GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 47

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Congruence and similarity

Shared core with Higher extensions

Separate same-size congruence from same-shape similarity. Match corresponding vertices carefully, justify triangle relationships and use length, area and volume scale factors in forward and reverse problems.

Similar: lengths k, areas k^2, volumes k^3

Congruent means same size as well as same shape.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Use ratios, solve simple equations and recognise triangle sides and angles.

Your route through this notebook

1. Congruence and triangle criteria
2. Writing a geometric proof
3. Similar shapes and length scale factors
4. Area and volume scales — Higher extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Congruence and triangle criteria

Congruent shapes have corresponding lengths and angles equal; they may be rotated, reflected or translated. For triangles, SSS, SAS, ASA or AAS, and RHS give sufficient conditions. RHS requires two right-angled triangles with equal hypotenuses and one equal corresponding shorter side.

SAS needs the included angle between the given sides. SSA is not a general congruence criterion: two different triangles can share two side lengths and a non-included angle. AAA fixes shape but allows scaling, so it proves similarity rather than congruence. State the correspondence in a consistent vertex order.

A reliable method

1. List established equal sides and angles with their reasons.
2. Match them to a valid sufficient criterion.
3. Name corresponding vertices in the same order.

Worked example 1

Two triangles each have side lengths 4 cm,5 cm,6 cm. Explain congruence.

STEP-BY-STEP SOLUTION

Their three corresponding sides are equal, so they are congruent by SSS, regardless of orientation.

Worked example 2

Two right triangles have hypotenuse 13 cm and a matching shorter side 5 cm. Explain congruence.

STEP-BY-STEP SOLUTION

Both have a right angle, equal hypotenuses and one equal corresponding side: RHS.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Three equal angles cannot prove triangles are the same size. An enlargement preserves all angles while changing every non-zero length.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Name the criterion for sides 5 and 8 with equal included angle 40°.

1.2 Name a criterion using two equal corresponding angles and a corresponding side.

1.3 Is AAA sufficient for congruence?

1.4 Why is a non-included angle insufficient for the general SAS claim?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Writing a geometric proof

A proof is a sequence of statements supported by reasons. Use given markings, shared sides, angle facts and parallel-line properties to establish the needed equalities, then invoke the correct triangle criterion. A common side equals itself; it must be identified, not guessed from appearance.

Once triangles are proved congruent, corresponding parts are equal. Avoid circular reasoning: do not assume the very equality you are trying to prove as one of the inputs to congruence. If labels are available, name the exact segments and angles. A neat diagram supports the argument, but the logical chain supplies the proof.

A reliable method

1. State the given and shared equalities explicitly.

2. Apply a valid congruence criterion.

3. Deduce the requested corresponding-part equality.

Worked example 1

In isosceles triangle ABC, AB = AC and D is midpoint of BC. Prove AD bisects angle BAC.

STEP-BY-STEP SOLUTION

AB = AC (given), BD = DC (midpoint), AD = AD (shared). Triangles ABD and ACD are congruent by SSS. Hence angles BAD and DAC are equal, so AD bisects angle BAC.

Worked example 2

A diagonal AC divides parallelogram ABCD. Prove triangles ABC and CDA congruent.

STEP-BY-STEP SOLUTION

AB = CD and BC = DA (opposite parallelogram sides), AC shared. SSS proves congruence with matching vertices $A \leftrightarrow C, B \leftrightarrow D, C \leftrightarrow A$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A diagram that looks symmetric does not establish symmetry. In the first example, midpoint and equal-side information justify the result.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 What equality follows when D is midpoint of BC?

2.2 Why can a shared side be used in two triangles?

2.3 After triangles are proved congruent, what can be said about corresponding angles?

2.4 Why is it circular to assume BAD = DAC when proving AD is an angle bisector using that equality?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Similar shapes and length scale factors

Similar shapes have equal corresponding angles and proportional corresponding lengths. Triangles are similar when two corresponding angles agree (AA), when corresponding sides are proportional (SSS similarity), or when two side ratios agree and their included angle is equal (SAS similarity).

Choose and label corresponding sides before forming a ratio. For a length multiplier k from shape A to B, every B length equals k times its corresponding A length. Reversing direction uses 1/k. Orientation does not determine correspondence; use angle labels or structure. Do not compare a short side in one shape with a long side in the other merely because they occupy similar positions on the page.

A reliable method

1. Match vertices and sides using angle or structural information.

2. Find target/source length factor k.

3. Apply k consistently or divide by it to reverse.

Worked example 1

Similar triangles have matching sides 6 cm and 15 cm. A second side in the smaller is 8 cm. Find its larger partner.

STEP-BY-STEP SOLUTION

k = 15/6 = 2.5. The partner is 8 × 2.5 = 20 cm.

Worked example 2

A larger shape’s matching length is 27 cm under factor 3 from the smaller. Find smaller length.

STEP-BY-STEP SOLUTION

Divide by 3: 9 cm.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Two equal-length sides in two triangles do not alone establish similarity. You need sufficient angle or side-ratio information for the whole shape relationship.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Similar lengths are in ratio 3:7. A smaller side is 12 cm. Find larger.

3.2 A large side 30 cm corresponds to a small side 18 cm. Find small-to-large factor.

3.3 Two triangles have angles 40°,60°,80°. Are they necessarily congruent?

3.4 A scale factor 1.5 takes A to B. Find factor B to A.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Area and volume scales — Higher extension

For similar shapes with length factor k, area factor is k^2 and volume factor is k^3. Every relevant linear dimension changes by k, so multiplying two or three dimensions produces the powers. To recover a length factor from an area ratio take a positive square root; from a volume ratio take a positive cube root.

The shapes must be similar for these direct power relationships. A box stretched in only one direction does not have the same factor in all dimensions. For ratios, square or cube both entries. A surface-area ratio of similar solids also uses the squared length factor, while their volume ratio uses the cubed factor.

A reliable method

1. Identify whether the given and required quantities are lengths, areas or volumes.
2. Find the positive length factor, using a root if necessary.
3. Apply the required squared or cubed factor.

Worked example 1

Similar solids have length ratio 2:5. Find surface-area and volume ratios.

STEP-BY-STEP SOLUTION

Surface areas 4:25; volumes 8:125.

Worked example 2

Similar solids have volumes 64 cm^3 and 216 cm^3. Find length and surface-area ratios.

STEP-BY-STEP SOLUTION

Volume ratio 8:27 after simplification. Cube roots give length 2:3; surface area 4:9.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

If area increases by 44%, the factor is 1.44, giving length factor $\sqrt{1.44} = 1.2$, a 20% length increase. Do not halve a percentage increase to reverse a squared factor.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 A shape area 18 cm² is enlarged by length factor 4. Find new area.

4.2 Similar shapes have area ratio 9:25. Find length ratio.

4.3 Similar solids have volume ratio 27:125. Find length ratio.

4.4 Similar solids have length factor 2. A small solid has area 40 cm² and volume 60 cm³. Find larger area and volume.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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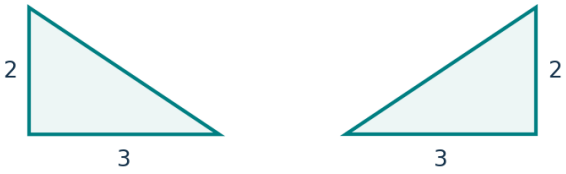
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

Read the geometry

Match corresponding parts before using scale factors

SSS congruence



Equal corresponding sides

Similar: length factor 2



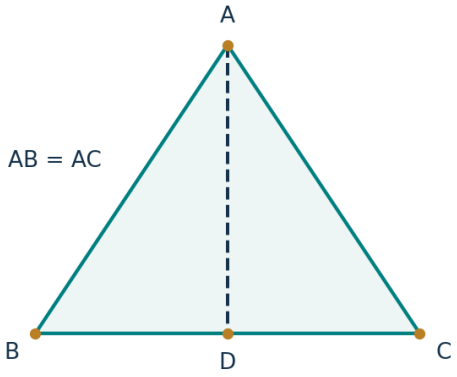
Scale factors

Lengths: k

Areas: k^2

Volumes: k^3

Congruent halves of an isosceles triangle



BD = DC; AD shared

WHAT THE FIGURES ESTABLISH

Congruent triangles can be reflected and still match exactly. Similar triangles need not be the same size. A length factor 2 gives area factor 4 and volume factor 8 when the relevant shapes or solids are similar. The final diagram supplies AB = AC and BD = DC; shared AD proves SSS congruence and hence equal vertex-half angles.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. State the triangle criterion for three equal corresponding sides. [1 mark]

R2. Explain why equal angles alone do not prove congruence. [2 marks]

R3. In AB = AC with D midpoint of BC, identify three side equalities for triangles ABD and ACD. [3 marks]

R4. Similar triangles have matching sides 8 and 20. A small side is 14. Find its larger match. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. A factor 4 takes small to large. A large side is 34 cm. Find small side. [2 marks]

R6. Higher: similar shapes have length ratio 5:8. Find area ratio. [1 mark]

R7. Higher: similar solids have volume ratio 125:343. Find length and surface-area ratios. [3 marks]

R8. Higher: area rises from 50 to 98 cm² for similar shapes. Find length factor. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Congruence and triangle criteria

1.1 Name the criterion for sides 5 and 8 with equal included angle 40°.

SAS.

1.2 Name a criterion using two equal corresponding angles and a corresponding side.

ASA or AAS, depending on side position.

1.3 Is AAA sufficient for congruence?

No; it establishes similarity only.

1.4 Why is a non-included angle insufficient for the general SAS claim?

It gives SSA rather than SAS and can allow two non-congruent triangles.

Lesson 2 • Writing a geometric proof

2.1 What equality follows when D is midpoint of BC?

BD = DC.

2.2 Why can a shared side be used in two triangles?

It is the same segment, so its length equals itself.

2.3 After triangles are proved congruent, what can be said about corresponding angles?

They are equal.

2.4 Why is it circular to assume BAD = DAC when proving AD is an angle bisector using that equality?

The assumed equality is exactly the desired conclusion, so it has not been established independently.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Similar shapes and length scale factors

3.1 Similar lengths are in ratio 3:7. A smaller side is 12 cm. Find larger.

12 × 7/3 = 28 cm.

3.2 A large side 30 cm corresponds to a small side 18 cm. Find small-to-large factor.

30/18 = 5/3.

3.3 Two triangles have angles 40°,60°,80°. Are they necessarily congruent?

No. They are similar by AA, but their sizes are not specified.

3.4 A scale factor 1.5 takes A to B. Find factor B to A.

1/1.5 = 2/3.

Lesson 4 • Area and volume scales — Higher extension

4.1 A shape area 18 cm² is enlarged by length factor 4. Find new area.

18 × 16 = 288 cm².

4.2 Similar shapes have area ratio 9:25. Find length ratio.

3:5.

4.3 Similar solids have volume ratio 27:125. Find length ratio.

3:5.

4.4 Similar solids have length factor 2. A small solid has area 40 cm² and volume 60 cm³. Find larger area and volume.

Area 160 cm²; volume 480 cm³.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. State the triangle criterion for three equal corresponding sides. [1 mark]

SSS. [1 for criterion.]

R2. Explain why equal angles alone do not prove congruence. [2 marks]

They fix shape, not size; an enlargement gives a similar non-congruent triangle. [1 for distinction; 1 for example/reason.]

R3. In AB = AC with D midpoint of BC, identify three side equalities for triangles ABD and ACD. [3 marks]

AB = AC; BD = DC; AD = AD. [1 each.]

R4. Similar triangles have matching sides 8 and 20. A small side is 14. Find its larger match. [2 marks]

k = 2.5; larger 35. [1 for factor; 1 for result.]

R5. A factor 4 takes small to large. A large side is 34 cm. Find small side. [2 marks]

34/4 = 8.5 cm. [1 for reverse division; 1 for result.]

R6. Higher: similar shapes have length ratio 5:8. Find area ratio. [1 mark]

25:64. [1 for squares.]

R7. Higher: similar solids have volume ratio 125:343. Find length and surface-area ratios. [3 marks]

Length 5:7; surface area 25:49. [1 for cube-root method; 1 for lengths; 1 for areas.]

R8. Higher: area rises from 50 to 98 cm² for similar shapes. Find length factor. [3 marks]

Area factor 98/50 = 1.96; length factor √1.96 = 1.4. [1 for ratio; 1 for root; 1 for result.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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