GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 46

46

Transformations and symmetry

Shared core with Higher extensions

Describe and carry out reflections, rotations, translations and enlargements precisely. Use coordinates, invariant features and complete descriptions to check images. Higher work includes fractional and negative enlargement factors.

Image = centre + k × (point − centre)

Give every parameter in a complete transformation.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Plot coordinates, measure angles and calculate with signed numbers and ratios.

Your route through this notebook

1. Reflections and symmetry
2. Rotations and rotational symmetry
3. Translations and vectors
4. Enlargements and combined transformations — extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Reflections and symmetry

A reflection places each image point on the opposite side of a mirror line at the same perpendicular distance. The segment joining a point and its reflection is perpendicular to the mirror line and is bisected by it. Points on the mirror line stay fixed. Reflections preserve lengths and angles but reverse orientation.

Coordinate rules follow from this geometry: reflection in the x-axis maps (x,y) to (x,−y), and in the y-axis to (−x,y). Reflection in y = x swaps coordinates. For a vertical mirror x = a, average original and image x coordinates to get a. A shape’s line of symmetry is a reflection line that maps the shape onto itself.

A reliable method

1. Identify or draw the mirror line.

2. Use perpendicular equal distances or a verified coordinate rule.

3. Join corresponding vertices in the original order.

Worked example 1

Reflect (3,−2) in the x-axis and then separately in the y-axis.

STEP-BY-STEP SOLUTION

In the x-axis: (3,2). In the y-axis: (−3,−2). Each operation changes the coordinate perpendicular to the mirror.

Worked example 2

Reflect (1,4) in x = 3.

STEP-BY-STEP SOLUTION

The point is 2 units left of the mirror, so the image is 2 units right: (5,4).

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The x-axis is the line y = 0, so reflecting in it changes y, not x. Name the line by its geometric position rather than copying the letter in its name.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Reflect (−4,5) in the y-axis.

1.2 Reflect (2,−3) in y = x.

1.3 Find the mirror line for (1,2) and its image (7,2).

1.4 State the number of lines of symmetry of a non-square rectangle.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Rotations and rotational symmetry

A rotation needs a centre, an angle and a direction. Corresponding points are the same distance from the centre and turn through the same angle. A centre may lie outside the shape or at one of its vertices. A 180° turn gives the same result clockwise or anticlockwise.

About the origin, 90° anticlockwise maps (x,y) to (−y,x), 90° clockwise to (y,−x), and 180° to (−x,−y). For another centre, first measure relative coordinates from that centre, rotate the relative vector and add the centre back. Rotational symmetry order counts how many times a shape fits itself in a full turn, including its starting orientation.

A reliable method

1. Locate the exact centre and relative position of each vertex.

2. Turn by the specified angle and direction.

3. Check centre distances and give a complete description.

Worked example 1

Rotate (4,1) by 90° anticlockwise about the origin.

STEP-BY-STEP SOLUTION

Use (−y,x): image (−1,4). The squared distance from the origin remains 17.

Worked example 2

Rotate (5,2) by 180° about (2,1).

STEP-BY-STEP SOLUTION

Relative vector is (3,1). Reverse it to (−3,−1), then add centre: (−1,0).

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An angle without its centre does not fully determine a rotation. Two rotations of 90° about different centres generally produce different images.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Rotate (2,−5) 90° clockwise about the origin.

2.2 Rotate (−3,4) 180° about the origin.

2.3 Find rotational symmetry order of a regular pentagon.

2.4 Rotate (3,4) 180° about (1,2).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Translations and vectors

A translation moves every point by the same vector. A vector (a,b) in coordinate-pair notation means a horizontal change a and vertical change b; it can also be written as a column vector. The transformation maps (x,y) to (x + a,y + b). A vector describes a displacement, not the location of a point.

Translations preserve lengths, angles and orientation. To describe one, subtract original coordinates from image coordinates in the same order. Check another corresponding pair: every point must have the same displacement. The reverse translation uses the negative vector.

A reliable method

1. Find horizontal and vertical changes from original to image.

2. Apply the same signed changes to every vertex.

3. Verify a second pair and state the vector.

Worked example 1

Translate (−2,5) by vector (4,−3).

STEP-BY-STEP SOLUTION

Add coordinates: (2,2). Four right and three down.

Worked example 2

A = (1,−4) maps to A′ = (−3,2). Find translation.

STEP-BY-STEP SOLUTION

A′ − A = (−4,6): four left and six up.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A translation vector has no compulsory starting point. The same arrow can be drawn anywhere while preserving its displacement.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Translate (5,1) by (−7,4).

3.2 Find vector mapping (−2,3) to (4,−1).

3.3 Find the inverse of translation (3,−8).

3.4 Apply (2,5) then (−6,1). Give a single translation.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Enlargements and combined transformations — extension

An enlargement is specified by centre C and scale factor k. Each point P maps to C + k(P − C). Positive factors keep image points on the same rays from the centre; a factor between 0 and 1 makes a smaller image. Negative factors place images on the opposite rays and use the magnitude of k for length scaling.

Lengths scale by |k|, areas by k^2. Enlargement preserves angles and produces similar shapes, but it is not generally congruent unless |k| = 1. For combinations, apply the transformations in the stated order. Reflection followed by translation need not match translation followed by reflection. Track a labelled vertex and then verify the whole shape.

A reliable method

1. Form the vector from centre to original point.
2. Multiply by k and add the centre back.
3. For a sequence, use each image as the input to the next operation.

Worked example 1

Enlarge (4,3) from centre (1,1) by factor 2.

STEP-BY-STEP SOLUTION

Relative vector (3,2) becomes (6,4). Image is (7,5).

Worked example 2

Higher: enlarge (3,−2) from the origin by factor −2.

STEP-BY-STEP SOLUTION

Multiply both coordinates by −2: image (−6,4), on the opposite ray and twice as far from the centre.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A negative enlargement factor does not mean a negative length. Its sign controls which side of the centre contains the image; distances use the absolute value.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Enlarge (6,4) from the origin by factor 1/2.

4.2 Higher: enlarge (5,3) from centre (1,1) by factor −1.

4.3 A shape of area 12 is enlarged by factor 3. Find image area.

4.4 Reflect (2,1) in the y-axis, then translate by (3,0). Find image.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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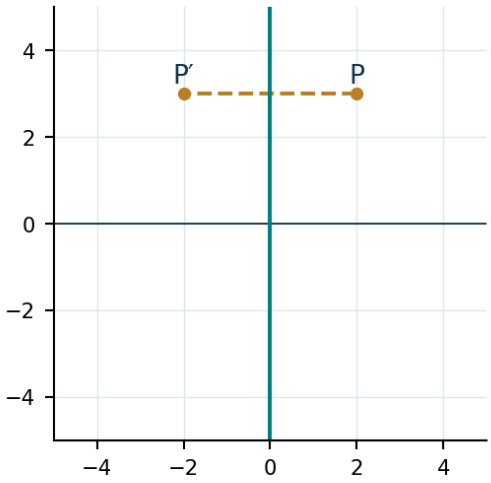
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

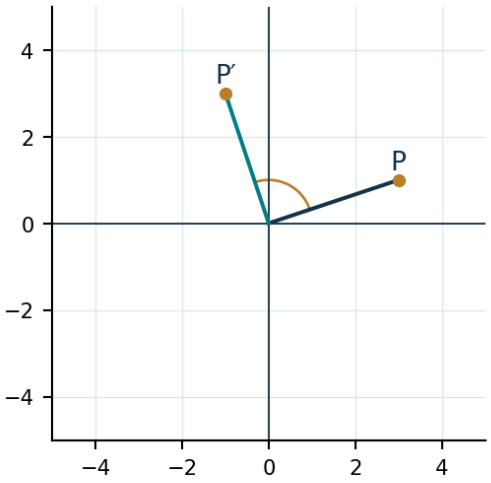
Read the geometry

Describe a transformation completely and track points

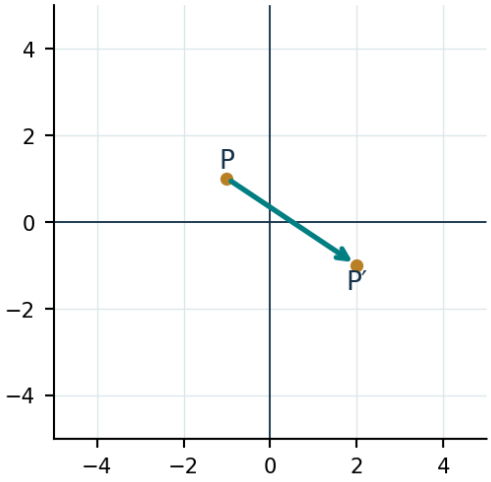
Reflection in the y-axis



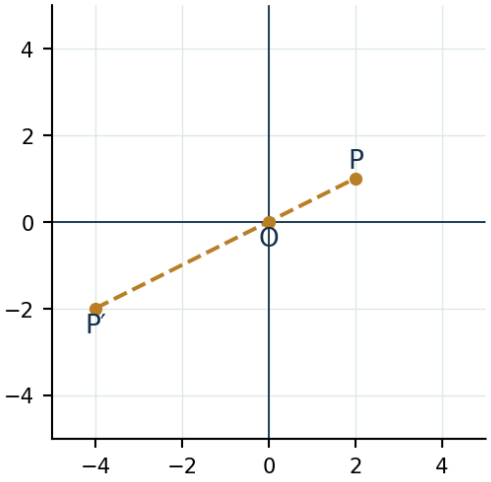
90° anticlockwise about O



Translation by (3, −2)



Enlargement: centre O, k = −2



WHAT THE FIGURES ESTABLISH

The reflected points have equal perpendicular distances from the mirror. Rotation preserves distance from its centre. The translation sends (−1,1) to (2,−1) by adding (3,−2). The negative enlargement sends (2,1) to (−4,−2): opposite ray and twice the distance, not a negative length.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Reflect (5,−3) in the x-axis. [1 mark]

R2. Reflect (−2,4) in x = 1. [2 marks]

R3. Rotate (3,2) 90° clockwise about the origin. [1 mark]

R4. Rotate (7,1) 180° about (2,3). [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Find the translation from (−4,2) to (1,−6). [2 marks]

R6. Enlarge (3,5) from centre (1,1) by factor 2. [2 marks]

R7. Higher: enlarge (4,−1) from centre (2,1) by factor −3. [3 marks]

R8. Translate (2,3) by (4,0), then reflect in the y-axis. Compare with the reversed order. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Reflections and symmetry

1.1 Reflect (−4,5) in the y-axis.

(4,5).

1.2 Reflect (2,−3) in y = x.

(−3,2).

1.3 Find the mirror line for (1,2) and its image (7,2).

x = 4.

1.4 State the number of lines of symmetry of a non-square rectangle.

Two: the perpendicular bisectors of opposite sides.

Lesson 2 • Rotations and rotational symmetry

2.1 Rotate (2,−5) 90° clockwise about the origin.

(−5,−2).

2.2 Rotate (−3,4) 180° about the origin.

(3,−4).

2.3 Find rotational symmetry order of a regular pentagon.

5.

2.4 Rotate (3,4) 180° about (1,2).

Image (2×1 − 3,2×2 − 4) = (−1,0).

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Translations and vectors

3.1 Translate (5,1) by (−7,4).

(−2,5).

3.2 Find vector mapping (−2,3) to (4,−1).

(6,−4).

3.3 Find the inverse of translation (3,−8).

(−3,8).

3.4 Apply (2,5) then (−6,1). Give a single translation.

Add vectors: (−4,6).

Lesson 4 • Enlargements and combined transformations — extension

4.1 Enlarge (6,4) from the origin by factor 1/2.

(3,2).

4.2 Higher: enlarge (5,3) from centre (1,1) by factor −1.

Relative (4,2) becomes (−4,−2); image (−3,−1).

4.3 A shape of area 12 is enlarged by factor 3. Find image area.

12 × 9 = 108 square units.

4.4 Reflect (2,1) in the y-axis, then translate by (3,0). Find image.

First (−2,1), then (1,1).

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Reflect (5,−3) in the x-axis. [1 mark]

(5,3). [1 for point.]

R2. Reflect (−2,4) in x = 1. [2 marks]

Three units left becomes three right: (4,4). [1 for distance; 1 for image.]

R3. Rotate (3,2) 90° clockwise about the origin. [1 mark]

(2,−3). [1 for point.]

R4. Rotate (7,1) 180° about (2,3). [2 marks]

(4 − 7,6 − 1) = (−3,5). [1 for method; 1 for point.]

R5. Find the translation from (−4,2) to (1,−6). [2 marks]

(5,−8). [1 for each component.]

R6. Enlarge (3,5) from centre (1,1) by factor 2. [2 marks]

(1,1) + 2(2,4) = (5,9). [1 for relative vector; 1 for image.]

R7. Higher: enlarge (4,−1) from centre (2,1) by factor −3. [3 marks]

Relative (2,−2); scaled (−6,6); image (−4,7). [1 each stage.]

R8. Translate (2,3) by (4,0), then reflect in the y-axis. Compare with the reversed order. [3 marks]

First order gives (−6,3). Reversed order gives (2,3). They differ. [1 for each image; 1 for comparison.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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