GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 23

23

Cubic, reciprocal and exponential graphs

Shared recognition with Higher extensions

Different rules produce recognisable families of curves. Learn their key points, intercepts, symmetry and long-term behaviour. Pay attention to values that are excluded from a graph’s domain.

x^3 $\frac{1}{x}$ 2^x

Different rules produce different graph families.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Plot coordinates, calculate powers and substitute positive and negative values.

Your route through this notebook

1. Cubic graphs
2. Reciprocal curves and asymptotes
3. Exponential growth and decay — Higher extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Cubic graphs

The basic cubic $y = x^3$ passes through the origin and has rotational symmetry of order two about it. Negative inputs give negative outputs and positive inputs give positive outputs. Unlike $y = x^2$, its two ends point in opposite directions. The curve flattens at the origin but continues increasing through it.

Adding a constant translates the graph vertically. Multiplying by a negative coefficient reflects it in the x-axis and reverses its overall direction. More general cubic graphs can have turning points, so not every cubic looks exactly like x^3. For sketching, identify intercepts and useful points rather than reproducing a shape from memory without reference to the equation.

A reliable method

1. Calculate values on both sides of zero.
2. Identify intercepts and the sign of the leading term.
3. Draw a smooth curve with the correct end behaviour.

Worked example 1

Give points on $y = x^3$ for $x = -2,-1,0,1,2$.

STEP-BY-STEP SOLUTION

The points are $(-2,-8)$, $(-1,-1)$, $(0,0)$, $(1,1)$, $(2,8)$.

Worked example 2

Describe $y = -x^3 + 2$.

STEP-BY-STEP SOLUTION

Reflect $y = x^3$ in the x-axis, then translate up 2. Its y-intercept is $(0,2)$, and it decreases overall.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An S-shaped cubic may have a stationary point without a maximum or minimum. At the origin, $y = x^3$ flattens but does not reverse direction.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Evaluate x^3 at $x = -3$.

1.2 Find the y-intercept of $y = x^3 - 5$.

1.3 Find the x-intercept of $y = x^3 - 8$.

1.4 Describe the symmetry of $y = x^3$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Reciprocal curves and asymptotes

For y = 1/x, x cannot be zero. Positive inputs give positive outputs and negative inputs give negative outputs, creating two separate branches. Near zero the magnitude of y grows very large; far from zero it approaches zero. The axes are asymptotes: lines approached by the curve but not reached in this basic example.

The graph y = k/x changes scale and, for negative k, changes the occupied quadrants. Do not join a line across x = 0. A plotted table with widely spaced values can hide the rapid change near the excluded input. Domain restrictions are part of the mathematics, not just a graphing inconvenience.

A reliable method

1. Exclude x = 0 and calculate points on both sides.

2. Sketch separate branches approaching the axes.

3. Use the sign of k to determine which quadrants contain the graph.

Worked example 1

Find points on y = 6/x for x = −3,−2,1,2,3.

STEP-BY-STEP SOLUTION

The points are (−3,−2), (−2,−3), (1,6), (2,3), (3,2).

Worked example 2

Describe the quadrants for y = −4/x.

STEP-BY-STEP SOLUTION

For positive x, y is negative; for negative x, y is positive. The branches are in quadrants IV and II.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The fact that y approaches zero does not mean y = 0 is a point on y = 1/x. No finite non-zero x makes 1/x equal zero.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find $y = 8/x$ at $x = -4$.

2.2 State the excluded input for $y = 3/x$.

2.3 State the asymptotes of $y = 5/x$.

2.4 Solve $6/x = 2$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Exponential growth and decay — Higher extension

In $y = a^x$, the input is in the exponent. For $a > 1$, values grow as x increases; for $0 < a < 1$ they decrease. Every positive-base graph passes through (0,1), because $a^0 = 1$. The outputs stay positive and the x-axis is an asymptote.

In $y = ka^x$, the initial value at $x = 0$ is k and a is a repeated multiplier for each unit increase in x. A 20% increase has multiplier 1.2; a 20% decrease has multiplier 0.8. This is multiplicative change, unlike the constant additive change of a straight line.

A reliable method

1. Identify the initial value and unit-step multiplier.
2. Calculate several inputs, including zero.
3. Distinguish exponential change from constant-rate linear change.

Worked example 1

Find $y = 2^x$ at $x = -2, -1, 0, 1, 2$.

STEP-BY-STEP SOLUTION

Values are 1/4, 1/2, 1, 2, 4. Negative exponents give reciprocals.

Worked example 2

A model is $P = 500(0.9)^t$. Interpret and evaluate at $t = 2$.

STEP-BY-STEP SOLUTION

Initial amount 500, with a 10% decrease per time unit. $P = 500 \times 0.81 = 405$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

$y = 2x$ and $y = 2^x$ are different rules. The first adds 2 to y for every unit of x; the second doubles y. They may share some points without being the same graph.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find the y-intercept of $y = 3^x$.

3.2 Evaluate 4×2^x at $x = 3$.

3.3 Does $y = (1/2)^x$ grow or decay as x increases?

3.4 Write a model for an initial 200 growing by 5% each whole time step.

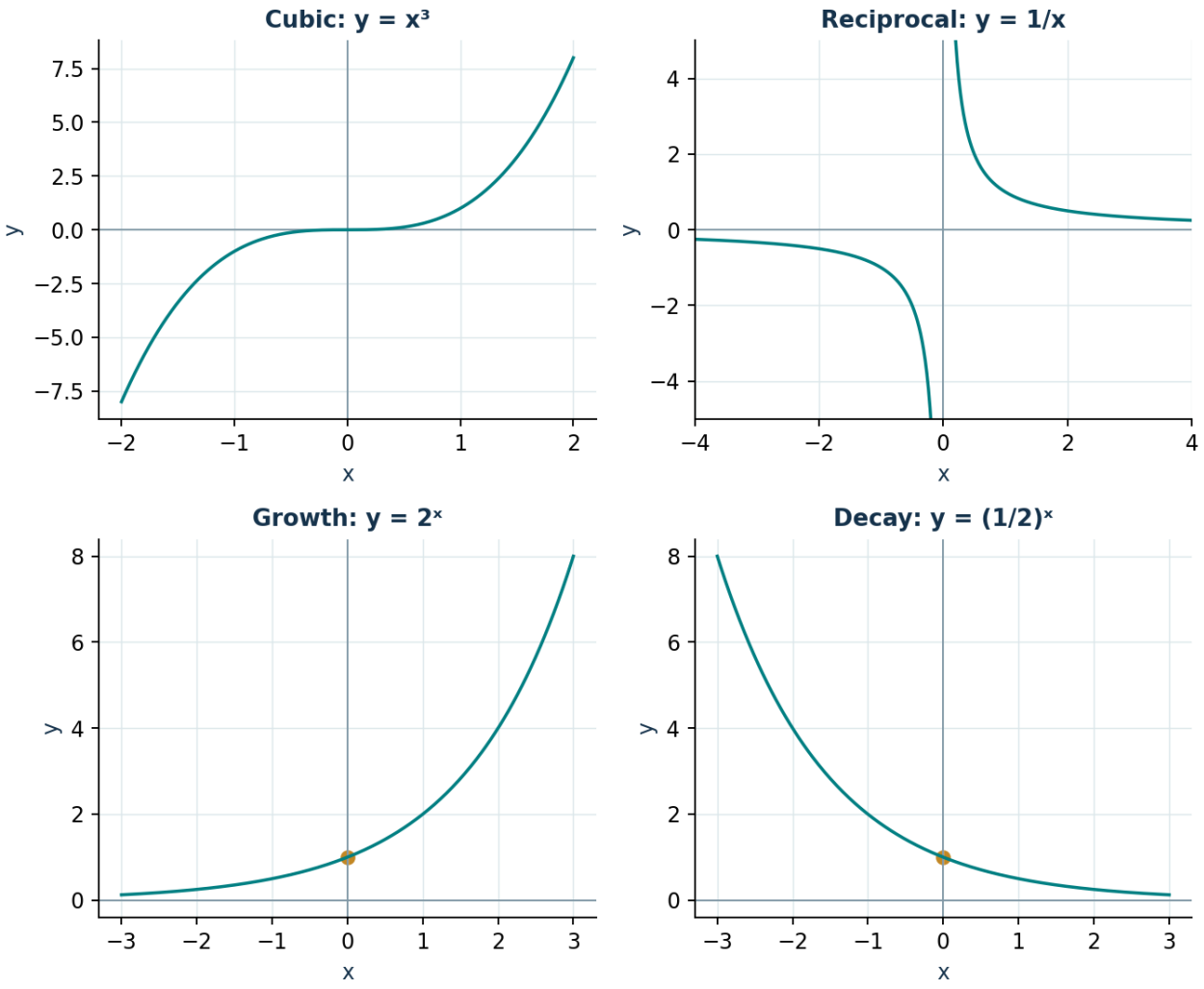
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

Recognise families from equations and key features



WHAT THE FIGURE SHOWS

The cubic takes negative and positive values. The reciprocal has two branches and excludes x = 0. Both exponential curves pass through (0,1), stay positive and approach the x-axis. Growth and decay differ in whether the multiplier is above or below 1.

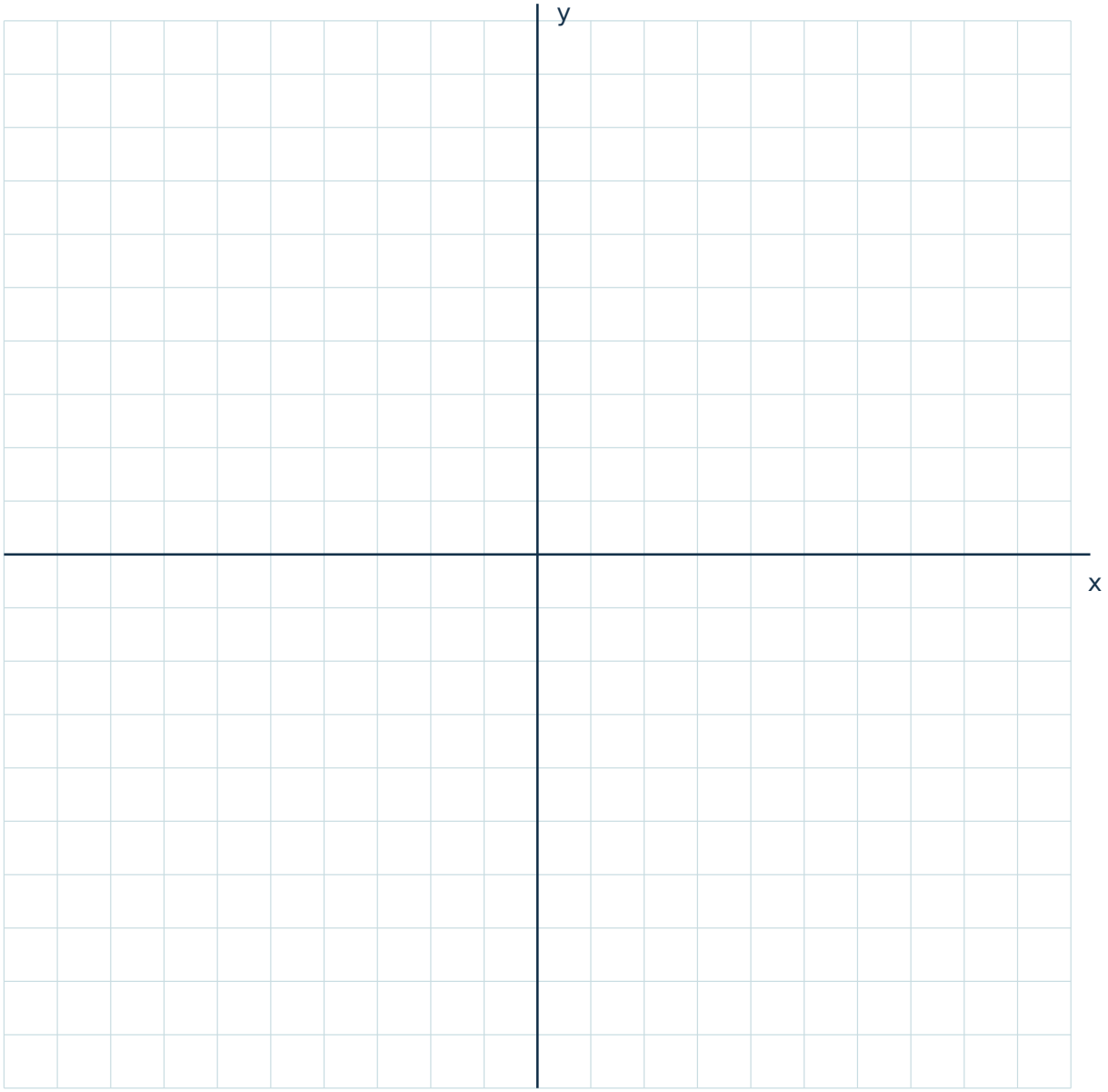
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find $y = x^3 + 1$ at $x = -2$. [2 marks]

R2. Find the root of $y = x^3 + 27$. [2 marks]

R3. For $y = 12/x$, find y at $x = -6$ and state the excluded input. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Explain why $y = 1/x$ has no x-intercept. [2 marks]

R5. Higher: evaluate $300(1.1)^2$. [2 marks]

R6. Higher: compare growth of $y = 3x$ and $y = 3^x$ at $x = 0,1,2$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Cubic graphs

1.1 Evaluate x^3 at $x = -3$.

-27.

1.2 Find the y-intercept of $y = x^3 - 5$.

$(0,-5)$.

1.3 Find the x-intercept of $y = x^3 - 8$.

$x^3 = 8$ gives $x = 2$, so $(2,0)$.

1.4 Describe the symmetry of $y = x^3$.

Rotational symmetry of order two about the origin.

Lesson 2 • Reciprocal curves and asymptotes

2.1 Find $y = 8/x$ at $x = -4$.

-2.

2.2 State the excluded input for $y = 3/x$.

$x = 0$.

2.3 State the asymptotes of $y = 5/x$.

$x = 0$ and $y = 0$.

2.4 Solve $6/x = 2$.

For $x \neq 0$, multiply by x: $6 = 2x$, so $x = 3$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Exponential growth and decay — Higher extension

3.1 Find the y-intercept of $y = 3^x$.

(0,1).

3.2 Evaluate 4×2^x at $x = 3$.

$4 \times 8 = 32$.

3.3 Does $y = (1/2)^x$ grow or decay as x increases?

It decays, because the multiplier is between 0 and 1.

3.4 Write a model for an initial 200 growing by 5% each whole time step.

$P = 200(1.05)^t$, with non-negative integer t for the stated step model.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find $y = x^3 + 1$ at $x = -2$. [2 marks]

$-8 + 1 = -7$. [1 for cube; 1 for result.]

R2. Find the root of $y = x^3 + 27$. [2 marks]

$x^3 = -27$, so $x = -3$. [1 for isolation; 1 for root.]

R3. For $y = 12/x$, find y at $x = -6$ and state the excluded input. [2 marks]

$y = -2$; $x \neq 0$. [1 each.]

R4. Explain why $y = 1/x$ has no x-intercept. [2 marks]

$1/x$ cannot be zero for any finite non-zero x. Multiplying $1/x = 0$ by an allowed x would give $1 = 0$. [1 for claim; 1 for justification.]

R5. Higher: evaluate $300(1.1)^2$. [2 marks]

$300 \times 1.21 = 363$. [1 for square; 1 for result.]

R6. Higher: compare growth of $y = 3x$ and $y = 3^x$ at $x = 0,1,2$. [3 marks]

Linear values: 0,3,6. Exponential values: 1,3,9. Linear adds 3; exponential multiplies by 3 per unit step. [1 for each list; 1 for interpretation.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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