GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 45

45

3D representations

Shared GCSE core

Interpret solids through nets, plans, elevations and cross-sections. Connect different drawings of the same object while recognising that one flat view may hide information. Use coordinates and cube arrangements to reason precisely.

Plan • Elevations • Nets • Sections

A single view may hide the depth or internal arrangement.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Recognise common solids, count square units and read simple grids.

Your route through this notebook

1. Faces, edges, vertices and solid properties
2. Nets and matching face dimensions
3. Plans, elevations and hidden information
4. Isometric drawings, coordinates and cross-sections

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Faces, edges, vertices and solid properties

A polyhedron has flat polygonal faces, straight edges and vertices. A cube has six square faces, twelve edges and eight vertices. A cuboid has the same counts but faces need not all be squares. A triangular prism has two triangular end faces and three rectangular lateral faces.

Curved solids require careful vocabulary: a cylinder has two flat circular faces and a curved surface; a cone has one flat circular base and a curved surface. Euler’s relation $V - E + F = 2$ applies to convex polyhedra, not indiscriminately to curved solids using an inconsistent counting convention. Count a shared edge once, not once for each adjoining face.

A reliable method

1. Identify the solid before counting.

2. Distinguish a flat face from a curved surface.

3. For a convex polyhedron, use Euler’s relation as a check.

Worked example 1

Count faces, edges and vertices of a triangular prism.

STEP-BY-STEP SOLUTION

Faces 5, edges 9, vertices 6. Check: $6 - 9 + 5 = 2$.

Worked example 2

A square-based pyramid has 5 vertices and 5 faces. Find edges.

STEP-BY-STEP SOLUTION

$5 - E + 5 = 2$ gives $E = 8$. Four base edges and four sloping edges confirm it.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A line visible inside a perspective sketch may represent a hidden edge or a face diagonal. A drawn diagonal is not automatically an edge of the solid.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Give cube face,edge,vertex counts.

1.2 Give square-based pyramid counts.

1.3 A convex polyhedron has 8 vertices and 12 edges. Find faces.

1.4 How many triangular end faces has a triangular prism?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Nets and matching face dimensions

A net is a flat arrangement that folds to make a solid without overlap. A closed cube net uses six congruent squares, but not every connected arrangement of six squares folds correctly. Track which faces meet and which will become opposite. A cuboid net must match edge lengths where adjoining faces fold.

A cylinder net consists of two equal circles and a rectangle whose length equals their circumference, not their diameter. A cone net uses a circular sector for the curved surface, whose arc length equals the base circumference; its sector radius is the cone’s slant height. Labels identify these relationships even when the net is not drawn to scale.

A reliable method

1. Identify each face and where folds occur.

2. Match lengths along edges that meet.

3. Check that every required surface is covered once without overlap.

Worked example 1

A cylinder has r = 3 cm and height 8 cm. State net dimensions.

STEP-BY-STEP SOLUTION

Two circles radius 3 cm and a rectangle of dimensions 6π cm by 8 cm.

Worked example 2

A cuboid measures 5 by 3 by 2 cm. List its net faces.

STEP-BY-STEP SOLUTION

Two 5×3 rectangles, two 5×2 rectangles and two 3×2 rectangles.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The rectangle in a cylinder net must wrap once around the circle. A length of 2r would span only a diameter and cannot make the curved surface.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 How many squares form a closed cube net?

2.2 A cylinder has diameter 10 cm and height 7 cm. Give rectangle dimensions.

2.3 What is the radius of a cone’s sector net?

2.4 What must the cone net sector’s arc length equal?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Plans, elevations and hidden information

A plan is a view from directly above. A front elevation looks from the stated front direction; a side elevation looks from a perpendicular side. These are orthographic views, so depth is not drawn as a sloping length. Align common dimensions: plan and front share width, front and side share height, and plan and side share depth.

For vertical stacks of unit cubes, the plan shows occupied floor cells, not the number of cubes in each stack. An elevation shows the maximum visible height along each viewing line. Different hidden arrangements can produce the same silhouettes. Therefore plans and elevations do not always determine a unique total cube count without additional information.

A reliable method

1. Mark the stated front direction.

2. Project along parallel viewing lines, preserving the relevant dimensions.

3. Use maximum stack heights for silhouettes and check what is hidden.

Worked example 1

Two adjacent stacks have heights 2 and 4 cubes. Viewed directly from the front with both side by side, what elevation heights appear?

STEP-BY-STEP SOLUTION

Two columns of heights 2 and 4. The plan has two occupied squares; total cubes 6.

Worked example 2

Two stacks of heights 2 and 4 lie directly behind each other from the front. What front height appears?

STEP-BY-STEP SOLUTION

Only a silhouette of height 4 in that viewing line. The front elevation alone cannot reveal that total cubes are 6.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An elevation silhouette hides cubes behind others. Do not multiply its area by a guessed depth unless the solid is known to be a filled prism.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A filled cuboid is 4 units wide,3 deep,2 high. Give plan dimensions.

3.2 Give its front elevation dimensions when width 4 is front-facing.

3.3 Give its side elevation dimensions.

3.4 A plan has 5 occupied unit cells and every stack is exactly 3 cubes high. Find cube count.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Isometric drawings, coordinates and cross-sections

An isometric sketch uses three directions to show width, depth and height without full perspective. Equal grid steps represent equal lengths along those directions. Count dimensions along the grid axes rather than measuring slanted lines as ordinary horizontal distances.

A cross-section is the shape produced by a plane cut through a solid. A cut parallel to a prism’s ends gives its constant cross-section. A plane parallel to a cone’s base gives a circle, while a vertical cut through its axis gives an isosceles triangle. A sphere cut by a plane through its interior gives a circle, with largest section through its centre. Different cutting directions can give different shapes from one solid.

A reliable method

1. Identify the three grid directions or coordinate axes.

2. Locate features using all three dimensions.

3. Describe a section together with its cutting plane.

Worked example 1

A filled cuboid is 3 cubes wide,2 deep and 4 high. Find number of unit cubes.

STEP-BY-STEP SOLUTION

$3 \times 2 \times 4 = 24$. Hidden cubes count too.

Worked example 2

Describe a cylinder cut parallel to its circular bases and a cut through its axis.

STEP-BY-STEP SOLUTION

Parallel-to-base section is a circle. An axial section is a rectangle of width equal to the diameter and height equal to the cylinder’s height.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Saying “a cone has a triangular cross-section” is incomplete. A triangle occurs for a plane through the axis; other cuts can give circles or other curves.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 State the constant cross-section of a triangular prism.

4.2 A sphere has radius 6 cm. Find the radius of its largest planar section.

4.3 A cuboid occupies $0 \le x \le 5, 0 \le y \le 3, 0 \le z \le 4$. Find dimensions of its section $z = 2$.

4.4 Why can a front elevation alone fail to determine volume?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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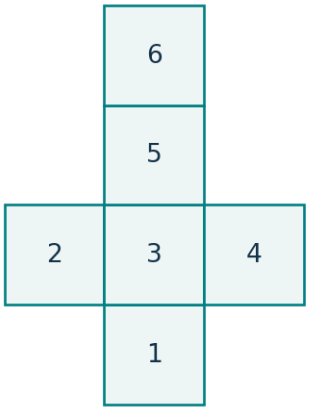
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

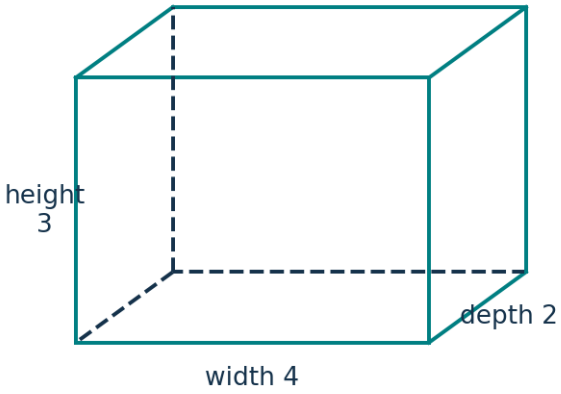
Read the geometry

One solid can have several useful flat representations

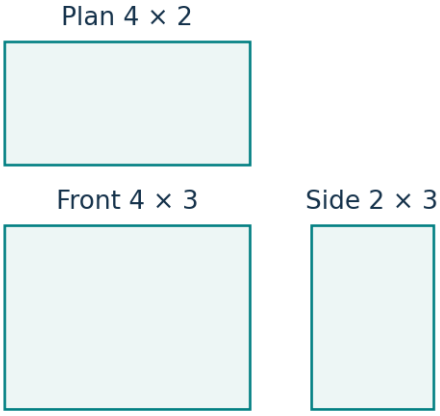
A valid cube net



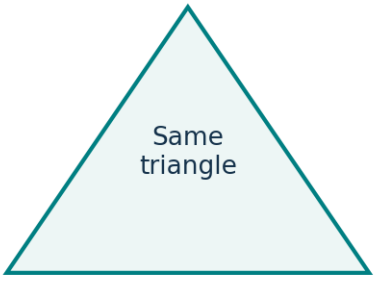
Cuboid: width, depth, height



Orthographic views



Prism cross-section



Cuts parallel to triangular ends

WHAT THE FIGURES ESTABLISH

The six-square net folds into a cube without overlapping faces. The oblique cuboid sketch and three orthographic rectangles describe the same width 4, depth 2, height 3 solid. The plan omits height, while the front and side omit different horizontal dimensions. A prism has congruent parallel cross-sections.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. A triangular prism has 6 vertices and 9 edges. Find faces using Euler’s relation. [2 marks]

R2. List the six faces of a 6 by 4 by 3 cm cuboid net. [3 marks]

R3. A cylinder has radius 4 cm and height 9 cm. Give curved rectangle dimensions in its net. [2 marks]

R4. A filled cuboid is 5 units wide,2 deep,3 high. Give plan,front and side dimensions. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Six occupied plan cells each have height 4 cubes. Find total cubes. [1 mark]

R6. Two stacks are aligned in one viewing line with heights 3 and 5. What silhouette height appears? [1 mark]

R7. Name sections of a right circular cone parallel to the base and through the axis. [2 marks]

R8. A cuboid occupies $0 \le x \le 8, 0 \le y \le 5, 0 \le z \le 3$. Find area of section $x = 4$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Faces, edges, vertices and solid properties

1.1 Give cube face,edge,vertex counts.

6 faces,12 edges,8 vertices.

1.2 Give square-based pyramid counts.

5 faces,8 edges,5 vertices.

1.3 A convex polyhedron has 8 vertices and 12 edges. Find faces.

$F = 2 - 8 + 12 = 6$.

1.4 How many triangular end faces has a triangular prism?

Two congruent parallel triangles.

Lesson 2 • Nets and matching face dimensions

2.1 How many squares form a closed cube net?

Six.

2.2 A cylinder has diameter 10 cm and height 7 cm. Give rectangle dimensions.

10π cm by 7 cm.

2.3 What is the radius of a cone’s sector net?

Its slant height.

2.4 What must the cone net sector’s arc length equal?

The circumference of the cone’s circular base.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Plans, elevations and hidden information

3.1 A filled cuboid is 4 units wide,3 deep,2 high. Give plan dimensions.

4 by 3.

3.2 Give its front elevation dimensions when width 4 is front-facing.

4 by 2.

3.3 Give its side elevation dimensions.

3 by 2.

3.4 A plan has 5 occupied unit cells and every stack is exactly 3 cubes high. Find cube count.

$5 \times 3 = 15$.

Lesson 4 • Isometric drawings, coordinates and cross-sections

4.1 State the constant cross-section of a triangular prism.

A triangle congruent to an end face.

4.2 A sphere has radius 6 cm. Find the radius of its largest planar section.

6 cm, for a plane through the centre.

4.3 A cuboid occupies $0 \le x \le 5, 0 \le y \le 3, 0 \le z \le 4$. Find dimensions of its section $z = 2$.

A 5 by 3 rectangle.

4.4 Why can a front elevation alone fail to determine volume?

It does not show how much material lies along the hidden depth direction.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. A triangular prism has 6 vertices and 9 edges. Find faces using Euler’s relation. [2 marks]

$F = 2 - 6 + 9 = 5$. [1 for relation; 1 for result.]

R2. List the six faces of a 6 by 4 by 3 cm cuboid net. [3 marks]

Two 6×4, two 6×3, two 4×3 rectangles. [1 for each pair.]

R3. A cylinder has radius 4 cm and height 9 cm. Give curved rectangle dimensions in its net. [2 marks]

8π cm by 9 cm. [1 each.]

R4. A filled cuboid is 5 units wide,2 deep,3 high. Give plan,front and side dimensions. [3 marks]

Plan 5×2; front 5×3; side 2×3, with width 5 front-facing. [1 each.]

R5. Six occupied plan cells each have height 4 cubes. Find total cubes. [1 mark]

24. [1 for product.]

R6. Two stacks are aligned in one viewing line with heights 3 and 5. What silhouette height appears? [1 mark]

5. [1 for maximum.]

R7. Name sections of a right circular cone parallel to the base and through the axis. [2 marks]

Circle and isosceles triangle. [1 each.]

R8. A cuboid occupies $0 \le x \le 8, 0 \le y \le 5, 0 \le z \le 3$. Find area of section $x = 4$. [3 marks]

The section spans y and z: 5 by 3, area 15 square units. [1 for plane interpretation; 1 for dimensions; 1 for area.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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