Use exact values where possible and check length plausibility.

Worked example 1

A right triangle has hypotenuse 13 cm and one leg 5 cm. Find the other.

STEP-BY-STEP SOLUTION

$b^2 = 169 - 25 = 144$, so $b = 12$ cm.

Worked example 2

Are side lengths 7,24,25 right-angled?

STEP-BY-STEP SOLUTION

Longest is 25. Since $7^2 + 24^2 = 49 + 576 = 625 = 25^2$, yes, by the converse.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Subtracting lengths before squaring changes the calculation: $\sqrt{(13^2 - 5^2)}$ is 12, while $\sqrt{((13 - 5)^2)}$ is 8. Preserve the original equation.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Hypotenuse 10, leg 6. Find other leg.

2.2 Hypotenuse 17, leg 8. Find other leg.

2.3 Are 6,8,11 right-angled?

2.4 A right triangle is said to have hypotenuse 7 and another side 9. Explain the problem.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Coordinates and 3D diagonals — extension

Two points in a coordinate plane form a right triangle using their horizontal and vertical differences. Distance is $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$. Differences can be negative, but squaring gives the same distance in either direction. This formula is Pythagoras expressed in coordinates.

For a cuboid, first find a face diagonal, then use that diagonal and the perpendicular third edge to find the space diagonal. The result is $\sqrt{(l^2 + w^2 + h^2)}$. This works because the edges are mutually perpendicular; it is not a general formula for every skew solid. Draw the internal right triangle instead of assuming a diagonal across a perspective sketch is a face edge.

A reliable method

1. Find perpendicular coordinate differences or a face diagonal.
2. Apply Pythagoras once or in successive perpendicular planes.
3. Retain unrounded intermediate values.

Worked example 1

Find distance between (−2,1) and (4,9).

STEP-BY-STEP SOLUTION

Differences 6 and 8. Distance $\sqrt{(36 + 64)} = 10$ units.

Worked example 2

Higher: find space diagonal of a cuboid 3 by 4 by 12 cm.

STEP-BY-STEP SOLUTION

Face diagonal is 5 cm. Space diagonal $\sqrt{(5^2 + 12^2)} = 13$ cm; directly $\sqrt{(9 + 16 + 144)} = 13$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Using a rounded face diagonal in the second stage can alter the final answer. Keep its squared exact value or full calculator precision.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find distance between (1,2) and (7,10).

3.2 Find distance between (−3,−1) and (2,11).

3.3 Higher: find space diagonal of a 2 by 3 by 6 cuboid.

3.4 Higher: a cube has side 5 cm. Give its space diagonal exactly.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Ladders, heights and exact reasoning

Applied problems require identifying a right triangle within a larger situation. A ladder against a vertical wall gives perpendicular wall and ground directions. An isosceles triangle can be split into two right triangles by dropping a perpendicular from the vertex to the base midpoint. A diagonal may similarly create a right triangle in a rectangle.

Use the mathematical model stated; a calculation alone is not a practical safety recommendation. Distinguish the requested distance from a related side, check units and state approximation accuracy. If a result remains a surd, simplify square factors where possible. Reject negative algebraic length solutions.

A reliable method

1. Sketch and mark the justified right angle.

2. Match the unknown with hypotenuse or shorter side.

3. Calculate, interpret and check against the geometric constraints.

Worked example 1

A 5 m ladder has its foot 1.4 m from a vertical wall. Find contact height to 2 decimal places.

STEP-BY-STEP SOLUTION

$h = \sqrt{(25 - 1.96)} = \sqrt{23.04} = 4.80$ m in the idealised right-triangle model.

Worked example 2

An isosceles triangle has equal sides 10 cm and base 12 cm. Find area.

STEP-BY-STEP SOLUTION

The height bisects the base into 6 cm pieces. $h = \sqrt{(100 - 36)} = 8$ cm. Area $= 12 \times 8/2 = 48$ cm^2.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The base-bisecting height in an isosceles triangle is justified by symmetry or congruence. A general triangle’s height does not necessarily hit its base midpoint.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 A 13 m ladder has foot 5 m from a vertical wall. Find height.

4.2 A rectangle is 9 by 12 cm. Find diagonal.

4.3 An isosceles triangle has equal sides 13 and base 10. Find height and area.

4.4 A right triangle has equal legs 6 cm. Find hypotenuse exactly.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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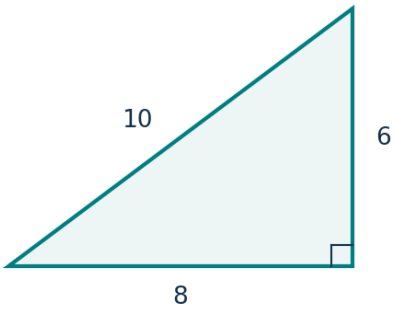
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

Read the geometry

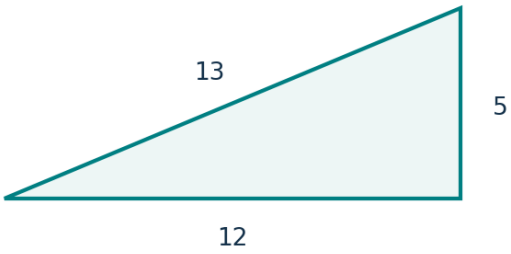
Pythagoras belongs to a justified right triangle

Find the hypotenuse



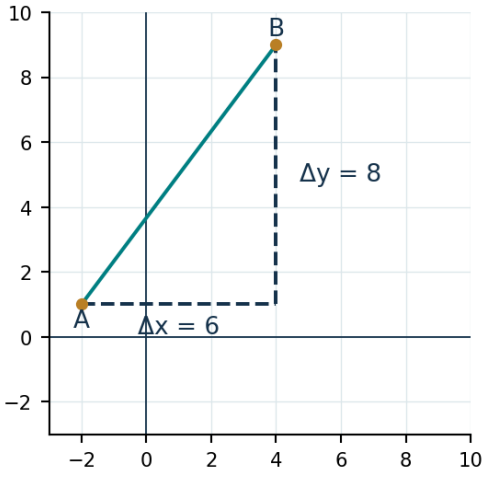
$6^2 + 8^2 = 10^2$

Find a shorter side



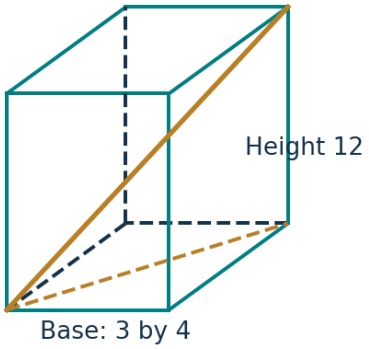
$12 = \sqrt{(13^2 - 5^2)}$

Coordinate distance



Space diagonal

$\sqrt{(3^2 + 4^2 + 12^2)} = 13$



WHAT THE FIGURES ESTABLISH

Add squares to find a hypotenuse and subtract to find a leg. Coordinate differences create perpendicular horizontal and vertical legs. In the cuboid example, true dimensions are 3,4,12; the oblique sketch is not to scale. The base diagonal is 5 and the space diagonal is 13, using two perpendicular triangles.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find hypotenuse for legs 9 cm and 12 cm. [2 marks]

R2. Find missing leg with hypotenuse 25 cm and other leg 7 cm. [2 marks]

R3. Test whether 9,40,41 form a right triangle. [2 marks]

R4. Find distance from (−1,−2) to (7,13). [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Higher: find diagonal of a cuboid 6 by 8 by 24 cm. [3 marks]

R6. A rectangle has diagonal 20 cm and width 12 cm. Find length. [2 marks]

R7. An isosceles triangle has equal sides 17 cm and base 16 cm. Find area. [4 marks]

R8. Give exact diagonal of a square side 7 cm. [2 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • The theorem and the hypotenuse

1.1 Find hypotenuse for sides 5 cm and 12 cm.

13 cm.

1.2 Find hypotenuse for sides 8 m and 15 m.

17 m.

1.3 Find hypotenuse for sides 4 and 7 exactly.

√65 units.

1.4 Where is the hypotenuse relative to the right angle?

Opposite it.

Lesson 2 • Finding a shorter side and checking triangles

2.1 Hypotenuse 10, leg 6. Find other leg.

8.

2.2 Hypotenuse 17, leg 8. Find other leg.

15.

2.3 Are 6,8,11 right-angled?

No: $6^2 + 8^2 = 100$, not 121.

2.4 A right triangle is said to have hypotenuse 7 and another side 9. Explain the problem.

The hypotenuse must be longest, so the data or labelling are inconsistent.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Coordinates and 3D diagonals — extension

3.1 Find distance between (1,2) and (7,10).

$\sqrt{(6^2 + 8^2)} = 10$ units.

3.2 Find distance between (−3,−1) and (2,11).

$\sqrt{(5^2 + 12^2)} = 13$ units.

3.3 Higher: find space diagonal of a 2 by 3 by 6 cuboid.

$\sqrt{(4 + 9 + 36)} = 7$ units.

3.4 Higher: a cube has side 5 cm. Give its space diagonal exactly.

$5\sqrt{3}$ cm.

Lesson 4 • Ladders, heights and exact reasoning

4.1 A 13 m ladder has foot 5 m from a vertical wall. Find height.

12 m.

4.2 A rectangle is 9 by 12 cm. Find diagonal.

15 cm.

4.3 An isosceles triangle has equal sides 13 and base 10. Find height and area.

Half-base 5; height 12; area 60 square units.

4.4 A right triangle has equal legs 6 cm. Find hypotenuse exactly.

$\sqrt{72} = 6\sqrt{2}$ cm.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find hypotenuse for legs 9 cm and 12 cm. [2 marks]

√(81 + 144) = 15 cm. [1 for squares; 1 for root.]

R2. Find missing leg with hypotenuse 25 cm and other leg 7 cm. [2 marks]

√(625 − 49) = 24 cm. [1 for subtraction; 1 for root.]

R3. Test whether 9,40,41 form a right triangle. [2 marks]

81 + 1600 = 1681 = 41², so yes. [1 for comparison; 1 for conclusion.]

R4. Find distance from (−1,−2) to (7,13). [3 marks]

Differences 8 and 15; distance 17. [1 for changes; 1 for squares; 1 for root.]

R5. Higher: find diagonal of a cuboid 6 by 8 by 24 cm. [3 marks]

√(36 + 64 + 576) = 26 cm. [1 for perpendicular model; 1 for sum; 1 for root.]

R6. A rectangle has diagonal 20 cm and width 12 cm. Find length. [2 marks]

√(400 − 144) = 16 cm. [1 for subtraction; 1 for root.]

R7. An isosceles triangle has equal sides 17 cm and base 16 cm. Find area. [4 marks]

Half-base 8; height √(289 − 64) = 15; area 16×15/2 = 120 cm². [1 for half-base; 1 for Pythagoras; 1 for height; 1 for area.]

R8. Give exact diagonal of a square side 7 cm. [2 marks]

√98 = 7√2 cm. [1 for Pythagoras; 1 for simplified surd.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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