GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 43

43

Circles, arcs and sectors

Shared core with Higher extensions

Connect radius, diameter, circumference and area, then calculate arcs, sectors and composite circular regions. Keep exact multiples of π until a decimal is requested and distinguish curved boundary length from area.

$$C = 2\pi r \qquad A = \pi r^2$$

Arc and sector fractions come from the central angle.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

INDEPENDENT GCSE MATHS • WEBSITE EDITION • NOVEMBER 2026

© 2026 GCSE Online Tutoring. All rights reserved.

GCSE ONLINE TUTORING / ABOUT THIS BOOK

Your learning guide

© 2026 GCSE Online Tutoring. All rights reserved.

Free for personal and household educational use. Tutors and schools must obtain permission before copying or distributing these books to learners. Resale, paid redistribution, removal of branding and uploading to third-party websites are prohibited without written permission. For permissions, visit www.gcseonlinetutoring.uk.

Independent educational resource

All explanations, examples, diagrams, questions and marking guidance are independently written. No exam-board questions or mark schemes are reproduced. This book is not affiliated with, approved by or endorsed by any awarding organisation.

Before you begin

Calculate with fractions and powers and use the area formulas for triangles and rectangles.

Your route through this notebook

1. Circle vocabulary and circumference
2. Circle area and circular regions
3. Arc lengths and sector areas
4. Composite regions and circular segments — Higher extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 1 / UNDERSTAND

Circle vocabulary and circumference

The radius joins the centre to the circumference; the diameter passes through the centre and is twice the radius. A chord joins two points on the circumference. A tangent touches a circle at one point and is perpendicular to its radius there. An arc is part of the circumference.

Circumference is the full boundary length, $C = \pi d = 2\pi r$. The constant π is the ratio of circumference to diameter. Use the calculator π key or retain π exactly rather than replacing it early by 3.14. If a diameter is supplied, either use πd directly or halve it before using a radius formula.

A reliable method

1. Identify whether the given length is a radius or diameter.

2. Use $C = \pi d$ or $C = 2\pi r$.

3. Retain π for exact answers and use length units.

Worked example 1

Find circumference for radius 7 cm exactly.

STEP-BY-STEP SOLUTION

$C = 2\pi \times 7 = 14\pi$ cm, approximately 43.98 cm.

Worked example 2

A circle has circumference 18π m. Find its diameter and radius.

STEP-BY-STEP SOLUTION

$\pi d = 18\pi$ gives $d = 18$ m and $r = 9$ m.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A chord need not pass through the centre. Only a diameter is always twice the radius; an arbitrary chord cannot be substituted for d.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find diameter when radius is 4.5 cm.

1.2 Find exact circumference for diameter 12 m.

1.3 Find radius for circumference 10π cm.

1.4 State the angle between a tangent and radius at contact.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 2 / UNDERSTAND

Circle area and circular regions

Circle area is $A = \pi r^2$. The radius is squared, so doubling radius multiplies area by four. A semicircle has half the area of its full circle. An annulus, or ring, has area $\pi R^2 - \pi r^2$ for outer radius R and inner radius r.

For a semicircle’s perimeter include the diameter as well as the curved half-circumference. For a ring’s total boundary length include both circles when the whole boundary is requested. Area subtraction does not imply the same subtraction for boundary length. Sketch and highlight the boundary separately from the shaded region.

A reliable method

1. Use the radius in every squared term.

2. For a region, add or subtract relevant areas.

3. For a perimeter, trace every exposed straight and curved edge.

Worked example 1

Find area and perimeter of a semicircle of radius 5 cm exactly.

STEP-BY-STEP SOLUTION

Area = $25\pi/2$ cm². Perimeter = half-circumference plus diameter = $5\pi + 10$ cm.

Worked example 2

A ring has outer radius 8 cm and inner radius 3 cm. Find area.

STEP-BY-STEP SOLUTION

$A = \pi(8^2 - 3^2) = 55\pi$ cm².

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

$\pi(R - r)^2$ is not a ring's area. Squaring the thickness ignores where that thickness is distributed. Subtract the areas of the complete circles.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find exact area of a circle with diameter 14 cm.

2.2 Find area of a semicircle of diameter 8 m.

2.3 Find ring area with radii 10 cm and 6 cm.

2.4 Find total boundary length of that ring.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 3 / UNDERSTAND

Arc lengths and sector areas

A sector is a fraction of a circle defined by a central angle. For angle θ in degrees, arc length is $(\theta/360)2\pi r$ and sector area is $(\theta/360)\pi r^2$. Both scale with the fraction of a full turn, but one is a length and the other an area.

A sector's perimeter normally includes two radii and its arc. A reflex sector uses an angle greater than 180°; use the specified region rather than always choosing the smaller angle. To recover an angle, compare the given arc or sector area with the corresponding full-circle quantity. These formulas here use degree measure, not radians.

A reliable method

1. Find the central-angle fraction θ/360.
2. Multiply by full circumference or full area as required.
3. For sector perimeter add both radii.

Worked example 1

Find arc length and sector area for r = 6 cm, θ = 120°.

STEP-BY-STEP SOLUTION

The fraction is 1/3. Arc = $(1/3)12\pi = 4\pi$ cm. Area = $(1/3)36\pi = 12\pi$ cm².

Worked example 2

A radius-9 sector has arc length 6π cm. Find its angle.

STEP-BY-STEP SOLUTION

Full circumference is 18π, so the arc is 1/3 of the circle. Angle = 120°.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An angle on the circumference is not the central angle for these formulas. If a circle theorem supplies a different angle, convert to the angle at the centre first.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find arc length for r = 8 cm and angle 90°.

3.2 Find sector area for r = 10 m and angle 72°.

3.3 Find perimeter of a sector with r = 4 cm and angle 90°.

3.4 A radius-6 sector has area 9π cm^2. Find angle.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 4 / UNDERSTAND

Composite regions and circular segments — Higher extension

A circular segment is bounded by a chord and an arc. For a minor segment with central angle below 180°, subtract the triangle formed by two radii from the sector. The triangle area is $r^2\sin\theta/2$, using the included central angle in degrees. A major segment is the full circle minus the minor segment.

Composite shapes may combine circular and straight-edged parts. Separate area from perimeter: a line where two shapes join is internal and does not count towards the outer boundary. Use exact expressions where possible, retain unrounded intermediate values and state the final accuracy. Explain which region is being removed rather than applying a memorised subtraction without reference to a sketch.

A reliable method

1. Identify the sector and the triangle or adjoining shape.

2. Add or subtract areas, using perpendicular or trigonometric triangle area.

3. Trace only exposed edges for boundary questions.

Worked example 1

A radius-6 circle has a 60° minor segment. Find its exact area.

STEP-BY-STEP SOLUTION

Sector = 6π cm². Triangle = $(1/2)36\sin60° = 9\sqrt{3}$ cm². Segment = $6\pi - 9\sqrt{3}$ cm².

Worked example 2

A 10 m by 6 m rectangle has a semicircle of diameter 6 m attached along one short side. Find area and perimeter.

STEP-BY-STEP SOLUTION

Area = $60 + 9\pi/2$ m². Perimeter excludes the shared diameter: $10 + 10 + 6 + 3\pi = 26 + 3\pi$ m.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For a minor segment, the triangle lies inside its sector when $0° < \theta < 180°$. For a reflex angle, use a carefully defined major-region method instead of subtracting a signed sine term blindly.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 A quarter-circle radius 4 has its right triangle removed. Find segment area.

4.2 A 12 by 8 rectangle contains a circular hole of radius 2. Find remaining area.

4.3 A sector has $r = 5$ and $\theta = 120°$. Express its minor segment area exactly.

4.4 For a rectangle with an attached semicircle, why omit the diameter from outer perimeter?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

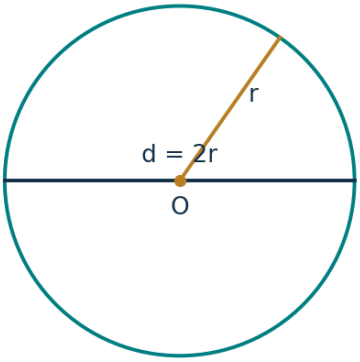
© 2026 GCSE Online Tutoring. All rights reserved.

VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

Read the geometry

Circle formulas use the radius and the correct region

Radius and diameter

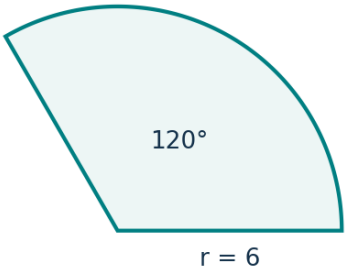


Semicircle boundary



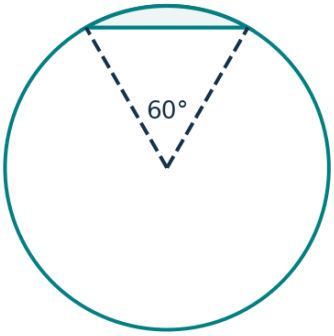
diameter 2r

120° sector, r = 6



Arc 4π; area 12π

60° minor segment



Segment = sector − triangle

WHAT THE FIGURES ESTABLISH

A semicircle perimeter includes its diameter. The 120° sector is one third of its full circle: with radius 6, arc length is 4π and sector area 12π. A minor segment is the area between chord and arc; subtract the triangle formed by the radii from its sector.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

© 2026 GCSE Online Tutoring. All rights reserved.

MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find exact circumference for diameter 18 cm. [1 mark]

R2. Find exact area for radius 9 cm. [1 mark]

R3. A circle has area 64π cm^2. Find radius. [2 marks]

R4. Find perimeter of a semicircle of radius 7 m. [2 marks]

© 2026 GCSE Online Tutoring. All rights reserved.

MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. Find arc and sector area for r = 12 cm and θ = 150°. [4 marks]

R6. A ring has radii 9 cm and 5 cm. Find exact area. [2 marks]

R7. Higher: find the minor segment area for r = 8 cm, θ = 90°. [3 marks]

R8. A 14 m by 10 m rectangle has a semicircle of diameter 10 m attached at a short side. Find area and perimeter exactly. [4 marks]

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Circle vocabulary and circumference

1.1 Find diameter when radius is 4.5 cm.

9 cm.

1.2 Find exact circumference for diameter 12 m.

12π m.

1.3 Find radius for circumference 10π cm.

2πr = 10π, so r = 5 cm.

1.4 State the angle between a tangent and radius at contact.

90°.

Lesson 2 • Circle area and circular regions

2.1 Find exact area of a circle with diameter 14 cm.

Radius 7 cm; area 49π cm².

2.2 Find area of a semicircle of diameter 8 m.

Radius 4 m; area 8π m².

2.3 Find ring area with radii 10 cm and 6 cm.

64π cm².

2.4 Find total boundary length of that ring.

Outer 20π plus inner 12π = 32π cm.

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Arc lengths and sector areas

3.1 Find arc length for r = 8 cm and angle 90°.

4π cm.

3.2 Find sector area for r = 10 m and angle 72°.

20π m².

3.3 Find perimeter of a sector with r = 4 cm and angle 90°.

Arc 2π cm plus 8 cm: 8 + 2π cm.

3.4 A radius-6 sector has area 9π cm². Find angle.

Fraction 9π/36π = 1/4, so 90°.

Lesson 4 • Composite regions and circular segments — Higher extension

4.1 A quarter-circle radius 4 has its right triangle removed. Find segment area.

Sector 4π, triangle 8; segment 4π − 8 square units.

4.2 A 12 by 8 rectangle contains a circular hole of radius 2. Find remaining area.

96 − 4π square units.

4.3 A sector has r = 5 and θ = 120°. Express its minor segment area exactly.

Sector 25π/3; triangle 25√3/4; segment 25π/3 − 25√3/4 square units.

4.4 For a rectangle with an attached semicircle, why omit the diameter from outer perimeter?

It is the internal joining edge and is not exposed on the outside boundary.

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find exact circumference for diameter 18 cm. [1 mark]

18π cm. [1 for result.]

R2. Find exact area for radius 9 cm. [1 mark]

81π cm². [1 for result.]

R3. A circle has area 64π cm². Find radius. [2 marks]

$r^2 = 64$, so positive r = 8 cm. [1 for isolation; 1 for root.]

R4. Find perimeter of a semicircle of radius 7 m. [2 marks]

7π + 14 m. [1 for arc; 1 for diameter.]

R5. Find arc and sector area for r = 12 cm and θ = 150°. [4 marks]

Arc = (150/360)24π = 10π cm; area = (150/360)144π = 60π cm². [1 for each setup; 1 for each answer.]

R6. A ring has radii 9 cm and 5 cm. Find exact area. [2 marks]

π(81 − 25) = 56π cm². [1 for subtraction; 1 for result.]

R7. Higher: find the minor segment area for r = 8 cm, θ = 90°. [3 marks]

Sector 16π; triangle 32; segment 16π − 32 cm². [1 each stage.]

R8. A 14 m by 10 m rectangle has a semicircle of diameter 10 m attached at a short side. Find area and perimeter exactly. [4 marks]

Area 140 + 25π/2 m². Perimeter 14 + 14 + 10 + 5π = 38 + 5π m. [1 for each area component; 1 for exposed straight edges; 1 for full perimeter.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

© 2026 GCSE Online Tutoring. All rights reserved.