GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 26

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Solving quadratic equations

Shared factorisation with Higher methods

Quadratic equations can have two, one or no real solutions. Start with factorisation and the zero-product property, then learn the quadratic formula and completing the square. Choose a method from the equation’s structure and check every solution.

$$ax^2 + bx + c = 0$$

Factorise, complete the square or use the formula.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Factorise quadratics, rearrange equations, simplify roots and use a calculator.

Your route through this notebook

1. Factorisation and the zero-product property
2. The quadratic formula — Higher
3. Completing the square and choosing a method — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Factorisation and the zero-product property

If a product equals zero, at least one factor must be zero. Therefore a factorised quadratic gives linear equations for its roots. First rearrange the original equation so one side is zero; this is essential because a product equal to a non-zero number does not have the same shortcut.

A common factor x must not be cancelled if doing so loses the solution $x = 0$. Factorise and set each factor equal to zero instead. A repeated factor produces a repeated root, not two distinct values. Numerical checks in the original equation can catch a rearrangement or sign error.

A reliable method

1. Rearrange into a quadratic equal to zero.

2. Factorise completely.

3. Set each factor equal to zero and check the resulting values.

Worked example 1

Solve $x^2 - 5x + 6 = 0$.

STEP-BY-STEP SOLUTION

$(x - 2)(x - 3) = 0$. Thus $x = 2$ or $x = 3$.

Worked example 2

Solve $x^2 = 4x$.

STEP-BY-STEP SOLUTION

$x^2 - 4x = x(x - 4) = 0$. Thus $x = 0$ or 4. Dividing immediately by x would lose zero.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

From $(x - 2)(x - 3) = 6$, you cannot conclude $x = 2$ or 3. Those values make the left side zero, not six. Expand and rearrange to zero first.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Solve $x^2 + 7x + 12 = 0$.

1.2 Solve $x^2 - 16 = 0$.

1.3 Solve $3x^2 - 12x = 0$.

1.4 Solve $x^2 - 6x + 9 = 0$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

The quadratic formula — Higher

For $ax^2 + bx + c = 0$ with $a \neq 0$, the quadratic formula is $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$. Read a,b,c with their signs from a correctly rearranged equation. The entire numerator is divided by 2a. Calculator brackets should reproduce this grouping.

The discriminant $D = b^2 - 4ac$ determines the real-root count: positive gives two distinct real roots, zero gives one repeated real root, and negative gives no real roots. Keep exact roots when requested; otherwise calculate both signs and round only at the end.

A reliable method

1. Put the equation in standard form and identify signed coefficients.

2. Calculate the discriminant before evaluating roots.

3. Use both ± branches and the full denominator.

Worked example 1

Solve $2x^2 + 3x - 1 = 0$ exactly.

STEP-BY-STEP SOLUTION

$a = 2$, $b = 3$, $c = -1$. $D = 9 + 8 = 17$. Thus $x = (-3 \pm \sqrt{17})/4$.

Worked example 2

How many real roots has $x^2 + 2x + 5 = 0$?

STEP-BY-STEP SOLUTION

$D = 2^2 - 4 \times 1 \times 5 = -16$. It has no real roots.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

If b = −5, then −b = +5 and $b^2 = 25$. Enter $(-5)^2$ rather than -5^2 when calculating the discriminant.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find D for $x^2 - 4x + 1 = 0$.

2.2 Solve $x^2 - 4x + 1 = 0$ exactly.

2.3 Find the root count for $4x^2 + 4x + 1 = 0$.

2.4 Solve $3x^2 - x - 2 = 0$ using the formula.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Completing the square and choosing a method — Higher

Completing the square turns a quadratic equation into a squared expression equal to a constant. For $x^2 + bx + c$, write $(x + b/2)^2 + c - (b/2)^2$. Isolate the square, take both square roots and solve. If the square would equal a negative number, there are no real solutions.

Choose an efficient method: factorisation is quick for obvious integer factors; a simple isolated square may need only square roots; the formula is systematic for any real quadratic. When modelling lengths or counts, solve algebraically first and then reject values that fail the contextual restrictions.

A reliable method

1. Create a complete square and isolate it.

2. Take both signs when the right side is positive.

3. Interpret algebraic solutions in the original domain.

Worked example 1

Solve $x^2 + 6x + 1 = 0$ by completing the square.

STEP-BY-STEP SOLUTION

$(x + 3)^2 - 8 = 0$, so $x + 3 = \pm\sqrt{8}$. Therefore $x = -3 \pm 2\sqrt{2}$.

Worked example 2

A rectangle has sides x and $x + 3$ with area 40. Find positive x.

STEP-BY-STEP SOLUTION

$x^2 + 3x - 40 = 0$ gives $(x + 8)(x - 5) = 0$. Values are -8 and 5; only $x = 5$ is a valid length.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The instruction “solve” needs values of x, not only completed-square form. Finish by isolating x and include every allowed branch.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Solve $(x - 4)^2 = 9$.

3.2 Solve $x^2 - 8x + 7 = 0$ by completing the square.

3.3 Solve $x^2 + 2x - 2 = 0$ exactly.

3.4 A positive length x satisfies $x(x + 2) = 15$. Find x.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Solve $x^2 - 9x + 20 = 0$. [3 marks]

R2. Solve $2x^2 = 10x$. [3 marks]

R3. Higher: solve $x^2 + 3x - 2 = 0$ exactly. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Higher: state the real-root count of $2x^2 + 4x + 7 = 0$. [2 marks]

R5. Higher: solve $x^2 - 4x - 5 = 0$ by completing the square. [3 marks]

R6. A rectangle has sides x and x + 5 and area 84. Find its dimensions. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Factorisation and the zero-product property

1.1 Solve $x^2 + 7x + 12 = 0$.

$(x + 3)(x + 4) = 0$, so $x = -3$ or -4.

1.2 Solve $x^2 - 16 = 0$.

$(x - 4)(x + 4) = 0$, so $x = \pm 4$.

1.3 Solve $3x^2 - 12x = 0$.

$3x(x - 4) = 0$, so $x = 0$ or 4.

1.4 Solve $x^2 - 6x + 9 = 0$.

$(x - 3)^2 = 0$ gives the repeated root $x = 3$.

Lesson 2 • The quadratic formula — Higher

2.1 Find D for $x^2 - 4x + 1 = 0$.

$16 - 4 = 12$.

2.2 Solve $x^2 - 4x + 1 = 0$ exactly.

$x = (4 \pm \sqrt{12})/2 = 2 \pm \sqrt{3}$.

2.3 Find the root count for $4x^2 + 4x + 1 = 0$.

$D = 16 - 16 = 0$: one repeated real root.

2.4 Solve $3x^2 - x - 2 = 0$ using the formula.

$D = 1 + 24 = 25$. $x = (1 \pm 5)/6$, giving 1 or $-2/3$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Completing the square and choosing a method — Higher

3.1 Solve $(x - 4)^2 = 9$.

$x - 4 = \pm 3$, so $x = 1$ or 7.

3.2 Solve $x^2 - 8x + 7 = 0$ by completing the square.

$(x - 4)^2 = 9$, so $x = 1$ or 7.

3.3 Solve $x^2 + 2x - 2 = 0$ exactly.

$(x + 1)^2 = 3$, so $x = -1 \pm \sqrt{3}$.

3.4 A positive length x satisfies $x(x + 2) = 15$. Find x.

$x^2 + 2x - 15 = (x + 5)(x - 3) = 0$. Positive $x = 3$.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Solve $x^2 - 9x + 20 = 0$. [3 marks]

$(x - 4)(x - 5) = 0$, so $x = 4$ or 5. [1 for factors; 1 for each root.]

R2. Solve $2x^2 = 10x$. [3 marks]

$2x(x - 5) = 0$, so $x = 0$ or 5. [1 for factorisation; 1 for each root.]

R3. Higher: solve $x^2 + 3x - 2 = 0$ exactly. [3 marks]

$D = 17$, so $x = (-3 \pm \sqrt{17})/2$. [1 for coefficients/discriminant; 1 for substitution; 1 for both branches.]

R4. Higher: state the real-root count of $2x^2 + 4x + 7 = 0$. [2 marks]

$D = 16 - 56 = -40$, so no real roots. [1 for D; 1 for conclusion.]

R5. Higher: solve $x^2 - 4x - 5 = 0$ by completing the square. [3 marks]

$(x - 2)^2 = 9$, so $x = -1$ or 5. [1 for square; 1 for ±; 1 for values.]

R6. A rectangle has sides x and $x + 5$ and area 84. Find its dimensions. [4 marks]

$x^2 + 5x - 84 = (x + 12)(x - 7) = 0$. Reject -12; sides are 7 and 12. [1 for model; 1 for factors; 1 for valid x; 1 for dimensions.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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