GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 28

28

Functions and graph transformations

Shared function machines with Higher extensions

A function assigns one output to each allowed input. Develop notation, composition and inversion, then connect changes in a function’s rule to movements of its graph. Domain restrictions are kept explicit.

f(g(x)): g acts first

Track inputs, outputs and the order of operations.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Substitute values, rearrange formulae and recognise basic graph families.

Your route through this notebook

1. Function notation and domain
2. Composite and inverse functions — Higher
3. Translations and reflections — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Function notation and domain

The notation f(x) means the output of function f for input x; it does not mean f multiplied by x. In f(x) = 3x − 2, replace every x by the supplied input, including an expression such as a + 1. A function must give a single output for each input in its domain.

The domain is the set of permitted inputs. For a rational expression, denominator zeros must be excluded. For a real square-root function, the quantity under the root must be non-negative. Context may impose tighter restrictions, such as non-negative whole-number ticket counts.

A reliable method

1. Read f(input) as a substitution instruction.

2. Bracket expression inputs before simplifying.

3. Check denominator and square-root restrictions.

Worked example 1

For $f(x) = 2x^2 - 1$, find $f(-3)$.

STEP-BY-STEP SOLUTION

$f(-3) = 2(-3)^2 - 1 = 17$.

Worked example 2

For f(x) = 3x + 4, find f(a − 2).

STEP-BY-STEP SOLUTION

f(a − 2) = 3(a − 2) + 4 = 3a − 2.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An input-output machine and function notation describe the same idea. The order of steps matters: multiply by 3 then add 4 is not the same as add 4 then multiply by 3.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 For $f(x) = 5x - 2$, find $f(4)$.

1.2 For $f(x) = x^2 + 3$, find $f(-2)$.

1.3 For $f(x) = 1/(x - 5)$, state the excluded input.

1.4 For $f(x) = \sqrt{(x + 2)}$, state the real domain.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Composite and inverse functions — Higher

A composite function applies one function and then another. In fg(x), meaning f(g(x)), g acts first. Substitute the entire output expression of g into f. Reversing the order generally changes the result. This guide explicitly uses fg for composition rather than multiplication.

An inverse reverses a one-to-one function. Write y = f(x), rearrange to express x in terms of y, then relabel the input. The notation $f^{-1}(x)$ means inverse function, not 1/f(x). A function such as x^2 needs a restricted domain to have a single-valued inverse, because positive and negative inputs otherwise share outputs.

A reliable method

1. For f(g(x)), work from the inner function outward.

2. For an inverse, rearrange y = f(x).

3. Check the domain and verify a round trip.

Worked example 1

Let f(x) = 2x + 1 and $g(x) = x^2$. Find f(g(x)) and g(f(x)).

STEP-BY-STEP SOLUTION

$f(g(x)) = 2x^2 + 1$. $g(f(x)) = (2x + 1)^2$. They differ in general.

Worked example 2

Find the inverse of f(x) = 3x − 7.

STEP-BY-STEP SOLUTION

$y = 3x - 7$ gives $x = (y + 7)/3$. Therefore $f^{-1}(x) = (x + 7)/3$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For $f(x) = x^2$ on $x \geq 0$, the inverse is $\sqrt{x}$ with input $x \geq 0$. Without the original restriction, the proposed inverse would not recover a negative original input.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 If $f(x) = x + 4$ and $g(x) = 2x$, find $f(g(x))$.

2.2 With those functions, find $g(f(x))$.

2.3 Find the inverse of $f(x) = 5x + 2$.

2.4 Find the inverse of $f(x) = x^2 + 1$ on $x \geq 0$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Translations and reflections — Higher

A vertical translation changes outputs: y = f(x) + a moves the graph up a. A horizontal translation changes inputs: y = f(x − a) moves it right a. The horizontal sign can feel reversed because the old input is reached when the new x is larger by a.

For y = −f(x), every output changes sign, giving a reflection in the x-axis. For y = f(−x), inputs change sign, giving a reflection in the y-axis. Track a known point instead of relying only on a verbal rule. Multiple changes can be understood by writing the new coordinate of a general original point.

A reliable method

1. Identify whether the change is outside or inside f.

2. Track a known point (p,q).

3. Check the new rule by substitution at the transformed point.

Worked example 1

The graph y = f(x) contains (2,5). Where does it go on y = f(x − 3) + 1?

STEP-BY-STEP SOLUTION

Right 3 and up 1: (5,6). At x = 5, the input to f is 2 and the output is 5 + 1.

Worked example 2

Reflect $y = x^2 - 2$ in the x-axis.

STEP-BY-STEP SOLUTION

Negate the entire output: $y = -(x^2 - 2) = -x^2 + 2$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

y = f(x + 4) moves left 4, not right. For a point originally at input 2, the new x must be −2 so that x + 4 still equals 2.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Describe $y = f(x) - 6$.

3.2 Describe $y = f(x + 2)$.

3.3 A point (−3,4) is reflected in the y-axis. Find its image.

3.4 The vertex of $y = x^2$ is (0,0). Find the vertex of $y = (x - 4)^2 - 3$.

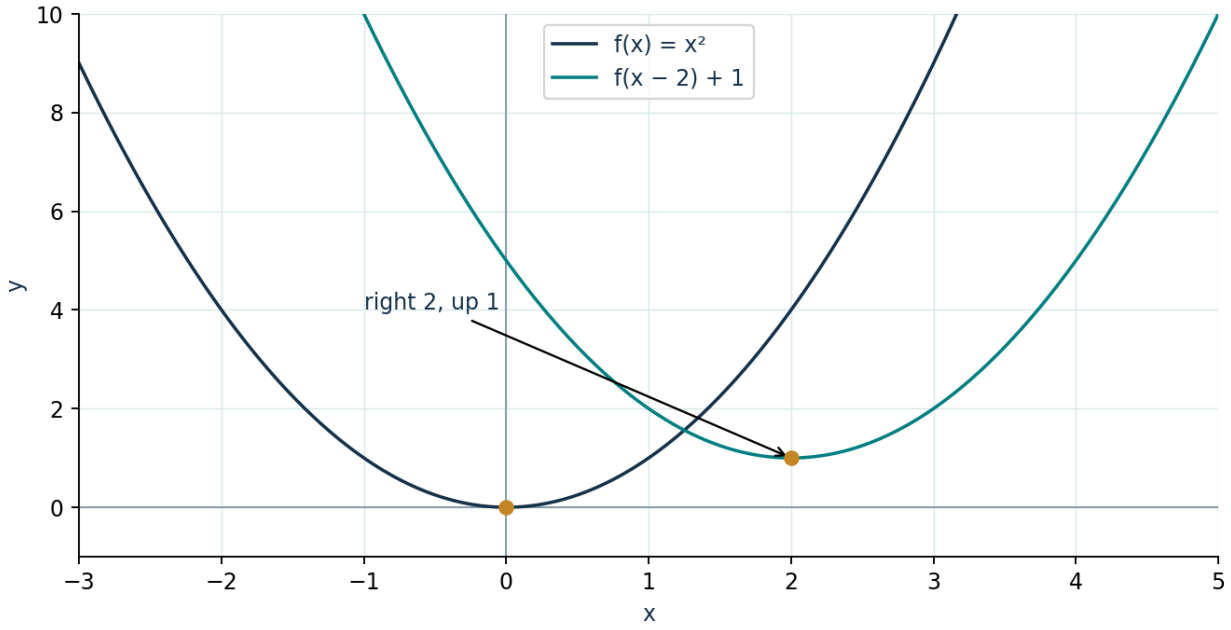
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

Track points when transforming a graph



WHAT THE FIGURE SHOWS

The original vertex (0,0) moves to (2,1). More generally, a point (p,q) moves to (p + 2,q + 1). The input x − 2 reaches the old input p when x = p + 2, explaining the horizontal sign.

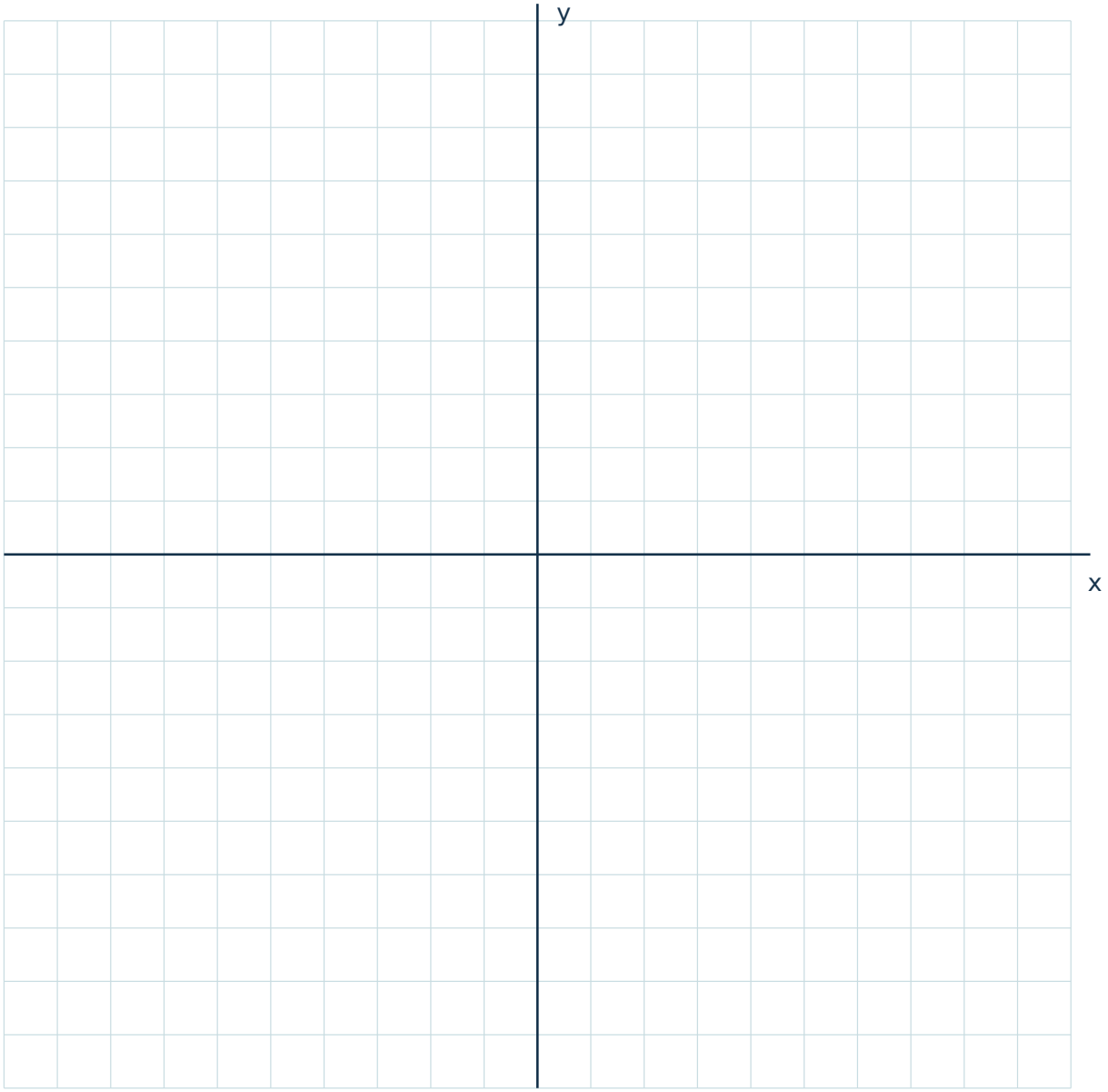
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. For $f(x) = 3x^2 + 2$, find $f(-2)$. [2 marks]

R2. For $f(x) = 2x - 5$, simplify $f(a + 3)$. [2 marks]

R3. Higher: $f(x) = 3x + 1$, $g(x) = x - 2$. Find $f(g(x))$. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Higher: find the inverse of $f(x) = 4x - 9$. [3 marks]

R5. Higher: describe $y = f(x - 5) + 2$. [2 marks]

R6. Higher: a point $(2,-3)$ lies on $y = f(x)$. Give its image on $y = -f(x + 1)$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Function notation and domain

1.1 For $f(x) = 5x - 2$, find $f(4)$.

18.

1.2 For $f(x) = x^2 + 3$, find $f(-2)$.

7.

1.3 For $f(x) = 1/(x - 5)$, state the excluded input.

$x \neq 5$.

1.4 For $f(x) = \sqrt{(x + 2)}$, state the real domain.

$x \geq -2$.

Lesson 2 • Composite and inverse functions — Higher

2.1 If $f(x) = x + 4$ and $g(x) = 2x$, find $f(g(x))$.

$2x + 4$.

2.2 With those functions, find $g(f(x))$.

$2x + 8$.

2.3 Find the inverse of $f(x) = 5x + 2$.

$f^{-1}(x) = (x - 2)/5$.

2.4 Find the inverse of $f(x) = x^2 + 1$ on $x \geq 0$.

$f^{-1}(x) = \sqrt{(x - 1)}$, with inverse input $x \geq 1$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Translations and reflections — Higher

3.1 Describe $y = f(x) - 6$.

Translation down 6.

3.2 Describe $y = f(x + 2)$.

Translation left 2.

3.3 A point $(-3,4)$ is reflected in the y-axis. Find its image.

$(3,4)$.

3.4 The vertex of $y = x^2$ is $(0,0)$. Find the vertex of $y = (x - 4)^2 - 3$.

$(4,-3)$.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. For $f(x) = 3x^2 + 2$, find $f(-2)$. [2 marks]

$3 \times 4 + 2 = 14$. [1 for substitution; 1 for result.]

R2. For $f(x) = 2x - 5$, simplify $f(a + 3)$. [2 marks]

$2(a + 3) - 5 = 2a + 1$. [1 for substitution; 1 for simplification.]

R3. Higher: $f(x) = 3x + 1$, $g(x) = x - 2$. Find $f(g(x))$. [2 marks]

$3(x - 2) + 1 = 3x - 5$. [1 for order; 1 for simplification.]

R4. Higher: find the inverse of $f(x) = 4x - 9$. [3 marks]

$y = 4x - 9$ gives $x = (y + 9)/4$, so $f^{-1}(x) = (x + 9)/4$. [1 for equation; 1 for rearrangement; 1 for notation.]

R5. Higher: describe $y = f(x - 5) + 2$. [2 marks]

Translation right 5 and up 2. [1 each.]

R6. Higher: a point $(2,-3)$ lies on $y = f(x)$. Give its image on $y = -f(x + 1)$. [3 marks]

Left 1 and reflect output: (1,3). [1 for x change; 1 for y change; 1 for paired coordinate.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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