GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 27

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Algebraic fractions

Higher-level GCSE material

Algebraic fractions use the same arithmetic principles as numerical fractions, with an extra responsibility: preserve restrictions from original denominators. Factorise before cancelling and distinguish factors from terms.

$$(x^2 - 9)/(x - 3) = x + 3, \quad x \neq 3$$

Cancel factors and preserve original restrictions.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Factorise quadratics, calculate with ordinary fractions and solve equations.

Your route through this notebook

1. Simplifying and preserving the domain
2. Four operations with algebraic fractions
3. Equations with variable denominators

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Simplifying and preserving the domain

A fraction simplifies by cancelling common factors of its whole numerator and denominator. Terms joined by addition are not factors unless the entire expression has first been factorised. For example, $(x^2 + 3x)/x = x(x + 3)/x = x + 3$ for $x \neq 0$. The simplified expression still inherits that restriction.

Cancelling can hide a point where the original expression was undefined. In $(x^2 - 9)/(x - 3)$, factorisation gives $x + 3$ for $x \neq 3$. It is not a licence to evaluate the original at $x = 3$. Denominator restrictions should be identified before any cancellation.

A reliable method

1. List values making any original denominator zero.
2. Factorise whole numerators and denominators.
3. Cancel matching non-zero factors and retain restrictions.

Worked example 1

Simplify $(x^2 - 16)/(x - 4)$.

STEP-BY-STEP SOLUTION

Factorise the numerator: $(x - 4)(x + 4)/(x - 4) = x + 4$, with $x \neq 4$.

Worked example 2

Simplify $(6x^2 + 9x)/(3x)$.

STEP-BY-STEP SOLUTION

$3x(2x + 3)/(3x) = 2x + 3$, with $x \neq 0$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

(x + 5)/x cannot simplify by crossing out x from only one term. It can be written 1 + 5/x for x ≠ 0, but not as 5.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Simplify $12x^2/(4x)$.

1.2 Simplify $(x^2 - 25)/(x + 5)$.

1.3 Simplify $(x^2 + 5x + 6)/(x + 2)$.

1.4 State the restriction for $1/(2x - 6)$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Four operations with algebraic fractions

Addition and subtraction require a common denominator. Multiply each numerator by whatever was used to change its denominator. Keep a subtraction bracket around an entire numerator, then simplify. Multiplication uses a product of numerators over a product of denominators; factorising first can reveal cancellation.

For division, multiply by the reciprocal of the divisor. The divisor itself must be non-zero, so there may be extra exclusions beyond the original denominators. Record restrictions from every stage that defines the original operation, not only from the final simplified result.

A reliable method

1. For sums, choose a common denominator and preserve numerator brackets.

2. For products, factorise and cancel before multiplying where useful.

3. For division, invert the divisor and exclude zeros of that divisor.

Worked example 1

Simplify $2/x + 3/(x + 1)$.

STEP-BY-STEP SOLUTION

Use $x(x + 1)$: $[2(x + 1) + 3x]/[x(x + 1)] = (5x + 2)/[x(x + 1)]$, with $x \neq 0, -1$.

Worked example 2

Simplify $(x^2 - 1)/(x + 2) \div (x - 1)/(x + 2)$.

STEP-BY-STEP SOLUTION

Multiply by $(x + 2)/(x - 1)$ and factor $x^2 - 1$. Result $x + 1$, with $x \neq -2$ from denominators and $x \neq 1$ because the divisor cannot be zero.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For $1/x - 1/(x + 1)$, the numerator is $(x + 1) - x = 1$. Neglecting the subtraction bracket is a common source of false terms.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Simplify $3/x + 5/x$.

2.2 Simplify $1/x - 1/(x + 2)$.

2.3 Simplify $(2x/3) \times (9/x^2)$.

2.4 Simplify $(x/4) \div (x^2/8)$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Equations with variable denominators

To solve an equation containing algebraic fractions, first state values excluded by the denominators. Multiply every term by a common denominator to form a polynomial equation. Solving that equation may produce values that were invalid in the original, so check all candidates against the restrictions.

A denominator with a variable can change sign, but for an equation multiplying by a non-zero expression preserves equality. Inequalities require additional sign reasoning and are not handled by blindly using the same step. Keep the focus here on equations and verify by substitution.

A reliable method

1. Write all denominator restrictions first.

2. Multiply every term by the common denominator.

3. Solve, reject excluded values and check the original equation.

Worked example 1

Solve $3/x = 2/(x + 1)$.

STEP-BY-STEP SOLUTION

$x \neq 0, -1$. Cross-multiplication gives $3(x + 1) = 2x$, so $x = -3$, which is allowed and checks.

Worked example 2

Solve $1/(x - 1) + 1 = 3/(x - 1)$.

STEP-BY-STEP SOLUTION

$x \neq 1$. Multiply by $x - 1$: $1 + x - 1 = 3$, so $x = 3$. Check: $1/2 + 1 = 3/2$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Clearing a denominator can produce an excluded candidate. From $(x^2 - 1)/(x - 1) = 2$, algebra leads to $x = 1$, but the original is undefined there. Hence the original equation has no solution.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Solve $5/x = 1$.

3.2 Solve $2/(x + 1) = 1$.

3.3 Solve $1/x + 1/2 = 1$.

3.4 Solve $(x^2 - 4)/(x - 2) = 4$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Simplify $(x^2 - 36)/(x - 6)$ and state restrictions. [3 marks]

R2. Simplify $(4x^2 + 8x)/(2x)$. [3 marks]

R3. Simplify $2/x + 1/(x - 1)$. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Simplify $((x + 1)/x) \times (x^2/(x^2 - 1))$. [3 marks]

R5. Solve $4/x = 3/(x + 2)$. [3 marks]

R6. Solve $(x^2 - 9)/(x - 3) = 6$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Simplifying and preserving the domain

1.1 Simplify $12x^2/(4x)$.

$3x$, with $x \neq 0$.

1.2 Simplify $(x^2 - 25)/(x + 5)$.

$x - 5$, with $x \neq -5$.

1.3 Simplify $(x^2 + 5x + 6)/(x + 2)$.

$(x + 2)(x + 3)/(x + 2) = x + 3$, with $x \neq -2$.

1.4 State the restriction for $1/(2x - 6)$.

$2x - 6 \neq 0$, so $x \neq 3$.

Lesson 2 • Four operations with algebraic fractions

2.1 Simplify $3/x + 5/x$.

$8/x$, $x \neq 0$.

2.2 Simplify $1/x - 1/(x + 2)$.

$2/[x(x + 2)]$, $x \neq 0, -2$.

2.3 Simplify $(2x/3) \times (9/x^2)$.

$6/x$, $x \neq 0$.

2.4 Simplify $(x/4) \div (x^2/8)$.

$(x/4)(8/x^2) = 2/x$, $x \neq 0$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Equations with variable denominators

3.1 Solve $5/x = 1$.

$x \neq 0$; $x = 5$.

3.2 Solve $2/(x + 1) = 1$.

$x \neq -1$; $2 = x + 1$ gives $x = 1$.

3.3 Solve $1/x + 1/2 = 1$.

$x \neq 0$. Multiply by $2x$: $2 + x = 2x$, so $x = 2$.

3.4 Solve $(x^2 - 4)/(x - 2) = 4$.

$x \neq 2$. Simplifies to $x + 2 = 4$, giving excluded $x = 2$. No solution.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Simplify $(x^2 - 36)/(x - 6)$ and state restrictions. [3 marks]

$x + 6$, $x \neq 6$. [1 for factors; 1 for cancellation; 1 for restriction.]

R2. Simplify $(4x^2 + 8x)/(2x)$. [3 marks]

$2x(x + 2)/(x)$ after coefficient simplification, or directly $2(x + 2) = 2x + 4$, $x \neq 0$. [1 for factorisation; 1 for cancellation; 1 for result/restriction.]

R3. Simplify $2/x + 1/(x - 1)$. [3 marks]

$[2(x - 1) + x]/[x(x - 1)] = (3x - 2)/[x(x - 1)]$, $x \neq 0,1$. [1 for denominator; 1 for numerator; 1 for simplified form/restrictions.]

R4. Simplify $((x + 1)/x) \times (x^2/(x^2 - 1))$. [3 marks]

Factor $x^2 - 1 = (x - 1)(x + 1)$. Result $x/(x - 1)$, with $x \neq 0,1,-1$. [1 for factors; 1 for cancellation; 1 for restrictions.]

R5. Solve $4/x = 3/(x + 2)$. [3 marks]

$x \neq 0,-2$; $4x + 8 = 3x$ gives $x = -8$. [1 for restrictions; 1 for equation; 1 for allowed result.]

R6. Solve $(x^2 - 9)/(x - 3) = 6$. [3 marks]

$x \neq 3$. Simplification gives $x + 3 = 6$, so the only candidate is excluded $x = 3$. No solution. [1 for restriction; 1 for candidate; 1 for rejecting it.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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