GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 40

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Constructions and loci

Shared GCSE core

Use ruler and compasses to construct exact geometric relationships, then describe paths and regions defined by distance conditions. Keep construction arcs visible and distinguish an exact construction from a measured sketch.

Construct. Keep the arcs. Explain the locus.

Equal distances determine the required line or region.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Measure lengths and angles and recognise perpendicular lines, bisectors and triangles.

Your route through this notebook

1. Perpendicular bisectors and equidistant points
2. Angle bisectors and perpendiculars
3. Constructing triangles from sufficient information
4. Loci and intersecting conditions

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Perpendicular bisectors and equidistant points

The perpendicular bisector of a segment meets it at 90° and divides it into equal lengths. Every point on this line is equidistant from the segment's endpoints. This property explains the compass method: intersections of equal-radius circles centred on the endpoints are equally far from both.

Open the compasses to more than half the segment length. Without changing that opening, draw arcs above and below the segment from both endpoints. Join the two arc intersections with a straightedge. A measured midpoint can be a useful check, but measuring and drawing a right angle with a protractor is not the same requested ruler-and-compass construction.

A reliable method

1. Use one opening greater than half the segment length.

2. Draw matching arcs from both endpoints above and below the segment.

3. Join intersections and retain the arcs as evidence.

Worked example 1

Construct the perpendicular bisector of AB, with AB = 6 cm.

STEP-BY-STEP SOLUTION

Draw AB = 6 cm. Use equal radii greater than 3 cm from A and B to create two intersections. Join those intersections. The line meets AB at its 3 cm midpoint at right angles.

Worked example 2

Describe all points equally distant from fixed points P and Q.

STEP-BY-STEP SOLUTION

They form the perpendicular bisector of PQ, extending indefinitely in both directions unless another boundary is given.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Arcs of radius less than half the endpoint separation cannot intersect, so they cannot locate the bisector. Different radii at the two endpoints would instead define unequal distances.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 For AB = 8 cm, can radius 3 cm construct its perpendicular bisector?

1.2 State two properties of a perpendicular bisector.

1.3 A point lies on the perpendicular bisector of AB. What equality follows?

1.4 Why should construction arcs remain visible?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Angle bisectors and perpendiculars

An angle bisector splits an angle into two equal angles. Draw an arc centred at the vertex to cut both arms, then equal-radius arcs from those cut points to meet inside the angle. Join the vertex to that intersection. Points on the internal angle bisector have equal perpendicular distances from the two arms.

To construct a perpendicular from an external point to a line, first mark two equal-distance points on the line with an arc centred at the external point. Their perpendicular bisector passes through the external point. To erect a perpendicular at a point on a line, mark equal distances along the line on both sides and use the same equal-radius intersection idea.

A reliable method

1. Create equal reference distances with an initial arc.

2. Use matching compass openings for the paired arcs.

3. Join the appropriate vertex or external point to the arc intersection.

Worked example 1

Construct the bisector of a 70° angle. What should a protractor check show?

STEP-BY-STEP SOLUTION

Use the compass procedure, then measure only as a check. Each resulting angle should be 35°, subject to drawing accuracy.

Worked example 2

Construct a perpendicular from point P outside line l.

STEP-BY-STEP SOLUTION

Draw an arc centred at P meeting l at A and B. Construct equal-radius arc intersections from A and B; their bisector line passes through P and crosses l at 90°.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A point’s distance from a line means the shortest, perpendicular distance. Equal sloping distances to selected points on the arms do not define the angle-bisector locus.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 An angle of 124° is bisected. Find each angle.

2.2 What distance is used for a point-to-line locus?

2.3 What stays equal when drawing arcs from the two marked points in an angle-bisector construction?

2.4 Why must the initial angle-bisector arc have its centre at the vertex?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Constructing triangles from sufficient information

A triangle can be determined by three sides (SSS), two sides and their included angle (SAS), or two angles and a corresponding side (ASA or AAS). With SSS, draw one side and use the remaining two side lengths as radii from its endpoints. An arc intersection locates the third vertex; the reflected position on the other side gives a congruent triangle.

The longest side must be shorter than the sum of the other two for a non-degenerate triangle. Three angles alone determine shape but not size. Two sides and a non-included angle do not always determine a unique triangle, so do not treat SSA as a general unique-construction condition.

A reliable method

1. Identify the supplied sides and included or corresponding angles.

2. Draw a convenient given side first.

3. Use compass intersections or measured rays, keeping the method visible.

Worked example 1

Construct a triangle with sides 5 cm,6 cm,7 cm.

STEP-BY-STEP SOLUTION

Draw base 7 cm. From one endpoint draw radius 5 cm; from the other draw radius 6 cm. Join either common arc intersection to both endpoints.

Worked example 2

Can a triangle have side lengths 3 cm,4 cm,8 cm?

STEP-BY-STEP SOLUTION

No. The two shorter sides total 7 cm, less than 8 cm, so their arcs cannot meet to form the triangle.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A sketch with three correct-looking angles can be enlarged without changing those angles. This is why AAA establishes similarity, not a unique triangle size.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Can sides 2 cm,5 cm,6 cm form a triangle?

3.2 Can sides 2 cm,3 cm,5 cm form a non-degenerate triangle?

3.3 Which angle must be given for an SAS construction?

3.4 A triangle has angles 45° and 65°. Find its third angle before construction.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 4 / UNDERSTAND

Loci and intersecting conditions

A locus is the set of all points satisfying a condition. Points a fixed distance r from a fixed point form a circle. Points a fixed perpendicular distance from a line form two parallel lines, one on each side, unless the domain restricts the side. Points equally distant from two intersecting lines lie on their two angle bisectors; inside a specified angle, use its internal bisector.

A region within r of a point is a filled disc, not only its circumference. To satisfy two conditions at once, take the overlap. An exact-distance condition may give a boundary, a within-distance condition a region, and combining them may give two points, one point or none. Label which region is wanted and include boundaries when equality is allowed.

A reliable method

1. Translate each condition into its geometric locus or region.

2. Draw every required boundary accurately.

3. Retain the intersection and state whether boundaries belong.

Worked example 1

Describe points within 3 cm of A and closer to A than B.

STEP-BY-STEP SOLUTION

Intersect the disc of radius 3 cm centred A with the A-side of the perpendicular bisector of AB. The circle boundary is included by within; the bisector is excluded by strictly closer.

Worked example 2

Describe points exactly 2 cm from line l and exactly 4 cm from P.

STEP-BY-STEP SOLUTION

Intersect the two parallels 2 cm from l with the circle of radius 4 cm centred P. The number of intersections depends on the placement.

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LESSON 4 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A circle and a disc are different sets. “Exactly 4 cm away” gives a circumference; “at most 4 cm away” includes the inside as well.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Describe the locus exactly 5 cm from a point O.

4.2 Describe points at most 2 cm from O.

4.3 Describe points equidistant from A and B and also on a given line l.

4.4 How do you combine the conditions inside region R and inside region S?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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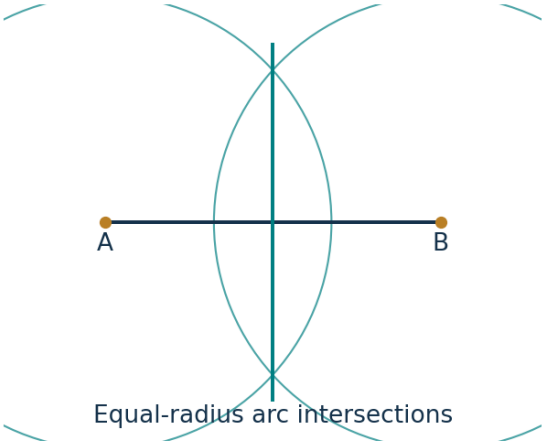
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

Read the geometry

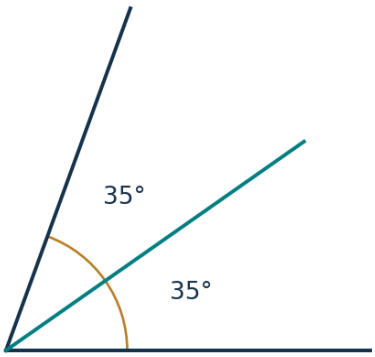
Compass constructions reveal equal-distance relationships

Perpendicular bisector

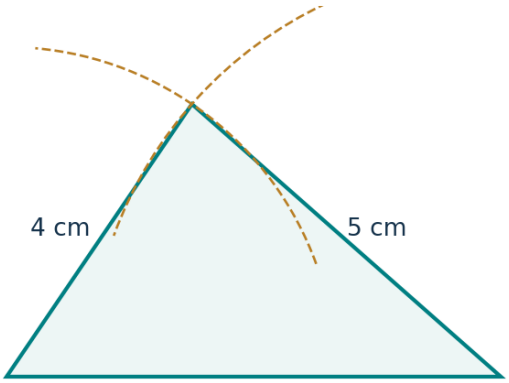


Equal-radius arc intersections

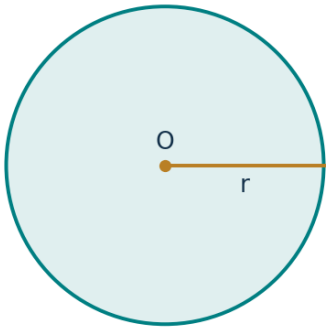
Angle bisector



SSS intersection



A locus and a region



Circle: exactly r
Disc: at most r

WHAT THE FIGURES ESTABLISH

Equal-radius intersections locate a perpendicular bisector. The angle-bisector panel shows the target 35° halves of a 70° angle; the lesson gives the compass steps. In the SSS example, a 6 cm base and arcs of radius 4 and 5 locate the third vertex. Construction working pages elsewhere use true centimetre lengths; these overview drawings are illustrative.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.

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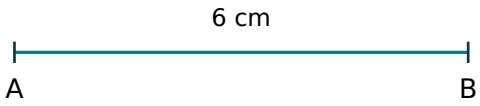
CONSTRUCTION LABORATORY / RULER & COMPASSES

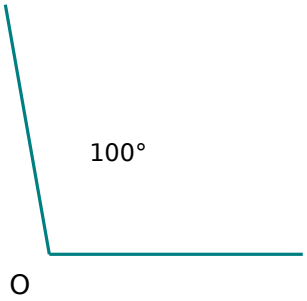
Build the relationship

Print at actual size (100%), not fit to page. Verify the segment AB measures 6 cm before using it as a length construction. If your print scaling differs, draw a fresh 6 cm base on the workspace. Use ruler and compasses; retain construction arcs.

1. Construct the perpendicular bisector of AB. Keep all arcs.



2. Construct the bisector of the printed 100° angle.



SELF-CHECK

The first line should cross the 6 cm segment at its 3 cm midpoint at 90°. The second bisector should create two 50° angles. Verify with a ruler or protractor only after completing the requested construction.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Draw AB = 6 cm and construct its perpendicular bisector with ruler and compasses. [3 marks]

R2. Construct the internal bisector of a 100° angle. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R3. Construct an SSS triangle with sides 4 cm,5 cm,6 cm. [3 marks]

R4. Explain why lengths 4 cm,5 cm,10 cm cannot form a triangle. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 3

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R5. Draw a straight line and construct a perpendicular from a point P not on it. [3 marks]

R6. Describe and sketch the locus of points exactly 3 cm from O. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 4

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R7. For AB = 8 cm, construct the region within 5 cm of A and closer to A than B. [4 marks]

R8. Inside a 90° angle, describe points equally distant from both arms and within 4 cm of its vertex. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Perpendicular bisectors and equidistant points

1.1 For AB = 8 cm, can radius 3 cm construct its perpendicular bisector?

No. The opening must exceed 4 cm to create two intersections.

1.2 State two properties of a perpendicular bisector.

It is at right angles to the segment and passes through its midpoint.

1.3 A point lies on the perpendicular bisector of AB. What equality follows?

Its distances to A and B are equal.

1.4 Why should construction arcs remain visible?

They show the equal-radius construction and provide evidence of the method, not just its final line.

Lesson 2 • Angle bisectors and perpendiculars

2.1 An angle of 124° is bisected. Find each angle.

62°.

2.2 What distance is used for a point-to-line locus?

Perpendicular distance.

2.3 What stays equal when drawing arcs from the two marked points in an angle-bisector construction?

The compass radius for those two arcs.

2.4 Why must the initial angle-bisector arc have its centre at the vertex?

It marks points equally far along the two arms, supplying the symmetric reference distances.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Constructing triangles from sufficient information

3.1 Can sides 2 cm,5 cm,6 cm form a triangle?

Yes: 2 + 5 > 6, and the other triangle inequalities also hold.

3.2 Can sides 2 cm,3 cm,5 cm form a non-degenerate triangle?

No. The shorter sides sum exactly to the longest, producing a straight line.

3.3 Which angle must be given for an SAS construction?

The included angle between the two given sides.

3.4 A triangle has angles 45° and 65°. Find its third angle before construction.

70°.

Lesson 4 • Loci and intersecting conditions

4.1 Describe the locus exactly 5 cm from a point O.

A circle of radius 5 cm centred O.

4.2 Describe points at most 2 cm from O.

The disc of radius 2 cm, including its boundary.

4.3 Describe points equidistant from A and B and also on a given line l.

The intersections of l with the perpendicular bisector of AB; these can be one point, none, or the whole line if they coincide.

4.4 How do you combine the conditions inside region R and inside region S?

Take their intersection, the overlap.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Draw AB = 6 cm and construct its perpendicular bisector with ruler and compasses. [3 marks]

Use equal-radius arcs greater than 3 cm above and below AB and join their intersections. [1 for base; 1 for visible matching arcs; 1 for perpendicular bisector through midpoint.]

R2. Construct the internal bisector of a 100° angle. [3 marks]

Draw the angle, use a vertex arc, then equal arcs from its cut points, and join vertex to the common intersection. Each angle is 50°. [1 for initial arc; 1 for matching arcs; 1 for bisector.]

R3. Construct an SSS triangle with sides 4 cm,5 cm,6 cm. [3 marks]

Draw base 6 cm; intersect radii 4 cm and 5 cm from its endpoints; join the vertex. [1 for base; 1 for correct arcs; 1 for triangle.]

R4. Explain why lengths 4 cm,5 cm,10 cm cannot form a triangle. [2 marks]

4 + 5 = 9 < 10, violating the triangle inequality. [1 for sum; 1 for conclusion.]

R5. Draw a straight line and construct a perpendicular from a point P not on it. [3 marks]

Use an arc centred P to cut the line twice, then construct the bisector of the intercepted segment through P. [1 for intercepts; 1 for equal arcs; 1 for perpendicular.]

R6. Describe and sketch the locus of points exactly 3 cm from O. [2 marks]

A circle centred O of radius 3 cm. [1 for centre; 1 for radius/circumference.]

R7. For AB = 8 cm, construct the region within 5 cm of A and closer to A than B. [4 marks]

Draw disc boundary radius 5 cm about A and bisector of AB at 4 cm from A. Shade their overlap on A side; exclude bisector for strictly closer and include circle for within. [1 for each boundary; 1 for overlap; 1 for inclusivity.]

R8. Inside a 90° angle, describe points equally distant from both arms and within 4 cm of its vertex. [3 marks]

The part of the internal 45° angle bisector from the vertex to the circle of radius 4 cm, including endpoints. [1 for bisector; 1 for distance limit; 1 for segment/endpoints.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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