GCSE ONLINE TUTORING

GEOMETRY & MEASURES / TOPIC GUIDE 39

39

# Angles and polygons

Shared GCSE core with deductive reasoning

Calculate unknown angles from stated geometric facts, then organise those facts into clear arguments. Work through lines, triangles, quadrilaterals and polygons without assuming a sketch has been drawn accurately.

**Triangle: 180° • Quadrilateral: 360°**

State a geometric reason for each angle calculation.

**UNDERSTAND THE IDEA. EXPLAIN THE METHOD.**

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

**Name:** _____

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GCSE ONLINE TUTORING / ABOUT THIS BOOK

# Your learning guide

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## Independent educational resource

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## Before you begin

Add and subtract angles, solve simple equations and recognise basic shapes.

## Your route through this notebook

1. Angle vocabulary and facts on lines
2. Parallel lines and transversal angles
3. Triangles, quadrilaterals and shape properties
4. Interior and exterior angles of polygons

## How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

**YOUR PROGRESS**

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.


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LESSON 1 / UNDERSTAND

# Angle vocabulary and facts on lines

An acute angle is between 0° and 90°, a right angle is 90°, an obtuse angle is between 90° and 180°, and a reflex angle is between 180° and 360°. Angles on a straight line sum to 180° and angles around a point sum to 360°. Vertically opposite angles formed by two intersecting straight lines are equal.

Name the reason each time you use a fact. Equal-looking angles need a mathematical justification, such as vertically opposite angles or a marked angle bisector. A right-angle marker gives exact information; appearance alone does not. If the unknown occurs in several expressions, form an equation from the total before solving.

## A reliable method

1. Identify a straight line, full turn or vertically opposite pair.

2. Write the relevant total or equality.

3. Solve and label the requested angle, which may differ from x.

## Worked example 1

Two adjacent angles on a straight line are 3x and 2x + 20. Find x.

**STEP-BY-STEP SOLUTION**

3x + 2x + 20 = 180, so 5x = 160 and x = 32. The actual angles are 96° and 84°.

## Worked example 2

Three angles around a point are 110°,95° and a. Find a.

**STEP-BY-STEP SOLUTION**

a = 360 − 110 − 95 = 155°.


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LESSON 1 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

Adjacent angles are not automatically supplementary. They sum to 180° only when their outer arms form a straight line.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find the supplement of 67°.

1.2 Find the complement of 38°.

1.3 An angle vertically opposite 124° is x. Find x.

1.4 Angles around a point are 80°,2x and 3x. Find x.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 2 / UNDERSTAND

# Parallel lines and transversal angles

When a transversal crosses parallel lines, corresponding angles are equal, alternate angles are equal and co-interior angles sum to 180°. These facts depend on the lines being parallel. Parallel markings or a stated relationship supply that condition.

Identify the pair by position rather than relying only on letter-shape mnemonics. Corresponding angles occupy matching positions at the intersections. Alternate angles lie on opposite sides of the transversal; the familiar interior pair lies between the parallel lines. Co-interior angles lie between the parallel lines on the same side. More than one valid chain of reasoning can reach the same answer.

## A reliable method

1. Confirm the relevant lines are parallel.

2. Identify a useful equal pair or supplementary pair.

3. State the reason, including the parallel-line condition.

## Worked example 1

An angle is 72°. Its corresponding angle on a parallel line is b. Find b.

**STEP-BY-STEP SOLUTION**

b = 72°, because corresponding angles between parallel lines are equal.

## Worked example 2

Co-interior angles are 4x + 10 and 2x + 20. Find x.

**STEP-BY-STEP SOLUTION**

They sum to 180: 6x + 30 = 180, giving x = 25. The angles are 110° and 70°.


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LESSON 2 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

Without parallel lines, a pair in corresponding positions need not be equal. A diagram's visual resemblance to parallel lines is not a sufficient reason.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 An alternate interior angle corresponds to a 53° angle between parallel lines. Find it.

2.2 One co-interior angle is 118°. Find the other.

2.3 Corresponding angles are $5x - 7$ and $3x + 19$. Find x.

2.4 Explain why a co-interior pair of 80° and 95° cannot belong to parallel lines.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 3 / UNDERSTAND

# Triangles, quadrilaterals and shape properties

Interior angles of a triangle sum to 180°. Isosceles triangles have equal base angles opposite equal sides, and equilateral triangles have three angles of 60°. An exterior angle at a triangle vertex equals the sum of the two opposite interior angles, because it supplements the remaining interior angle.

A quadrilateral has interior-angle sum 360°. A parallelogram has equal opposite angles and supplementary adjacent angles. A rectangle has four right angles; a rhombus has four equal sides but need not have four right angles. Use the defining and stated properties, not an assumption that every special quadrilateral is a square. A square is both a rectangle and a rhombus. Parallelogram diagonals bisect each other; rectangle diagonals are also equal, while rhombus diagonals are perpendicular and bisect its angles. A kite has adjacent pairs of equal sides and perpendicular diagonals, with its symmetry diagonal bisecting the other. An isosceles trapezium has equal base angles. These extra properties can justify further angle deductions.

## A reliable method

1. Identify the relevant shape and marked equal sides or parallel sides.
2. Apply angle totals and any equal-angle properties.
3. Keep a written chain of reasons for multi-step problems.

## Worked example 1

An isosceles triangle has vertex angle 44°. Find its base angles.

**STEP-BY-STEP SOLUTION**

The equal base angles total $180 - 44 = 136°$, so each is 68°.

## Worked example 2

A parallelogram has one angle 112°. Find the other three.

**STEP-BY-STEP SOLUTION**

Opposite angle 112°. Each adjacent angle is $180 - 112 = 68°$. Thus 68°,112°,68° around the remaining vertices.


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LESSON 3 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

The equal angles in an isosceles triangle are opposite the marked equal sides. Do not divide a remaining total by two until you have identified which pair is equal.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A triangle has angles 47° and 68°. Find the third.

3.2 An isosceles triangle has a base angle of 73°. Find its vertex angle.

3.3 A quadrilateral has angles 80°,95°,110° and x. Find x.

3.4 A triangle exterior angle is 125° and one opposite interior angle is 48°. Find the other opposite angle.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 4 / UNDERSTAND

# Interior and exterior angles of polygons

An n-sided convex polygon can be split into n − 2 triangles from one vertex, so its interior-angle sum is (n − 2) × 180°. For a regular polygon, divide that sum by n to find each interior angle. Regular means equal sides and equal angles, not merely an attractive-looking shape.

The sum of the exterior turning angles of a convex polygon, taking one at each vertex in the same direction, is 360°. A regular n-gon therefore has exterior angle 360/n, and each interior angle supplements it. Recover n using 360/exterior angle and check that it is a whole number at least three.

## A reliable method

1. Decide whether you need a total or one angle of a regular polygon.

2. Use (n − 2)180 for the interior sum or 360/n for a regular exterior angle.

3. For a reverse problem, find the exterior angle before n.

## Worked example 1

Find each interior angle of a regular decagon.

**STEP-BY-STEP SOLUTION**

Exterior = 360/10 = 36°. Interior = 180 − 36 = 144°.

## Worked example 2

A regular polygon has interior angle 156°. Find its number of sides.

**STEP-BY-STEP SOLUTION**

Exterior = 24°. Thus n = 360/24 = 15.


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LESSON 4 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

The expression (n − 2)180 gives a total, not one angle. Division by n is justified only when all interior angles are equal.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

4.1 Find the interior-angle sum of a heptagon.

4.2 Find each exterior angle of a regular octagon.

4.3 A regular polygon has exterior angle 18°. Find n.

4.4 Could a regular polygon have exterior angle 50°? Explain.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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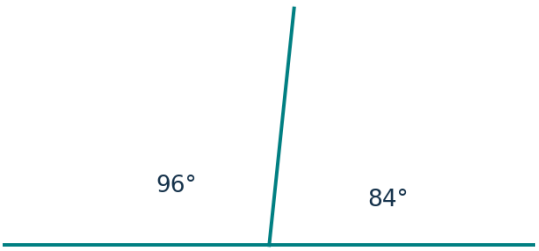
VISUAL EXPLAINER / RELATIONSHIPS, NOT GUESSWORK

# Read the geometry

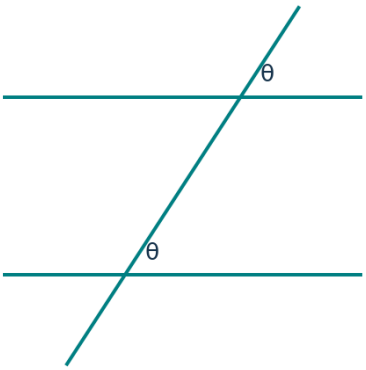
## Angle facts need a reason and the right configuration

### Straight-line total



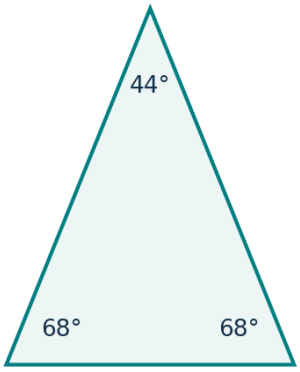

96 + 84 = 180

### Parallel lines

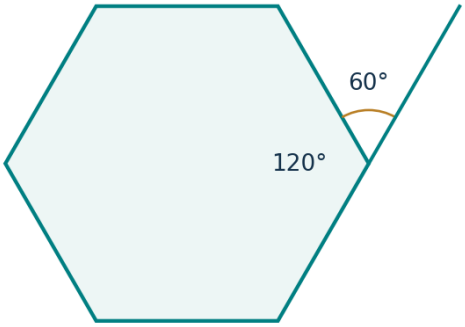



Corresponding angles equal

### Isosceles triangle




### Regular hexagon




**WHAT THE FIGURES ESTABLISH**

The first pair lies on a straight line. Corresponding-angle equality requires the two horizontal lines to be parallel, as stated in the panel. The isosceles example has two equal sloping sides. A regular hexagon has 60° exterior turns and 120° interior angles.

Discuss: identify the given information, the geometric condition and the conclusion. Recalculate a labelled result or explain a theorem without measuring the sketch. Unless explicitly stated, diagrams are illustrative and are not measurement questions.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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

# Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Angles on a straight line are $2x + 15$ and $3x - 10$. Find x. [3 marks]

R2. Angles around a point are 95°,145° and x. Find x. [1 mark]

R3. Parallel lines have co-interior angles x and 137°. Find x with a reason. [2 marks]

R4. An isosceles triangle has vertex angle 38°. Find each base angle. [2 marks]


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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

# Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R5. A parallelogram has angle 73°. Find its adjacent angle. [2 marks]

R6. Find the interior-angle sum of a nonagon. [2 marks]

R7. Find each interior angle of a regular 12-gon. [2 marks]

R8. A regular polygon has interior angle 162°. Find its number of sides. [3 marks]


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SOLUTIONS / LESSON PRACTICE

# Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

## Lesson 1 • Angle vocabulary and facts on lines

1.1 Find the supplement of 67°.

180 − 67 = 113°.

1.2 Find the complement of 38°.

90 − 38 = 52°.

1.3 An angle vertically opposite 124° is x. Find x.

124°, vertically opposite angles are equal.

1.4 Angles around a point are 80°,2x and 3x. Find x.

80 + 5x = 360, so x = 56.

## Lesson 2 • Parallel lines and transversal angles

2.1 An alternate interior angle corresponds to a 53° angle between parallel lines. Find it.

53°, by alternate-angle equality.

2.2 One co-interior angle is 118°. Find the other.

62°.

2.3 Corresponding angles are 5x − 7 and 3x + 19. Find x.

5x − 7 = 3x + 19 gives x = 13.

2.4 Explain why a co-interior pair of 80° and 95° cannot belong to parallel lines.

Their sum is 175°, not 180°; the parallel-line condition fails.


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SOLUTIONS / LESSON PRACTICE

# Check the reasoning • continued

## Lesson 3 • Triangles, quadrilaterals and shape properties

3.1 A triangle has angles 47° and 68°. Find the third.

65°.

3.2 An isosceles triangle has a base angle of 73°. Find its vertex angle.

$180 - 2 \times 73 = 34°$.

3.3 A quadrilateral has angles 80°,95°,110° and x. Find x.

$360 - 285 = 75°$.

3.4 A triangle exterior angle is 125° and one opposite interior angle is 48°. Find the other opposite angle.

$125 - 48 = 77°$.

## Lesson 4 • Interior and exterior angles of polygons

4.1 Find the interior-angle sum of a heptagon.

$(7 - 2) \times 180 = 900°$.

4.2 Find each exterior angle of a regular octagon.

45°.

4.3 A regular polygon has exterior angle 18°. Find n.

20.

4.4 Could a regular polygon have exterior angle 50°? Explain.

No, because 360/50 = 7.2 is not an integer number of sides.


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SOLUTIONS / MIXED REVIEW

# Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Angles on a straight line are 2x + 15 and 3x − 10. Find x. [3 marks]

5x + 5 = 180 gives x = 35. [1 for equation; 1 for isolation; 1 for result.]

R2. Angles around a point are 95°,145° and x. Find x. [1 mark]

120°. [1 for result.]

R3. Parallel lines have co-interior angles x and 137°. Find x with a reason. [2 marks]

43°, because co-interior angles on parallel lines total 180°. [1 for value; 1 for reason.]

R4. An isosceles triangle has vertex angle 38°. Find each base angle. [2 marks]

(180 − 38)/2 = 71°. [1 for total; 1 for division.]

R5. A parallelogram has angle 73°. Find its adjacent angle. [2 marks]

107°, adjacent angles sum to 180°. [1 for result; 1 for reason.]

R6. Find the interior-angle sum of a nonagon. [2 marks]

(9 − 2)180 = 1260°. [1 for formula; 1 for result.]

R7. Find each interior angle of a regular 12-gon. [2 marks]

Exterior 30°, so interior 150°. [1 for exterior; 1 for interior.]

R8. A regular polygon has interior angle 162°. Find its number of sides. [3 marks]

Exterior 18°; n = 360/18 = 20. [1 for supplement; 1 for division; 1 for n.]

## Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.


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