GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 21

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Coordinates and straight-line graphs

Shared core with Higher extensions

Coordinates connect algebra to position. Learn to plot lines, interpret gradient and intercept, and find equations from points. Higher extensions include perpendicular lines. Use equal coordinate units and read every scale carefully.

$$y = mx + c$$

Gradient m measures change; c is the intercept.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Substitute values, solve linear equations and calculate with negative numbers.

Your route through this notebook

1. Coordinates and tables of values
2. Gradient, intercept and line equations
3. Parallel and perpendicular lines — extension

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Coordinates and tables of values

An ordered pair (x,y) gives horizontal position first and vertical position second. The origin is (0,0). Negative x is left of the vertical axis and negative y is below the horizontal axis. The order matters: (2,5) and (5,2) usually name different points.

For a graph y = f(x), choose x values and calculate the corresponding y values. Plot the pairs accurately. A straight-line rule gives collinear points, so use a ruler and extend the line as appropriate. At least a third point helps check an error when two points alone would always define some line.

A reliable method

1. Read x horizontally, then y vertically.

2. Prepare a table with substitutions shown where signs are difficult.

3. Plot accurately and check the shape against the rule.

Worked example 1

Find points on $y = 2x - 1$ for $x = -1,0,2$.

STEP-BY-STEP SOLUTION

Substitution gives $y = -3,-1,3$. Plot $(-1,-3)$, $(0,-1)$, $(2,3)$.

Worked example 2

Find the midpoint of $(-2,4)$ and $(6,10)$.

STEP-BY-STEP SOLUTION

Average coordinates: $((-2 + 6)/2, (4 + 10)/2) = (2,7)$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

On a graph, one small square need not mean one unit. Read labelled ticks on each axis separately; horizontal and vertical scales may differ.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Give the coordinates of a point 3 left and 5 up from the origin.

1.2 For $y = 3x + 2$, find y at $x = -2,0,2$.

1.3 Does (4,9) lie on $y = 2x + 1$?

1.4 Find the midpoint of $(1,-3)$ and (7,5).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Gradient, intercept and line equations

In $y = mx + c$, m is the gradient and c is the y-intercept. Gradient is vertical change divided by horizontal change, using the same order of points in both differences. A positive gradient rises left to right; a negative gradient falls. A horizontal line has gradient zero. A vertical line has equation $x = k$ and no finite gradient.

To find a line from two points, calculate m, substitute one point into $y = mx + c$ and solve for c. The y-intercept is the y value at $x = 0$, not an arbitrary point where the line crosses another graph.

A reliable method

1. Calculate $m = (y_2 - y_1)/(x_2 - x_1)$.

2. Substitute a known point to find c.

3. Check the second point in the completed equation.

Worked example 1

Find the line through (1,5) and (4,11).

STEP-BY-STEP SOLUTION

$m = (11 - 5)/(4 - 1) = 2$. Use $5 = 2 \times 1 + c$, so $c = 3$. Equation: $y = 2x + 3$.

Worked example 2

Find the x-intercept of $y = -3x + 12$.

STEP-BY-STEP SOLUTION

At the x-axis $y = 0$. Solve $0 = -3x + 12$ to get $x = 4$. Intercept (4,0).

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Gradient is a ratio of coordinate changes, not the visual angle on a distorted display. A rise of 4 for a run of 2 gives gradient 2, whatever the printed scale.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 State gradient and y-intercept of $y = -4x + 7$.

2.2 Find the gradient through $(-1,2)$ and $(3,10)$.

2.3 Find the line of gradient 3 through $(2,8)$.

2.4 Write the horizontal line through $(5,-2)$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Parallel and perpendicular lines — extension

Distinct non-vertical parallel lines have equal gradients and different intercepts. A line through a given point parallel to another line uses the same gradient; substitute the point to find its intercept. Coincident lines have both the same gradient and the same intercept.

For non-zero finite gradients, perpendicular lines have gradients whose product is −1. Therefore a perpendicular gradient is the negative reciprocal. A line of gradient 2 is perpendicular to one of gradient −1/2. Horizontal and vertical lines form a separate perpendicular pair. These statements assume the usual equal-unit Cartesian geometry.

A reliable method

1. Put a line into y = mx + c before reading its gradient.

2. For parallel lines keep m; for perpendicular lines use −1/m.

3. Use the supplied point to determine the new intercept.

Worked example 1

Find a line parallel to y = 4x − 1 through (2,11).

STEP-BY-STEP SOLUTION

Use y = 4x + c. Then 11 = 8 + c gives c = 3. Result: y = 4x + 3.

Worked example 2

Higher: find a line perpendicular to y = 2x + 5 through (4,1).

STEP-BY-STEP SOLUTION

New gradient −1/2. In 1 = (−1/2)4 + c, c = 3. Result: y = −x/2 + 3.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The perpendicular gradient is not merely the negative gradient. For m = 3 it is −1/3, not −3. Multiply the two gradients to check for −1.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find a line parallel to y = −2x + 6 through (1,3).

3.2 Higher: find the perpendicular gradient to 5.

3.3 Higher: find the perpendicular gradient to −2/3.

3.4 Higher: find the line perpendicular to y = −x/3 + 4 through (2,1).

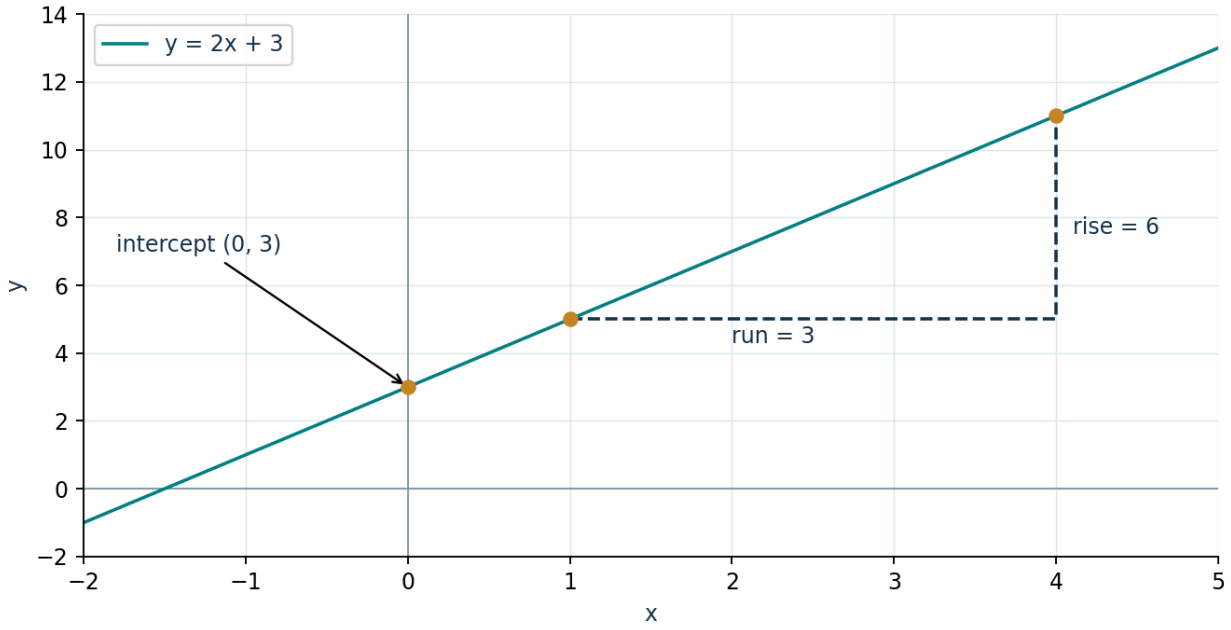
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

Gradient and intercept tell the line’s story



WHAT THE FIGURE SHOWS

The line passes through (1,5) and (4,11). Its gradient is rise/run = 6/3 = 2, and its y-intercept is (0,3). The dashed triangle measures coordinate changes; it is not a second graph.

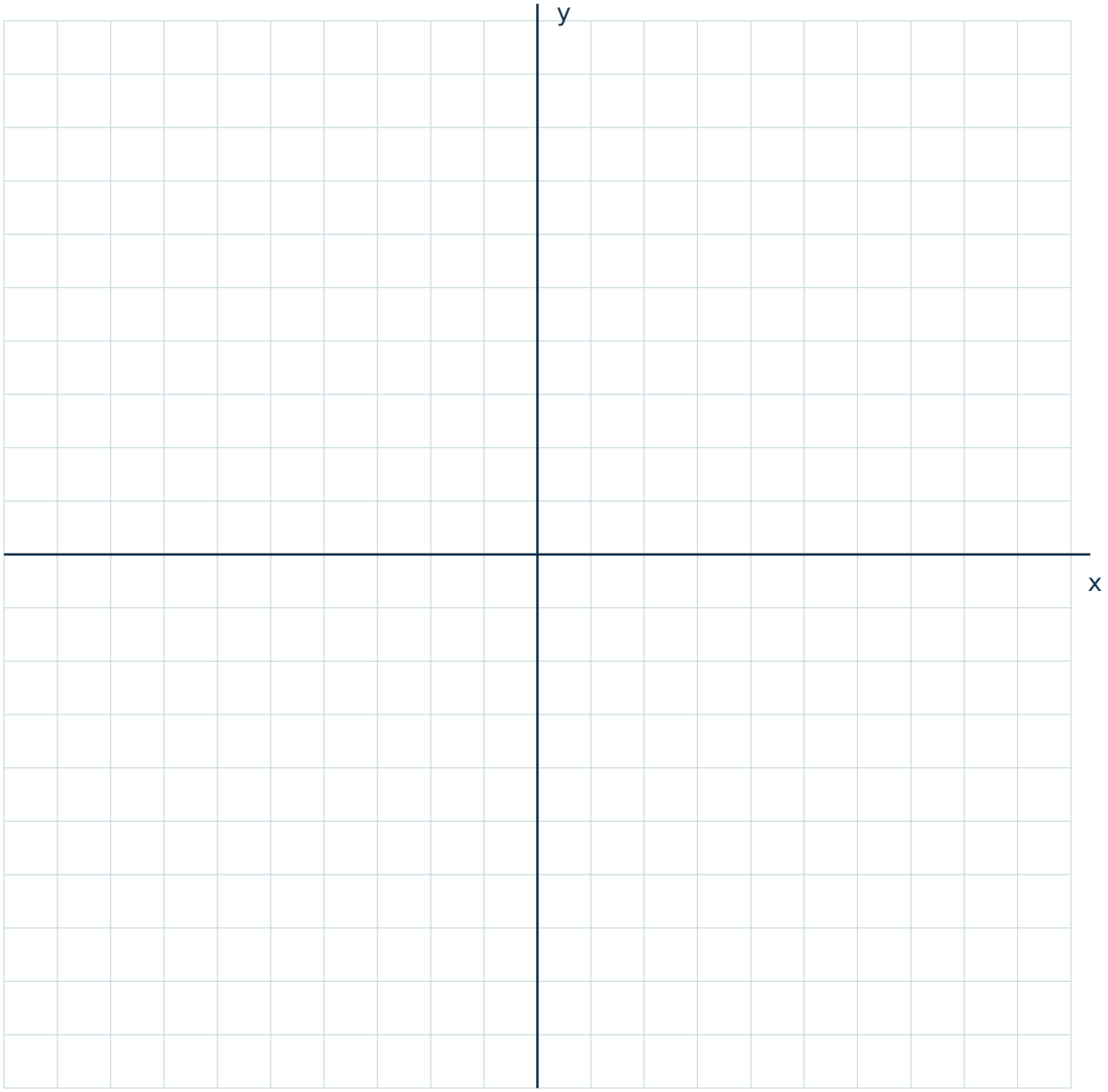
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find y on $y = 4x - 3$ at $x = -2$. [2 marks]

R2. Find the gradient through (2,1) and (6,9). [2 marks]

R3. Find the equation of the line through (0,−2) and (3,7). [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Find the x-intercept of $y = 5x - 15$. [2 marks]

R5. Find a parallel line to $y = 2x + 7$ through (3,4). [3 marks]

R6. Higher: find the perpendicular line to $y = 4x - 1$ through (4,2). [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Coordinates and tables of values

1.1 Give the coordinates of a point 3 left and 5 up from the origin.

$(-3,5)$.

1.2 For $y = 3x + 2$, find y at $x = -2,0,2$.

$-4,2,8$, giving $(-2,-4)$, $(0,2)$, $(2,8)$.

1.3 Does $(4,9)$ lie on $y = 2x + 1$?

Yes: $2 \times 4 + 1 = 9$.

1.4 Find the midpoint of $(1,-3)$ and $(7,5)$.

$(4,1)$.

Lesson 2 • Gradient, intercept and line equations

2.1 State gradient and y-intercept of $y = -4x + 7$.

Gradient -4; y-intercept $(0,7)$.

2.2 Find the gradient through $(-1,2)$ and $(3,10)$.

$(10 - 2)/(3 - (-1)) = 8/4 = 2$.

2.3 Find the line of gradient 3 through $(2,8)$.

$8 = 3 \times 2 + c$ gives $c = 2$. Thus $y = 3x + 2$.

2.4 Write the horizontal line through $(5,-2)$.

$y = -2$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Parallel and perpendicular lines — extension

3.1 Find a line parallel to y = −2x + 6 through (1,3).

y = −2x + c; 3 = −2 + c, so y = −2x + 5.

3.2 Higher: find the perpendicular gradient to 5.

−1/5.

3.3 Higher: find the perpendicular gradient to −2/3.

3/2.

3.4 Higher: find the line perpendicular to y = −x/3 + 4 through (2,1).

New gradient 3. Then 1 = 6 + c, so y = 3x − 5.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find y on $y = 4x - 3$ at $x = -2$. [2 marks]

$y = -8 - 3 = -11$. [1 for substitution; 1 for result.]

R2. Find the gradient through (2,1) and (6,9). [2 marks]

$(9 - 1)/(6 - 2) = 2$. [1 for differences; 1 for quotient.]

R3. Find the equation of the line through (0,−2) and (3,7). [3 marks]

Gradient $9/3 = 3$ and intercept -2, so $y = 3x - 2$. [1 for gradient; 1 for intercept; 1 for equation.]

R4. Find the x-intercept of $y = 5x - 15$. [2 marks]

Set $y = 0$: $x = 3$, giving (3,0). [1 for setting zero; 1 for point.]

R5. Find a parallel line to $y = 2x + 7$ through (3,4). [3 marks]

$4 = 2 \times 3 + c$ gives $c = -2$, so $y = 2x - 2$. [1 for gradient; 1 for c; 1 for equation.]

R6. Higher: find the perpendicular line to $y = 4x - 1$ through (4,2). [3 marks]

Gradient $-1/4$; $2 = -1 + c$ gives $c = 3$. Thus $y = -x/4 + 3$. [1 for gradient; 1 for c; 1 for equation.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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