GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 44

44

Volume and surface area

Shared core with Higher extensions

Measure space inside solids and the area of their exposed surfaces. Work through prisms, cylinders, pyramids, cones and spheres, then combine solids and capacity changes. Diagrams distinguish vertical height, cross-section and slant height.

Prism: V = Ah • Pyramid: V = Ah/3

Volume and surface area count different quantities.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Calculate plane areas, use powers and roots, and convert volume and capacity units.

Your route through this notebook

1. Prisms, cuboids and cross-sections
2. Cylinders and curved surfaces
3. Pyramids and cones
4. Spheres, hemispheres and composite solids

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Prisms, cuboids and cross-sections

A prism has a constant cross-section along its length. Its volume equals cross-sectional area multiplied by the perpendicular length through which that section is extended. A cuboid is a rectangular prism, so V = lwh. A triangular prism uses the triangle’s area, not the product of all three visible edge labels.

Surface area counts every exposed face once. For a closed cuboid, S = 2(lw + lh + wh). A prism’s lateral rectangular faces can be found from cross-section perimeter times prism length, then its end faces added. Open containers or joined solids require omitting unexposed faces.

A reliable method

1. Identify the constant cross-section and its perpendicular extension length.

2. Calculate section area before volume.

3. For surface area, list exposed faces and their multiplicities.

Worked example 1

A triangular prism has triangle base 6 cm, triangle height 4 cm and prism length 10 cm. Find volume.

STEP-BY-STEP SOLUTION

Cross-section area = 6 × 4/2 = 12 cm². Volume = 12 × 10 = 120 cm³.

Worked example 2

Find surface area of a closed 5 by 4 by 3 cm cuboid.

STEP-BY-STEP SOLUTION

2(5×4 + 5×3 + 4×3) = 2(20 + 15 + 12) = 94 cm².

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The word length can name different dimensions. Label the triangular height separately from the prism’s extension length before substituting.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find volume of a 7 by 5 by 4 cm cuboid.

1.2 A prism has section area 18 cm^2 and length 12 cm. Find volume.

1.3 Find surface area of a cube side 6 cm.

1.4 An open-top 8 by 5 by 4 cm cuboid has no lid. Find exposed material area, ignoring thickness.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Cylinders and curved surfaces

A cylinder is a circular prism with volume $\pi r^2 h$. Its curved surface unwraps to a rectangle with width equal to the base circumference $2\pi r$ and height h. Curved area is therefore $2\pi rh$. A closed cylinder adds two circular ends, giving total area $2\pi rh + 2\pi r^2$.

Use radius rather than diameter in squared terms and keep length units consistent. A pipe or open cylinder may expose one end, no ends or inner surfaces as well as outer ones, depending on the question. State which surfaces are counted. Capacity usually refers to internal volume, so external dimensions require a thickness model before they determine capacity.

A reliable method

1. Find the radius and perpendicular height.

2. Use $\pi r^2 h$ for volume and $2\pi rh$ for curved area.

3. Add only the stated exposed circular or annular ends.

Worked example 1

A closed cylinder has r = 3 cm, h = 10 cm. Find volume and total area exactly.

STEP-BY-STEP SOLUTION

Volume $9\pi \times 10 = 90\pi$ cm³. Area $2\pi \times 3 \times 10 + 2\pi \times 9 = 78\pi$ cm².

Worked example 2

A cylinder has volume 200π cm³ and radius 5 cm. Find height.

STEP-BY-STEP SOLUTION

$25\pi h = 200\pi$, so h = 8 cm.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Volume and curved surface area use similar symbols but different dimensions. $\pi r^2 h$ has cubic units; $2\pi rh$ has square units.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find cylinder volume for r = 4 cm,h = 7 cm.

2.2 Find curved area for r = 5 m,h = 6 m.

2.3 An open-top cylinder has r = 2 cm,h = 9 cm. Find material area excluding thickness.

2.4 A cylinder diameter is 10 cm and height 12 cm. Find volume.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Pyramids and cones

A pyramid or cone has volume one third of base area times perpendicular height. For a cone, $V = \pi r^2 h/3$. The vertical height is not the same as its slant height. For a right circular cone, $l^2 = r^2 + h^2$, so Pythagoras links them.

A cone's curved surface area is πrl, using slant height l; total area adds its circular base when exposed. A pyramid's surface area is found from its actual triangular faces and any exposed base. A face's perpendicular triangle height may differ from the solid's vertical height. Draw the relevant right triangle to identify which length is needed.

A reliable method

1. Use perpendicular solid height for volume.

2. Use each face's perpendicular height, or cone slant height, for surface area.

3. Find missing heights with Pythagoras when the geometry is right-angled.

Worked example 1

A cone has $r = 3$ cm,$h = 4$ cm. Find volume and closed total area.

STEP-BY-STEP SOLUTION

Volume $= (1/3)9\pi \times 4 = 12\pi$ cm³. Slant $l = 5$ cm. Total area $= \pi \times 3 \times 5 + 9\pi = 24\pi$ cm².

Worked example 2

A square-based pyramid has base side 6 cm and vertical height 4 cm. Find volume and surface area.

STEP-BY-STEP SOLUTION

Volume $= 36 \times 4/3 = 48$ cm³. Face height $= \sqrt{(4^2 + 3^2)} = 5$ cm. Four faces total $4 \times (6 \times 5/2) = 60$ cm²; with base, area 96 cm².

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For the pyramid face-height triangle, the horizontal distance is from the base centre to the midpoint of an edge, not to a corner. Using the diagonal half-length would find a different sloping edge.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find volume of a cone with r = 6 cm,h = 8 cm.

3.2 For that cone, find slant height and curved area.

3.3 A pyramid has base area 45 m^2 and height 9 m. Find volume.

3.4 A cone has slant height 13 cm and radius 5 cm. Find vertical height.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Spheres, hemispheres and composite solids

A sphere has volume $4\pi r^3/3$ and surface area $4\pi r^2$. A hemisphere has half the volume and half the curved surface, but its total closed surface also includes a circular base. Thus hemisphere volume is $2\pi r^3/3$, curved area $2\pi r^2$ and closed total area $3\pi r^2$.

For joined solids, add volumes if they occupy non-overlapping regions and subtract any removed volume. Surface area counts only exposed surfaces: two joined matching circles are internal. A hollow shape may need inner and outer surfaces plus rims. Keep exact expressions as long as possible, and check whether a final answer asks for capacity, exposed area or remaining material.

A reliable method

1. Use the stated radius in squared and cubed formulas.

2. Add or subtract non-overlapping volumes.

3. List exposed surfaces separately and omit internal joins.

Worked example 1

Find exact volume and surface area of a sphere radius 3 cm.

STEP-BY-STEP SOLUTION

Volume = $4\pi\times27/3 = 36\pi$ cm³. Area = $4\pi\times9 = 36\pi$ cm². Equal numerical coefficients here do not mean the physical quantities are the same.

Worked example 2

A cylinder r = 2 cm,h = 5 cm has a matching hemisphere on top and a flat base below. Find volume and exterior area.

STEP-BY-STEP SOLUTION

Volume = $20\pi + 16\pi/3 = 76\pi/3$ cm³. Exterior = cylinder curved 20π + hemisphere curved 8π + bottom $4\pi = 32\pi$ cm².

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Halving a sphere’s surface area gives only the curved area of a hemisphere. A separate flat base must be added if the hemisphere is closed.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Find sphere volume for r = 6 cm.

4.2 Find closed hemisphere total area for r = 4 cm.

4.3 A sphere radius 5 cm is removed from a 12 cm cube. Find remaining volume exactly.

4.4 A solid cylinder has a matching cone on top. Should the joining circle be included in exterior area?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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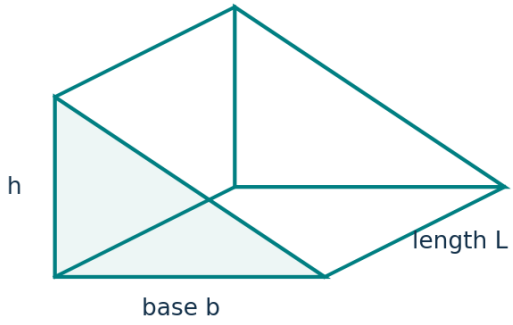
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

Read the geometry

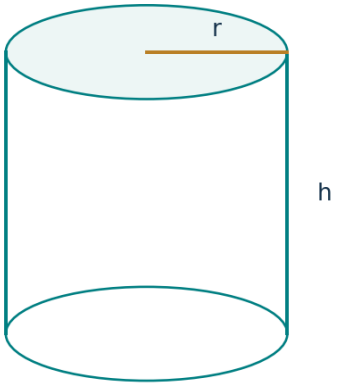
Different solid dimensions belong in different formulas

Triangular prism

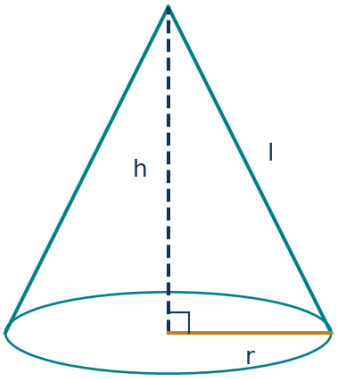
V = (bh/2)L



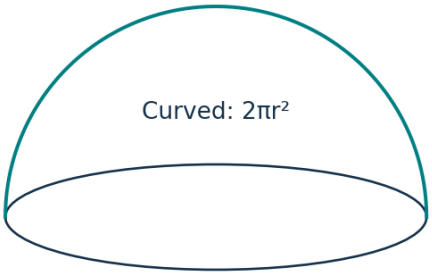
Cylinder



Right circular cone



Hemisphere: curved plus base



Closed total: 3πr²

WHAT THE FIGURES ESTABLISH

A triangular prism uses triangle area times extension length. A cone's volume uses perpendicular height h, while curved area uses slant height l, with $l^2 = r^2 + h^2$. The closed hemisphere includes its circular base: curved area $2\pi r^2$ plus base πr^2. Perspective sketches are illustrative.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find volume of a prism with section area 24 cm² and length 15 cm. [2 marks]

R2. Find closed cuboid surface area for dimensions 9,5,4 cm. [3 marks]

R3. Find closed cylinder volume and area for r = 4 cm,h = 10 cm. [4 marks]

R4. A cone has r = 5 cm,h = 12 cm. Find volume and curved area. [4 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. A square pyramid has base side 10 cm and vertical height 12 cm. Find volume. [2 marks]

R6. Find sphere surface area and volume for r = 2 cm. [4 marks]

R7. Find closed hemisphere total area and volume for r = 3 cm. [4 marks]

R8. A cylinder r = 3 cm,h = 8 cm has a matching hemisphere on top and exposed bottom. Find volume and exterior area. [5 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Prisms, cuboids and cross-sections

1.1 Find volume of a 7 by 5 by 4 cm cuboid.

140 cm^3.

1.2 A prism has section area 18 cm^2 and length 12 cm. Find volume.

216 cm^3.

1.3 Find surface area of a cube side 6 cm.

$6 \times 36 = 216$ cm^2.

1.4 An open-top 8 by 5 by 4 cm cuboid has no lid. Find exposed material area, ignoring thickness.

Base 40; two 8×4 faces total 64; two 5×4 faces total 40. Total 144 cm^2.

Lesson 2 • Cylinders and curved surfaces

2.1 Find cylinder volume for r = 4 cm,h = 7 cm.

112π cm^3.

2.2 Find curved area for r = 5 m,h = 6 m.

60π m^2.

2.3 An open-top cylinder has r = 2 cm,h = 9 cm. Find material area excluding thickness.

Curved 36π plus one base $4\pi = 40\pi$ cm^2.

2.4 A cylinder diameter is 10 cm and height 12 cm. Find volume.

Radius 5 cm, volume 300π cm^3.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Pyramids and cones

3.1 Find volume of a cone with r = 6 cm,h = 8 cm.

96π cm³.

3.2 For that cone, find slant height and curved area.

l = 10 cm; curved area 60π cm².

3.3 A pyramid has base area 45 m² and height 9 m. Find volume.

135 m³.

3.4 A cone has slant height 13 cm and radius 5 cm. Find vertical height.

√(13² − 5²) = 12 cm.

Lesson 4 • Spheres, hemispheres and composite solids

4.1 Find sphere volume for r = 6 cm.

288π cm³.

4.2 Find closed hemisphere total area for r = 4 cm.

3π×16 = 48π cm².

4.3 A sphere radius 5 cm is removed from a 12 cm cube. Find remaining volume exactly.

1728 − 500π/3 cm³.

4.4 A solid cylinder has a matching cone on top. Should the joining circle be included in exterior area?

No. It is internal; add only exposed curved and end surfaces.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find volume of a prism with section area 24 cm² and length 15 cm. [2 marks]

360 cm³. [1 for multiplication; 1 for units/result.]

R2. Find closed cuboid surface area for dimensions 9,5,4 cm. [3 marks]

2(45 + 36 + 20) = 202 cm². [1 for face areas; 1 for pairs; 1 for total.]

R3. Find closed cylinder volume and area for r = 4 cm,h = 10 cm. [4 marks]

V = 160π cm³; S = 80π + 32π = 112π cm². [1 for each formula; 1 for each result.]

R4. A cone has r = 5 cm,h = 12 cm. Find volume and curved area. [4 marks]

l = 13 cm; V = 100π cm³; curved area 65π cm². [1 for slant; 1 for volume method; 1 for volume; 1 for curved area.]

R5. A square pyramid has base side 10 cm and vertical height 12 cm. Find volume. [2 marks]

100 × 12/3 = 400 cm³. [1 for formula; 1 for result.]

R6. Find sphere surface area and volume for r = 2 cm. [4 marks]

Area 16π cm²; volume 32π/3 cm³. [1 for each formula; 1 for each result.]

R7. Find closed hemisphere total area and volume for r = 3 cm. [4 marks]

Area 27π cm²; volume 18π cm³. [1 for curved/base distinction; 1 for area; 1 for half-sphere volume; 1 for result.]

R8. A cylinder r = 3 cm,h = 8 cm has a matching hemisphere on top and exposed bottom. Find volume and exterior area. [5 marks]

Volume = 72π + 18π = 90π cm³. Area = 48π + 18π + 9π = 75π cm². [1 for each volume; 1 for sum; 1 for exposed surfaces; 1 for area.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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