GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 22

22

Quadratic graphs

Shared core with Higher extensions

A quadratic graph is a parabola. Connect tables and sketches with roots, intercepts and symmetry, then use completed-square form to identify turning points. Actual plotted examples accompany the algebra.

$$y = (x - 3)^2 - 4$$

Roots, intercepts and the turning point fit together.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Substitute signed numbers, expand brackets and factorise simple quadratics.

Your route through this notebook

1. Tables, plotting and parabolic shape
2. Roots, intercepts and graphical solutions
3. Turning points and completed-square form — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Tables, plotting and parabolic shape

A quadratic has form $y = ax^2 + bx + c$ with $a \neq 0$. Its graph is a smooth parabola, opening upwards if $a > 0$ and downwards if $a < 0$. Use brackets when squaring negative x values. Plot enough points on both sides of the turning point to show the shape.

A curve should pass smoothly through accurate points rather than zigzag between them. Equal x values either side of the symmetry line produce equal y values. A restricted x interval shows only part of the graph, so do not infer the full range from a short table alone.

A reliable method

1. Choose x values covering both sides of the vertex.

2. Calculate each y using the original rule.

3. Draw a smooth curve and check symmetry.

Worked example 1

Make a table for $y = x^2 - 4$ at $x = -2,-1,0,1,2$.

STEP-BY-STEP SOLUTION

The y values are $0,-3,-4,-3,0$. The minimum is $(0,-4)$ and the graph is symmetric about $x = 0$.

Worked example 2

Describe $y = -2x^2 + 3$.

STEP-BY-STEP SOLUTION

The negative leading coefficient makes it open downwards. It crosses the y-axis at $(0,3)$, which is also its maximum point.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For $x = -2$, x^2 is 4, not -4. A missing pair of substitution brackets can turn an upward parabola into an incorrect table.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find $y = x^2 + 1$ at $x = -2, -1, 0, 1, 2$.

1.2 State the opening direction of $y = 3x^2 - 2$.

1.3 State the y-intercept of $y = x^2 - 5x + 6$.

1.4 Find the symmetry line of $y = (x - 2)^2 + 1$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Roots, intercepts and graphical solutions

Roots are x values where y = 0, so they are the x-coordinates of x-axis intersections. Factorisation can identify exact roots; a graph can give approximate readings. The y-intercept is found by substituting x = 0. These are different kinds of intercept.

To solve $x^2 - 4 = 5$ using the graph $y = x^2 - 4$, draw the horizontal line y = 5 and read the intersection x values. To solve two graph rules simultaneously, read their intersections. A tangent touching the axis gives a repeated root; a graph missing the axis has no real roots.

A reliable method

1. Set y = 0 for roots, and x = 0 for the y-intercept.
2. For y = k, use the horizontal line at height k.
3. State graph readings to an accuracy justified by the scale.

Worked example 1

Find roots and y-intercept of $y = x^2 - 5x + 6$.

STEP-BY-STEP SOLUTION

Factorise $(x - 2)(x - 3)$, giving roots 2 and 3. The y-intercept is (0,6).

Worked example 2

Use $y = x^2 - 4$ to solve $x^2 - 4 = 5$ exactly.

STEP-BY-STEP SOLUTION

Intersections with y = 5 satisfy $x^2 = 9$, so $x = -3$ or 3. A plotted graph should show these two intersections.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A root is usually reported as an x value; its coordinate point is (x,0). Do not describe the y-intercept (0,c) as a root unless c is zero.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find roots of $y = x^2 - 9$.

2.2 Find roots of $y = (x + 2)(x - 5)$.

2.3 How many real roots has $y = x^2 + 4$?

2.4 Solve $x^2 - 1 = 8$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Turning points and completed-square form — Higher

In $y = a(x - h)^2 + k$, the squared expression is zero at $x = h$. The turning point is therefore (h,k), with symmetry line $x = h$. For $a > 0$ it is a minimum; for $a < 0$ it is a maximum. The sign inside the bracket is opposite the coordinate h.

Completing the square rewrites an expanded quadratic into this revealing form. For $x^2 + bx$, take half of b and square it: $x^2 + bx = (x + b/2)^2 - (b/2)^2$. Add the original constant afterwards. This also helps find roots and the range of y values.

A reliable method

1. Read or create $a(x - h)^2 + k$.

2. State the turning point and symmetry line.

3. Use the sign of a to identify a minimum or maximum.

Worked example 1

Find the turning point of $y = x^2 - 6x + 5$.

STEP-BY-STEP SOLUTION

Complete the square: $(x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$. Minimum point $(3,-4)$; symmetry $x = 3$.

Worked example 2

Describe the turning point and range of $y = -2(x + 1)^2 + 7$.

STEP-BY-STEP SOLUTION

Turning point $(-1,7)$, a maximum. Since the squared term is non-negative, $y \leq 7$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For $y = (x + 4)^2 - 2$, the turning point is $(-4,-2)$, not $(4,-2)$. Set the bracket equal to zero to decide its horizontal position.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find the turning point of $y = (x - 5)^2 + 3$.

3.2 Complete the square for $x^2 + 4x + 1$.

3.3 Find the minimum of $y = x^2 + 8x + 10$.

3.4 Find the roots of $y = (x - 2)^2 - 9$.

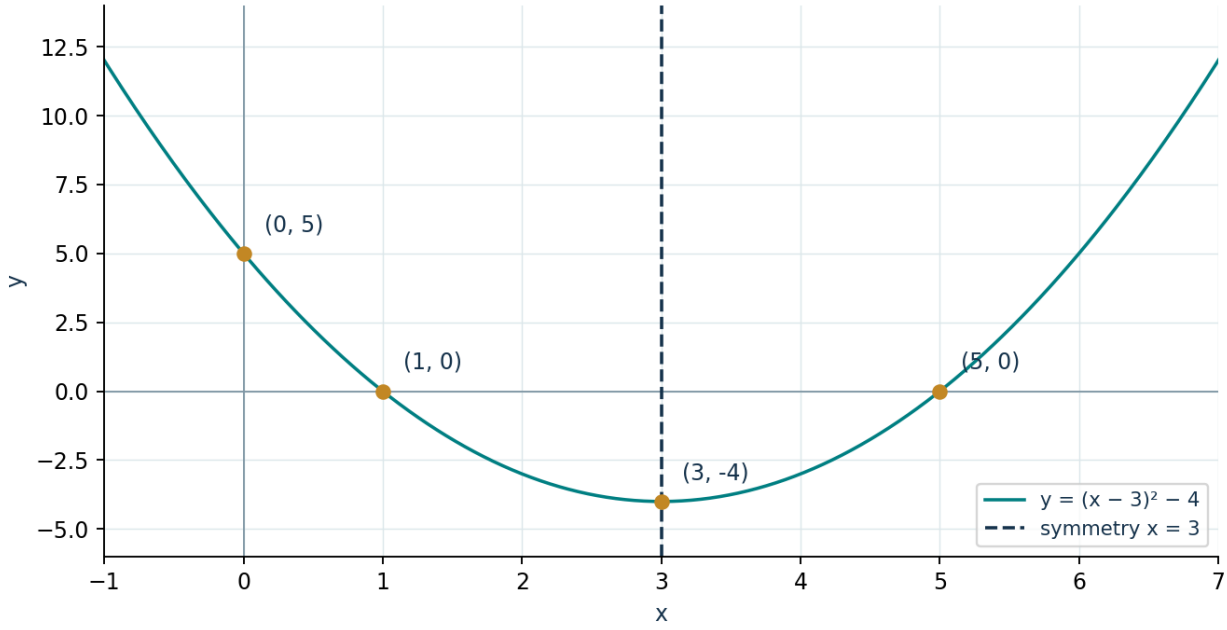
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

Roots, turning point and symmetry



WHAT THE FIGURE SHOWS

Expanding gives $y = x^2 - 6x + 5$. Roots are 1 and 5; the minimum is $(3,-4)$; the y-intercept is $(0,5)$. Equal horizontal steps either side of $x = 3$ give equal heights. These features should agree with any table of values.

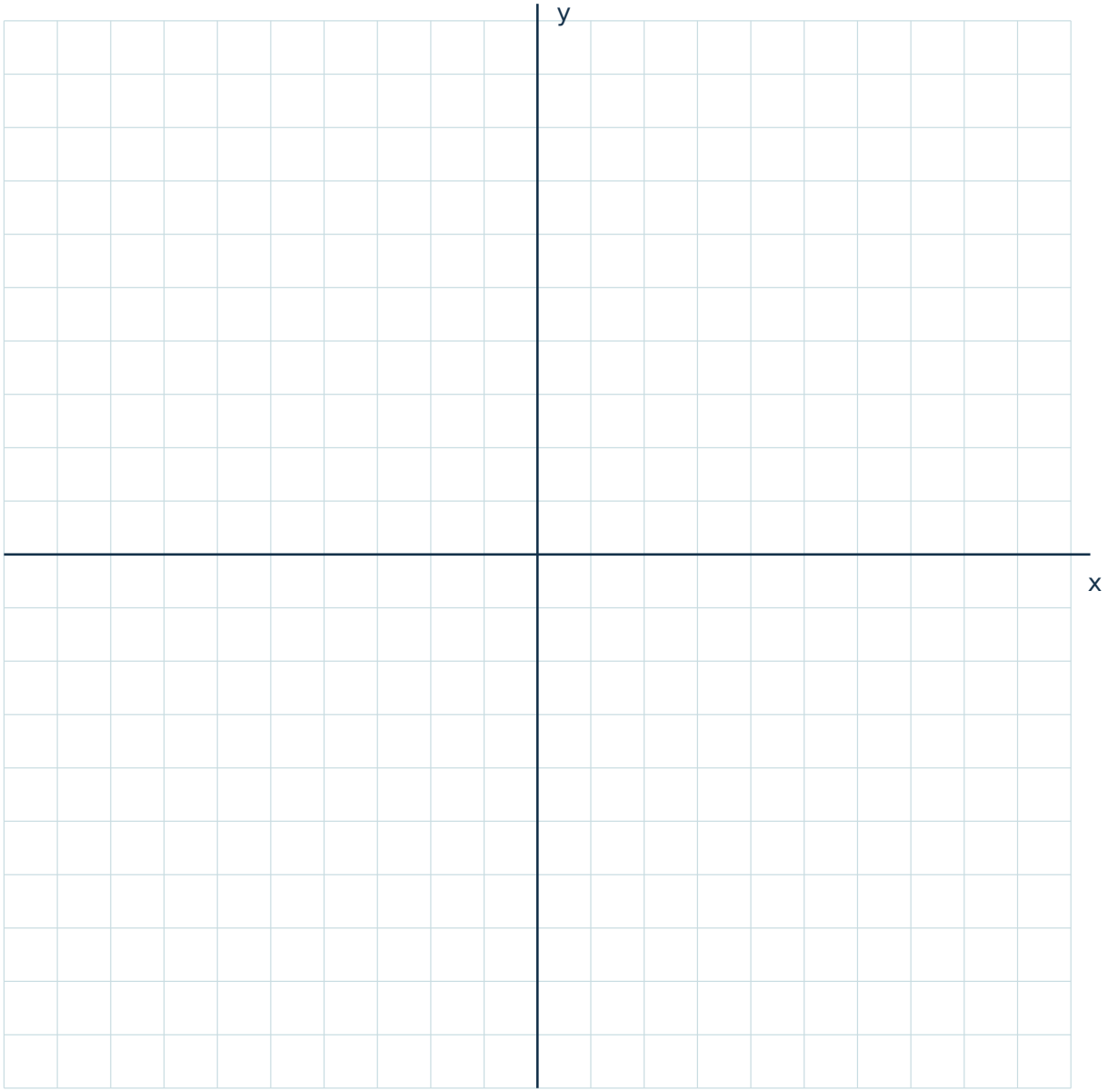
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Calculate $y = x^2 - 2x$ for $x = -2$. [2 marks]

R2. Find the y-intercept of $y = 2x^2 + 3x - 5$. [1 mark]

R3. Find roots of $y = x^2 - 7x + 10$. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Solve $x^2 - 4 = 12$. [2 marks]

R5. Higher: complete the square for $x^2 - 10x + 21$. [3 marks]

R6. Higher: state the turning point, symmetry line and range of $y = -(x - 3)^2 + 8$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Tables, plotting and parabolic shape

1.1 Find $y = x^2 + 1$ at $x = -2,-1,0,1,2$.

5,2,1,2,5.

1.2 State the opening direction of $y = 3x^2 - 2$.

Upwards, because $3 > 0$.

1.3 State the y-intercept of $y = x^2 - 5x + 6$.

At $x = 0$, $y = 6$, so (0,6).

1.4 Find the symmetry line of $y = (x - 2)^2 + 1$.

$x = 2$.

Lesson 2 • Roots, intercepts and graphical solutions

2.1 Find roots of $y = x^2 - 9$.

$x = -3$ and 3.

2.2 Find roots of $y = (x + 2)(x - 5)$.

$x = -2$ and 5.

2.3 How many real roots has $y = x^2 + 4$?

None: its minimum y is 4, above the x-axis.

2.4 Solve $x^2 - 1 = 8$.

$x^2 = 9$, so $x = -3$ or 3.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Turning points and completed-square form — Higher

3.1 Find the turning point of $y = (x - 5)^2 + 3$.

(5,3), a minimum.

3.2 Complete the square for $x^2 + 4x + 1$.

$(x + 2)^2 - 3$.

3.3 Find the minimum of $y = x^2 + 8x + 10$.

$y = (x + 4)^2 - 6$, so the minimum value is -6 at $x = -4$.

3.4 Find the roots of $y = (x - 2)^2 - 9$.

$(x - 2)^2 = 9$, so $x - 2 = \pm 3$. Roots -1 and 5.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Calculate $y = x^2 - 2x$ for $x = -2$. [2 marks]

$4 + 4 = 8$. [1 for substitution/signs; 1 for result.]

R2. Find the y-intercept of $y = 2x^2 + 3x - 5$. [1 mark]

$(0,-5)$. [1 for point.]

R3. Find roots of $y = x^2 - 7x + 10$. [3 marks]

$(x - 2)(x - 5) = 0$, so roots 2 and 5. [1 for factors; 1 for each root.]

R4. Solve $x^2 - 4 = 12$. [2 marks]

$x^2 = 16$, so $x = \pm 4$. [1 for isolation; 1 for both values.]

R5. Higher: complete the square for $x^2 - 10x + 21$. [3 marks]

$(x - 5)^2 - 4$. [1 for half coefficient; 1 for constant adjustment; 1 for final form.]

R6. Higher: state the turning point, symmetry line and range of $y = -(x - 3)^2 + 8$. [3 marks]

Maximum point (3,8), symmetry $x = 3$, range $y \leq 8$. [1 each.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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