GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 25

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# Velocity–time graphs and rates of change

Higher-level GCSE material

Graph gradients describe rates of change, while areas under velocity graphs describe displacement. Learn exact calculations for straight sections and numerical estimates for curves, keeping speed, velocity, distance and displacement distinct.

## gradient → acceleration

Signed area under velocity gives displacement.

**UNDERSTAND THE IDEA. EXPLAIN THE METHOD.**

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

**Name:** _____

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GCSE ONLINE TUTORING / ABOUT THIS BOOK

# Your learning guide

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## Independent educational resource

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## Before you begin

Calculate gradients and areas of triangles and trapezia. Read time and velocity units.

## Your route through this notebook

1. Gradient as acceleration
2. Area, displacement and distance
3. Curved graphs, tangents and numerical area

## How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

**YOUR PROGRESS**

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.


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LESSON 1 / UNDERSTAND

# Gradient as acceleration

Velocity includes direction, unlike speed. On a velocity–time graph, gradient is acceleration: change in velocity divided by elapsed time. With velocity in m/s and time in seconds, acceleration is in m/s². A straight sloping section means constant acceleration; a horizontal section means constant velocity.

A negative gradient does not always mean slowing down. If velocity is already negative and becomes more negative, speed increases in the negative direction. Compare the magnitude of velocity with zero to decide whether the object speeds up. A horizontal velocity graph may represent motion, whereas a horizontal distance graph represents rest.

## A reliable method

1. Read the velocity change and elapsed time for the chosen section.

2. Divide change in velocity by change in time.

3. Use the sign and velocity values to describe the motion.

## Worked example 1

Velocity rises from 4 m/s to 16 m/s in 6 s. Find acceleration.

**STEP-BY-STEP SOLUTION**

$a = (16 - 4)/6 = 2$ m/s².

## Worked example 2

Velocity changes from −2 m/s to −10 m/s in 4 s. Interpret.

**STEP-BY-STEP SOLUTION**

Acceleration $= (-10 - (-2))/4 = -2$ m/s². Speed grows from 2 to 10 m/s, so it speeds up in the negative direction.


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LESSON 1 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

Deceleration is often used for acceleration opposing the motion. A negative acceleration is not sufficient on its own: the sign of the velocity also matters.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Velocity rises from 0 to 18 m/s in 9 s. Find acceleration.

1.2 Velocity falls from 20 to 8 m/s in 4 s. Find acceleration.

1.3 Describe a horizontal graph at v = 5 m/s.

1.4 An object has v = −6 m/s and acceleration +2 m/s². Initially does it speed up or slow down?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 2 / UNDERSTAND

# Area, displacement and distance

For a constant velocity, displacement equals velocity multiplied by time, which is the rectangular area under the graph. This extends to changing velocity: signed area between a velocity–time graph and the time axis gives displacement. Straight sections create rectangles, triangles and trapezia whose areas can be found exactly.

Area below the time axis is negative for displacement. Total distance uses the absolute area of each section, so add their magnitudes. If a straight segment crosses the axis, split it at the zero-velocity time before calculating distance. Units provide a check: (m/s) × s = m.

## A reliable method

1. Split the region into simple shapes at changes and zero crossings.

2. Use signed area for displacement.

3. For distance, add absolute areas.

## Worked example 1

Velocity rises from 0 to 12 m/s in 4 s, then stays at 12 for 3 s. Find displacement.

**STEP-BY-STEP SOLUTION**

Triangle: 1/2 × 4 × 12 = 24 m. Rectangle: 3 × 12 = 36 m. Total = 60 m.

## Worked example 2

Velocity is +4 m/s for 3 s, then −2 m/s for 5 s. Find displacement and distance.

**STEP-BY-STEP SOLUTION**

Signed areas are +12 m and −10 m. Displacement = 2 m; distance = 12 + 10 = 22 m.


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LESSON 2 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

A negative displacement does not mean a negative distance. It describes the net change of position in the chosen coordinate direction.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find displacement at constant 7 m/s for 8 s.

2.2 Velocity rises linearly from 4 to 10 m/s in 6 s. Find displacement.

2.3 Velocity falls linearly from 8 to 0 m/s in 5 s. Find distance.

2.4 Velocity is −3 m/s for 4 s and +5 m/s for 2 s. Find displacement and distance.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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LESSON 3 / UNDERSTAND

# Curved graphs, tangents and numerical area

For a curved graph, the gradient changes. A chord between two points gives an average rate of change. A tangent at one point gives an estimate of instantaneous rate. Draw a tangent that follows the local direction, then use two well-separated points on that tangent to calculate its gradient.

Areas under curves can be estimated by rectangles or trapezia. Trapezia join adjacent sampled heights with straight segments. For equal time intervals h, add each trapezium area $h(v_1 + v_2)/2$. Whether an estimate is high or low depends on the curve shape and the chosen method; it is not always an underestimate.

## A reliable method

1. Use a chord for an average rate and a tangent for an instantaneous rate.

2. Read two points on the drawn tangent, not arbitrary points on the curve.

3. For numerical area, include every interval once and retain units.

## Worked example 1

A tangent to a velocity curve passes through (2,3) and (6,11). Estimate acceleration.

**STEP-BY-STEP SOLUTION**

Gradient = $(11 - 3)/(6 - 2) = 2$ m/s².

## Worked example 2

At t = 0,2,4 s, velocities are 0,6,8 m/s. Estimate displacement using trapezia.

**STEP-BY-STEP SOLUTION**

First area = $2(0 + 6)/2 = 6$ m. Second = $2(6 + 8)/2 = 14$ m. Total estimate = 20 m.


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LESSON 3 / DISCUSS & PRACTISE

# Explain it, then try it

**DISCUSSION • TEST THE IDEA**

A tangent is a local straight-line approximation, not necessarily a line that touches the entire graph only once. Your gradient accuracy depends on a careful drawing and readable scale.

## Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 A tangent passes through (1,4) and (5,14). Find its gradient.

3.2 Velocities at 0,3,6 s are 2,5,8 m/s. Find the trapezium estimate.

3.3 How does a chord differ from a tangent in rate interpretation?

3.4 For a curve bowing below its straight chord, is the chord-based trapezium area an overestimate?

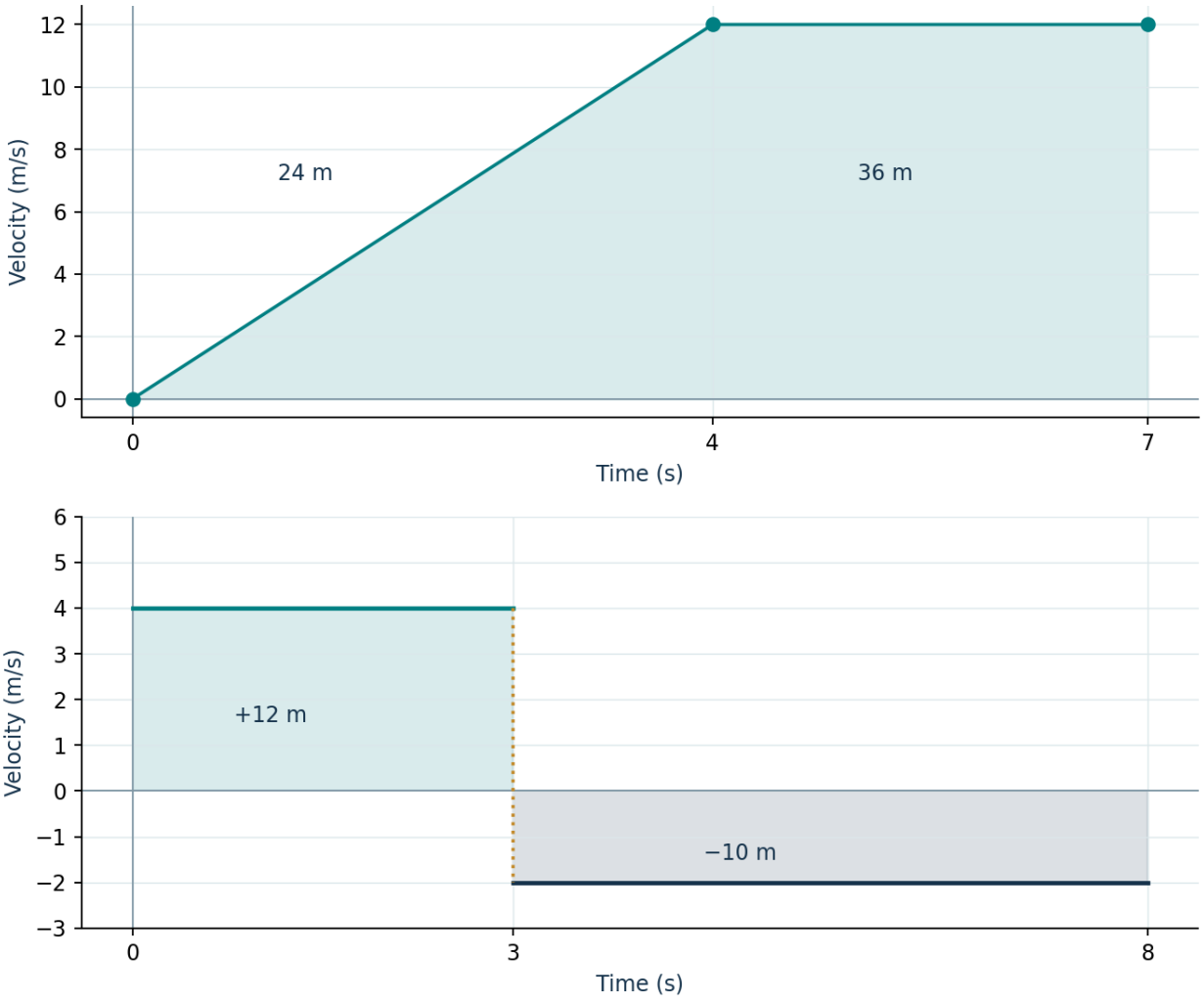
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?


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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

# Read the mathematics

Gradient is acceleration; signed area is displacement




## WHAT THE FIGURE SHOWS

Top: acceleration for the first 4 seconds is 12/4 = 3 m/s² and total displacement is 24 + 36 = 60 m. Bottom: an idealised instantaneous velocity change at 3 s separates positive and negative motion. Displacement is 12 − 10 = 2 m; distance is 12 + 10 = 22 m. The dotted jump marks the idealisation, not a finite acceleration segment.

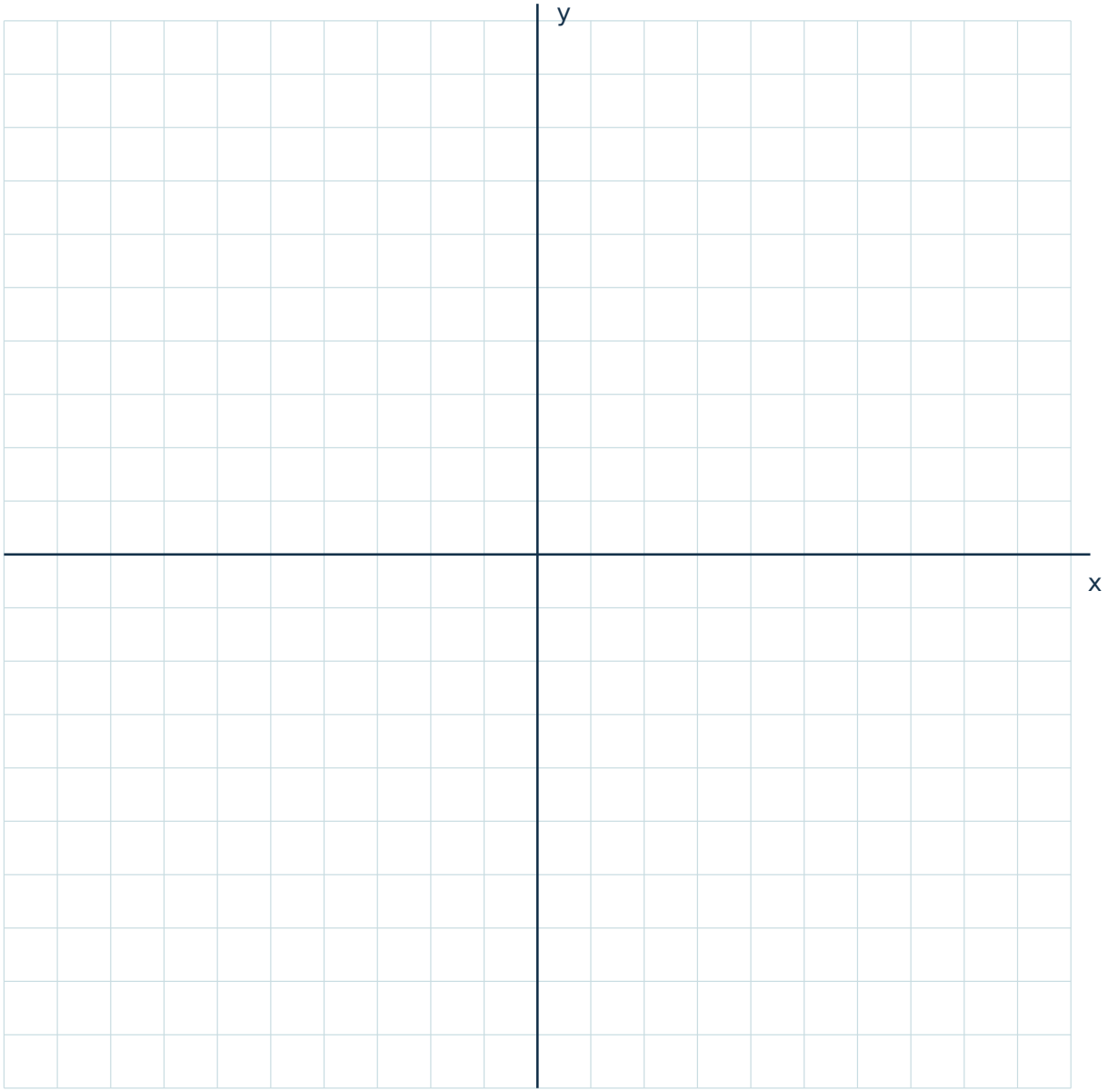
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.


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GRAPH WORKSPACE / RECREATE AND EXPLAIN

# Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.




**SELF-CHECK**

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.


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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

# Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Velocity rises from 3 to 15 m/s in 4 s. Find acceleration. [2 marks]

R2. Velocity is 6 m/s for 7 s. Find displacement. [2 marks]

R3. Velocity rises linearly from 2 to 14 m/s in 5 s. Find displacement. [3 marks]


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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

# Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Velocity is +5 m/s for 4 s and −4 m/s for 3 s. Find displacement and distance. [3 marks]

R5. A tangent through (2,5) and (8,17) is drawn on a velocity–time graph. Estimate acceleration. [2 marks]

R6. Velocities at times 0,2,4,6 s are 0,4,7,8 m/s. Use trapezia for displacement. [4 marks]


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SOLUTIONS / LESSON PRACTICE

# Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

## Lesson 1 • Gradient as acceleration

1.1 Velocity rises from 0 to 18 m/s in 9 s. Find acceleration.

2 m/s².

1.2 Velocity falls from 20 to 8 m/s in 4 s. Find acceleration.

$(8 - 20)/4 = -3$ m/s².

1.3 Describe a horizontal graph at $v = 5$ m/s.

Constant positive velocity of 5 m/s; acceleration is zero.

1.4 An object has $v = -6$ m/s and acceleration +2 m/s². Initially does it speed up or slow down?

It slows down initially because the acceleration moves its negative velocity towards zero.

## Lesson 2 • Area, displacement and distance

2.1 Find displacement at constant 7 m/s for 8 s.

$7 \times 8 = 56$ m.

2.2 Velocity rises linearly from 4 to 10 m/s in 6 s. Find displacement.

Trapezium area $= (4 + 10)/2 \times 6 = 42$ m.

2.3 Velocity falls linearly from 8 to 0 m/s in 5 s. Find distance.

Triangle area $= 1/2 \times 5 \times 8 = 20$ m.

2.4 Velocity is −3 m/s for 4 s and +5 m/s for 2 s. Find displacement and distance.

Displacement $-12 + 10 = -2$ m. Distance $12 + 10 = 22$ m.


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SOLUTIONS / LESSON PRACTICE

# Check the reasoning • continued

## Lesson 3 • Curved graphs, tangents and numerical area

3.1 A tangent passes through (1,4) and (5,14). Find its gradient.

$(14 - 4)/(5 - 1) = 2.5$ in the graph's rate units.

3.2 Velocities at 0,3,6 s are 2,5,8 m/s. Find the trapezium estimate.

$3(2 + 5)/2 + 3(5 + 8)/2 = 10.5 + 19.5 = 30$ m.

3.3 How does a chord differ from a tangent in rate interpretation?

A chord gives average change across an interval; a tangent gives instantaneous change at a point.

3.4 For a curve bowing below its straight chord, is the chord-based trapezium area an overestimate?

Yes, provided the comparison is over the same interval: the straight top lies above the curve.


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SOLUTIONS / MIXED REVIEW

# Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Velocity rises from 3 to 15 m/s in 4 s. Find acceleration. [2 marks]

$(15 - 3)/4 = 3$ m/s². [1 for changes; 1 for result/unit.]

R2. Velocity is 6 m/s for 7 s. Find displacement. [2 marks]

$6 \times 7 = 42$ m. [1 for area/model; 1 for result.]

R3. Velocity rises linearly from 2 to 14 m/s in 5 s. Find displacement. [3 marks]

$(2 + 14)/2 \times 5 = 40$ m. [1 for trapezium; 1 for substitution; 1 for result.]

R4. Velocity is +5 m/s for 4 s and −4 m/s for 3 s. Find displacement and distance. [3 marks]

Signed areas +20 and −12: displacement 8 m; distance 32 m. [1 for areas; 1 for each total.]

R5. A tangent through (2,5) and (8,17) is drawn on a velocity–time graph. Estimate acceleration. [2 marks]

$12/6 = 2$ m/s². [1 for gradient method; 1 for answer.]

R6. Velocities at times 0,2,4,6 s are 0,4,7,8 m/s. Use trapezia for displacement. [4 marks]

Areas 4,11,15 m; sum 30 m. [1 for each trapezium; 1 for total/unit.]

## Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.


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