GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 19

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Sequences

Shared GCSE core

A sequence is an ordered list generated by a rule. Learn to distinguish term-to-term rules from position-to-term rules, find linear nth terms and test membership. Include familiar number patterns without assuming every pattern is linear.

5, 8, 11, 14, … → 3n + 2

A linear nth term has a constant first difference.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Independent educational resource

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Before you begin

Use signed arithmetic, multiplication and simple linear equations.

Your route through this notebook

1. Generating terms and recognising patterns
2. Finding a linear nth term
3. Membership, missing terms and number patterns

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Generating terms and recognising patterns

A term-to-term rule describes how to get the next term from earlier terms. It needs starting information: “add 4” alone does not fix a sequence. A position-to-term rule calculates a term directly from its position n, usually starting at n = 1. These two kinds of rule answer different questions.

Familiar patterns include square, cube and triangular numbers. Fibonacci-type sequences add the previous two terms, so they need two starting values. A finite list can fit many possible rules; use the stated context or rule rather than claiming a unique continuation from a few terms alone.

A reliable method

1. Identify the starting value or values.

2. Apply the given rule consistently.

3. Distinguish the position n from the value of the nth term.

Worked example 1

Start at 7 and subtract 3 each time. Write five terms.

STEP-BY-STEP SOLUTION

7, 4, 1, −2, −5.

Worked example 2

The nth term is $n^2 + 1$. Find the first four terms.

STEP-BY-STEP SOLUTION

Substitute n = 1,2,3,4: 2, 5, 10, 17.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The sequence 2,4,6,8 has a natural “add 2” continuation, but four terms alone do not prove it. In an exam-style task, use the intended simple pattern and state your rule.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Start at −2 and add 5. Write five terms.

1.2 Find the first four terms of $3n - 1$.

1.3 A Fibonacci-type sequence starts 2,5. Write the next four terms.

1.4 Write the first five positive square numbers.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Finding a linear nth term

A linear sequence has a constant first difference d. Its nth term has form dn + c. The coefficient d is the difference; the constant c shifts the multiples of d to the required sequence. Compare the first term with d × 1 to find c.

Negative and fractional differences work in the same way. An nth-term formula allows direct access to a distant term without generating every earlier term. Check at least the first and another listed position. The formula describes positive integer positions unless another domain is stated.

A reliable method

1. Find the constant difference d.

2. Compare the sequence with d, 2d, 3d, … to find c.

3. Check the rule using known positions.

Worked example 1

Find the nth term of 5,8,11,14,… .

STEP-BY-STEP SOLUTION

Difference is 3. Multiples of 3 are 3,6,9,12,…; add 2. Rule: 3n + 2.

Worked example 2

Find the nth term of 12,7,2,−3,… .

STEP-BY-STEP SOLUTION

Difference is −5. Using −5n + c, the first term gives −5 + c = 12, so c = 17. Rule: 17 − 5n.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The common difference is not normally the whole nth-term formula. For 5,8,11,..., writing 3n gives 3,6,9,... . A constant shift is also needed.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Find the nth term of 4,9,14,19,... .

2.2 Find the nth term of 10,8,6,4,... .

2.3 Find the 30th term of 4n + 7.

2.4 Find the nth term of 1/2,1,3/2,2,... .

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Membership, missing terms and number patterns

To test whether a number belongs to a sequence, set the nth-term expression equal to it and solve for n. Membership requires a permitted position, usually a positive integer. A fractional or non-positive solution does not identify a term in a sequence beginning at n = 1.

Consecutive triangular numbers increase by 2,3,4,... after the first term, while square numbers increase by successive odd numbers. These patterns are not linear because their first differences change. Distinguishing the pattern matters more than forcing every sequence into dn + c.

A reliable method

1. Set the rule equal to the proposed term.

2. Solve for n and check its domain.

3. Inspect differences before deciding a sequence is linear.

Worked example 1

Is 83 a term of 5n + 3?

STEP-BY-STEP SOLUTION

5n + 3 = 83 gives n = 16, a positive integer. Yes, it is the 16th term.

Worked example 2

Is 42 a term of 4n + 1?

STEP-BY-STEP SOLUTION

4n + 1 = 42 gives n = 41/4 = 10.25. This is not an integer position, so no.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A formula can produce values at n = 0 or n = 1/2 algebraically, but those inputs do not belong to a sequence indexed by positive integers. Domain is part of the rule.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Is 61 a term of 6n + 1?

3.2 Is 50 a term of 3n + 2?

3.3 Find the missing term in a linear sequence 8, __, 20, 26.

3.4 Continue the triangular numbers 1,3,6,10,15 by two terms.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Write the next three terms after 4,9,14,19 using the same linear rule. [1 mark]

R2. Find the first three terms of $7n - 3$. [2 marks]

R3. Find the nth term of 6,10,14,18,… . [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Find the nth term of 15,12,9,6,... . [2 marks]

R5. Is 100 a term of 7n + 2? Justify. [3 marks]

R6. A row pattern has 5 tiles in row 1 and 3 extra tiles in each next row. Find row 20. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Generating terms and recognising patterns

1.1 Start at −2 and add 5. Write five terms.

−2, 3, 8, 13, 18.

1.2 Find the first four terms of 3n − 1.

2, 5, 8, 11.

1.3 A Fibonacci-type sequence starts 2,5. Write the next four terms.

7, 12, 19, 31.

1.4 Write the first five positive square numbers.

1, 4, 9, 16, 25.

Lesson 2 • Finding a linear nth term

2.1 Find the nth term of 4,9,14,19,… .

5n − 1.

2.2 Find the nth term of 10,8,6,4,… .

12 − 2n.

2.3 Find the 30th term of 4n + 7.

4 × 30 + 7 = 127.

2.4 Find the nth term of 1/2,1,3/2,2,… .

n/2.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Membership, missing terms and number patterns

3.1 Is 61 a term of 6n + 1?

6n = 60, so n = 10: yes.

3.2 Is 50 a term of 3n + 2?

3n = 48, so n = 16: yes.

3.3 Find the missing term in a linear sequence 8, __, 20, 26.

The difference is 6, so the missing term is 14.

3.4 Continue the triangular numbers 1,3,6,10,15 by two terms.

Add 6 then 7: 21, 28.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Write the next three terms after 4,9,14,19 using the same linear rule. [1 mark]

24, 29, 34. [1 for all three.]

R2. Find the first three terms of $7n - 3$. [2 marks]

4, 11, 18. [1 for substitution method; 1 for all terms.]

R3. Find the nth term of 6,10,14,18,… . [2 marks]

Difference 4, offset 2: $4n + 2$. [1 for 4n; 1 for constant.]

R4. Find the nth term of 15,12,9,6,… . [2 marks]

$18 - 3n$. [1 for difference; 1 for offset.]

R5. Is 100 a term of $7n + 2$? Justify. [3 marks]

$7n + 2 = 100$ gives $n = 14$, so yes, the 14th term. [1 for equation; 1 for n; 1 for conclusion.]

R6. A row pattern has 5 tiles in row 1 and 3 extra tiles in each next row. Find row 20. [3 marks]

Rule $3n + 2$; at $n = 20$ it gives 62 tiles. [1 for difference; 1 for rule; 1 for output.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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