GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 20

20

Quadratic and geometric sequences

Higher-level GCSE material

Changing differences and repeated multiplication produce patterns beyond linear sequences. Develop quadratic nth terms systematically and use geometric rules with powers, including fractional ratios.

$an^2 + bn + c$ • second difference $2a$

Quadratic differences and geometric multipliers.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Find linear nth terms, use powers and substitute into expressions.

Your route through this notebook

1. Recognising quadratic sequences
2. Finding a quadratic nth term
3. Geometric sequences and exponential rules

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Recognising quadratic sequences

A quadratic sequence has nth term $an^2 + bn + c$ with $a \neq 0$. Its first differences change, but its second differences are constant. For consecutive integer positions, the constant second difference is $2a$. This gives a direct connection between a number pattern and the coefficient of n^2.

A constant second difference is evidence for the intended quadratic pattern, not proof that every possible continuation of a finite list must be quadratic. Check the supplied terms and context. Negative second differences produce negative leading coefficients, so the sequence can eventually turn downwards.

A reliable method

1. Find first differences, then differences of those differences.

2. Use a = half the constant second difference.

3. Compare with an^2 to find the remaining linear part.

Worked example 1

Find the second difference of 3,8,15,24,35.

STEP-BY-STEP SOLUTION

First differences: 5,7,9,11. Second differences: 2,2,2. Therefore the intended quadratic has $a = 1$.

Worked example 2

A quadratic sequence has second difference 6. What is its n^2 coefficient?

STEP-BY-STEP SOLUTION

Since $2a = 6$, $a = 3$. The remaining coefficients still require term information.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A constant first difference signals a linear sequence; a non-zero constant second difference signals a quadratic pattern. Do not use a linear formula merely because the first few terms increase.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Find the first and second differences of 2,7,14,23,34.

1.2 Find a when the second difference is 8.

1.3 Find a when the second difference is −4.

1.4 Continue 1,4,9,16,25 using the square-number rule.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Finding a quadratic nth term

After finding a, subtract the sequence an^2 term by term. The remainders form a linear sequence $bn + c$. Find its nth term using the constant difference method, then add an^2 back. This separates the changing part of the differences from the remaining constant shift and linear growth.

Alternatively, substitute $n = 1,2,3$ into $an^2 + bn + c$ and solve the resulting equations. The subtraction method is often clearer for short GCSE sequences. Check the final formula against at least three supplied terms.

A reliable method

1. Find a from second differences.

2. Subtract an^2 at matching positions.

3. Find the linear remainder and combine the expressions.

Worked example 1

Find the nth term of 4,9,16,25,36.

STEP-BY-STEP SOLUTION

Second difference 2 gives $a = 1$. Subtract 1,4,9,16,25 to leave 3,5,7,9,11, whose rule is $2n + 1$. Result: $n^2 + 2n + 1$.

Worked example 2

Find the nth term of 5,14,29,50.

STEP-BY-STEP SOLUTION

Second differences are 6, so $a = 3$. Subtract 3,12,27,48 to leave 2,2,2,2. Result: $3n^2 + 2$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Subtract corresponding positions. The first comparison uses a $\times 1^2$, not a $\times 0^2$, when the sequence begins at $n = 1$.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find the nth term of 2,5,10,17,26.

2.2 Find the nth term of 3,10,21,36.

2.3 Find the 12th term of $n^2 + 3n - 2$.

2.4 Is 50 in the sequence $n^2 + 1$, $n \geq 1$?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Geometric sequences and exponential rules

A geometric sequence multiplies by a fixed ratio r to get the next term. If its first term is a, the nth term is ar^{n-1}. The exponent is n − 1 because reaching the first term requires no multiplications by r. With a positive ratio below 1, positive terms decrease; with ratio greater than 1, they grow.

Square-root ratios work exactly like other ratios. Some sequences use negative ratios and alternate signs; recognise them even when the focus is positive-ratio GCSE patterns. A recursive rule and a direct nth-term rule are different descriptions of the same sequence.

A reliable method

1. Divide consecutive non-zero terms to find the ratio.

2. Use first term × ratio^{n-1}.

3. Check n = 1 and n = 2 before using a distant position.

Worked example 1

Find the nth term of 3,6,12,24,... .

STEP-BY-STEP SOLUTION

First term 3, ratio 2. Rule: $3 \times 2^{n-1}$. The 5th term is 3 × 16 = 48.

Worked example 2

Find the 6th term of 80,40,20,10,... .

STEP-BY-STEP SOLUTION

The ratio is 1/2. Term 6 = $80 \times (1/2)^5 = 2.5$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A geometric sequence is not recognised by equal differences. For 2,6,18,54 the ratios are all 3 while the differences are 4,12,36. Compare the correct relationship.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Find the nth term of 5,15,45,135,... .

3.2 Find the 5th term of 48,24,12,... .

3.3 Find the ratio of $2,2\sqrt{3},6,6\sqrt{3},\ldots$.

3.4 Find the first four terms of $4 \times 3^{n-1}$.

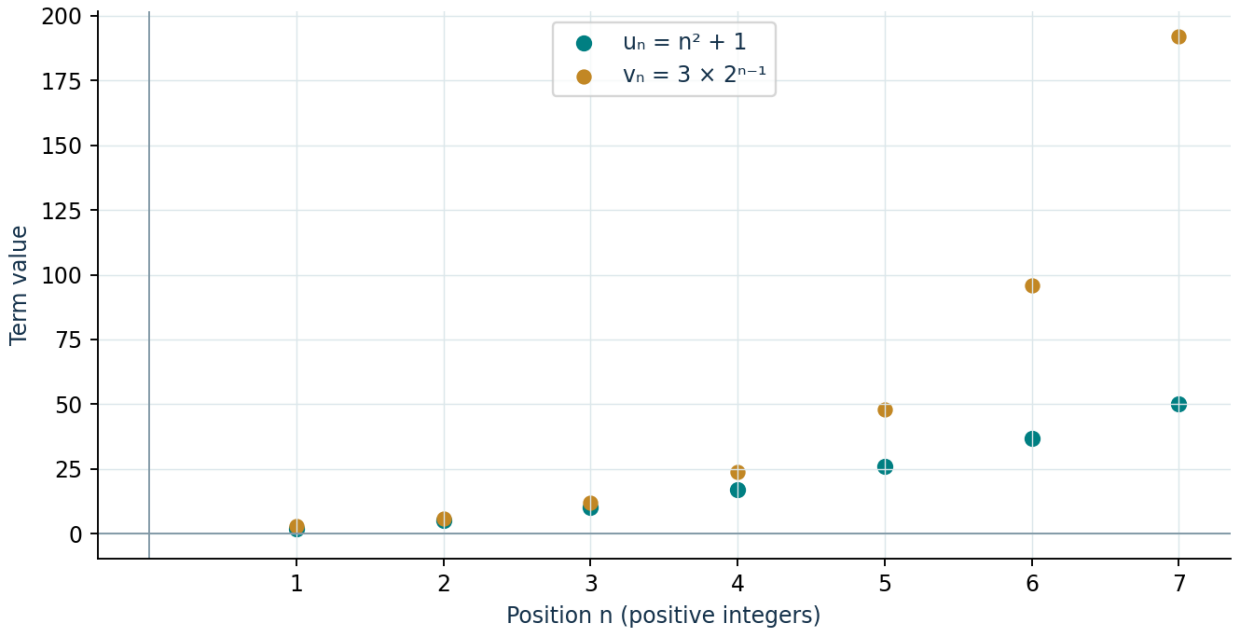
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

Position is discrete: plot terms as points



WHAT THE FIGURE SHOWS

These are sequences indexed by positive integers, so the plotted terms are separate points. The quadratic rule adds changing differences; the geometric rule doubles each term. The geometric values eventually overtake the quadratic values.

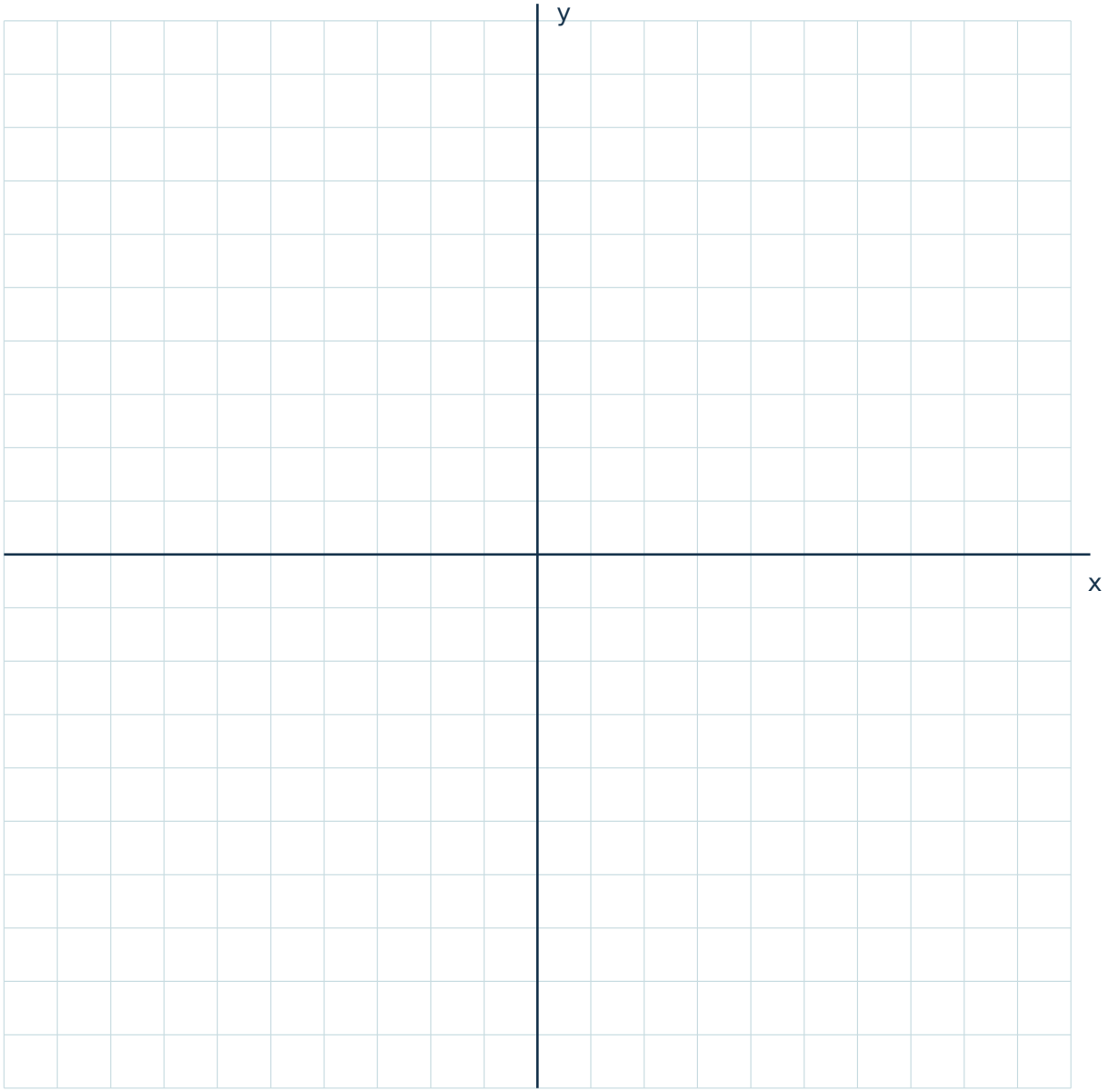
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Find the second difference of 1,6,15,28,45. [2 marks]

R2. Find the nth term of 2,8,18,32,50. [3 marks]

R3. Find the nth term of 6,11,18,27,38. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Find term 15 of $2n^2 - n + 4$. [2 marks]

R5. Find the nth term of 7,21,63,... . [2 marks]

R6. Find the 8th term of 128,64,32,... . [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Recognising quadratic sequences

1.1 Find the first and second differences of 2,7,14,23,34.

First: 5,7,9,11. Second: 2,2,2.

1.2 Find a when the second difference is 8.

$a = 4$.

1.3 Find a when the second difference is -4.

$a = -2$.

1.4 Continue 1,4,9,16,25 using the square-number rule.

36, 49, corresponding to 6^2 and 7^2.

Lesson 2 • Finding a quadratic nth term

2.1 Find the nth term of 2,5,10,17,26.

Subtract n^2: 1,1,1,1,1. Rule: $n^2 + 1$.

2.2 Find the nth term of 3,10,21,36.

Second difference 4 gives $2n^2$. Remainder 1,2,3,4 gives n. Rule: $2n^2 + n$.

2.3 Find the 12th term of $n^2 + 3n - 2$.

$144 + 36 - 2 = 178$.

2.4 Is 50 in the sequence $n^2 + 1$, $n \geq 1$?

$n^2 + 1 = 50$ gives $n^2 = 49$ and positive position $n = 7$. Yes.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Geometric sequences and exponential rules

3.1 Find the nth term of 5,15,45,135,... .

$5 \times 3^{n-1}$.

3.2 Find the 5th term of 48,24,12,... .

$48 \times (1/2)^4 = 3$.

3.3 Find the ratio of $2,2\sqrt{3},6,6\sqrt{3},\ldots$.

The common ratio is $\sqrt{3}$.

3.4 Find the first four terms of $4 \times 3^{n-1}$.

4,12,36,108.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Find the second difference of 1,6,15,28,45. [2 marks]

First differences 5,9,13,17; second difference 4. [1 for each stage.]

R2. Find the nth term of 2,8,18,32,50. [3 marks]

Second difference 4 gives $2n^2$; there is no remainder. [1 for differences; 1 for coefficient; 1 for rule.]

R3. Find the nth term of 6,11,18,27,38. [3 marks]

Second difference 2; subtract n^2 to leave 5,7,9,11,13 = 2n + 3. Rule $n^2 + 2n + 3$. [1 for coefficient; 1 for remainder; 1 for rule.]

R4. Find term 15 of $2n^2 - n + 4$. [2 marks]

$2 \times 225 - 15 + 4 = 439$. [1 for substitution; 1 for result.]

R5. Find the nth term of 7,21,63,… . [2 marks]

$7 \times 3^{n-1}$. [1 for ratio; 1 for correct exponent and first term.]

R6. Find the 8th term of 128,64,32,… . [3 marks]

$128 \times (1/2)^7 = 1$. [1 for ratio; 1 for exponent; 1 for result.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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