GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 18

18

Simultaneous equations

Shared core with Higher extensions

Two equations can constrain the same pair of unknowns. Find values that satisfy both using elimination, substitution and graph intersections. Higher work combines a line with a quadratic.

$y = x^2$ and $y = x + 2$

Intersections satisfy both equations at once.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Solve linear equations, expand brackets and factorise simple quadratics.

Your route through this notebook

1. Elimination with linear equations
2. Substitution, models and graphical meaning
3. Linear–quadratic systems — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Elimination with linear equations

A solution of simultaneous equations must make both equations true at once. Elimination creates a new equation with only one unknown by adding or subtracting suitable multiples of the original equations. This is valid because equal quantities can be added to equal quantities.

First choose which variable to eliminate. Matching coefficients are cancelled by subtraction; opposite coefficients cancel by addition. If scaling is needed, multiply every term on that equation, including the right side. After finding one variable, substitute into an original equation and check the pair in both originals.

A reliable method

1. Align x terms, y terms and constants.

2. Scale to equal or opposite coefficients, then eliminate.

3. Back-substitute and check both equations.

Worked example 1

Solve $x + y = 9$ and $x - y = 3$.

STEP-BY-STEP SOLUTION

Add equations: $2x = 12$, so $x = 6$. Then $y = 3$. Both equations check.

Worked example 2

Solve $2x + 3y = 17$ and $3x + 2y = 18$.

STEP-BY-STEP SOLUTION

Multiply the first by 3 and second by 2: $6x + 9y = 51$; $6x + 4y = 36$. Subtract: $5y = 15$, so $y = 3$. Then $2x + 9 = 17$ gives $x = 4$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Subtract an entire equation, not just its first term. Writing brackets around the row being subtracted helps protect negative signs.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Solve $x + y = 11$ and $x - y = 5$.

1.2 Solve $2x + y = 10$ and $x + y = 7$.

1.3 Solve $3x + 2y = 16$ and $x + 2y = 8$.

1.4 Solve $2x + 5y = 9$ and $3x - 5y = 11$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Substitution, models and graphical meaning

Substitution replaces one unknown by an equivalent expression from the other equation. It is especially efficient when an equation already has x or y isolated. Substitute the whole expression with brackets, solve the resulting one-variable equation, then recover the second unknown.

Each linear equation represents a line. An intersection supplies a pair satisfying both. Parallel distinct lines give no solution; the same line written in two ways gives infinitely many solutions. In modelling, define the quantities and use two independent relationships, such as a total count and a total cost.

A reliable method

1. Isolate a convenient variable.

2. Substitute the entire expression into the other equation.

3. Interpret the pair as a common solution or intersection.

Worked example 1

Solve $y = 2x + 1$ and $x + y = 10$.

STEP-BY-STEP SOLUTION

$x + (2x + 1) = 10$ gives $3x = 9$ and $x = 3$. Then $y = 7$.

Worked example 2

Three adult tickets and two child tickets cost £34. One adult and two children cost £18. Find each price.

STEP-BY-STEP SOLUTION

Let adult price be a and child price c. Subtract $a + 2c = 18$ from $3a + 2c = 34$: $2a = 16$, so $a = 8$. Then $2c = 10$, so $c = 5$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

A graph gives approximate readings unless intersections fall exactly on readable grid points. An algebraic solution can provide exact values. Always distinguish a coordinate pair (x,y) from two unrelated answers.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Solve $y = x + 4$ and $2x + y = 13$.

2.2 Solve $x = 3y$ and $x + y = 20$.

2.3 How many solutions have $y = 2x + 1$ and $y = 2x - 3$?

2.4 How many solutions have $x + y = 6$ and $2x + 2y = 12$?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Linear–quadratic systems — Higher

A line and a quadratic can intersect twice, once or not at all over the real numbers. Substitute the linear expression into the quadratic relationship to obtain a quadratic equation. Solve it and find a matching second coordinate for each first coordinate.

Keep pairs together. A list of two x values and two y values without pairing can create false combinations. If a quadratic has no real roots, the graphs have no real intersection. A repeated root corresponds to tangency in the usual line–parabola situation.

A reliable method

1. Use the line to express one variable.

2. Substitute and solve the resulting quadratic.

3. Find and check a complete pair for each root.

Worked example 1

Solve $y = x + 2$ and $y = x^2$.

STEP-BY-STEP SOLUTION

Set $x^2 = x + 2$: $(x - 2)(x + 1) = 0$. For $x = 2$, $y = 4$; for $x = -1$, $y = 1$. Solutions: $(2,4)$, $(-1,1)$.

Worked example 2

Solve $x + y = 5$ and $xy = 6$.

STEP-BY-STEP SOLUTION

$y = 5 - x$, so $x(5 - x) = 6$. Rearrange $x^2 - 5x + 6 = 0$, giving $x = 2$ or 3. Pairs are $(2,3)$, $(3,2)$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

An intersection question is asking where both graph rules hold, not where either graph crosses an axis. Equate the y expressions or substitute one relation into the other.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Solve $y = x + 6$ and $y = x^2$.

3.2 Solve $y = 2x$ and $y = x^2$.

3.3 Solve $y = 2x - 1$ and $y = x^2$.

3.4 Solve $y = -1$ and $y = x^2$ over real numbers.

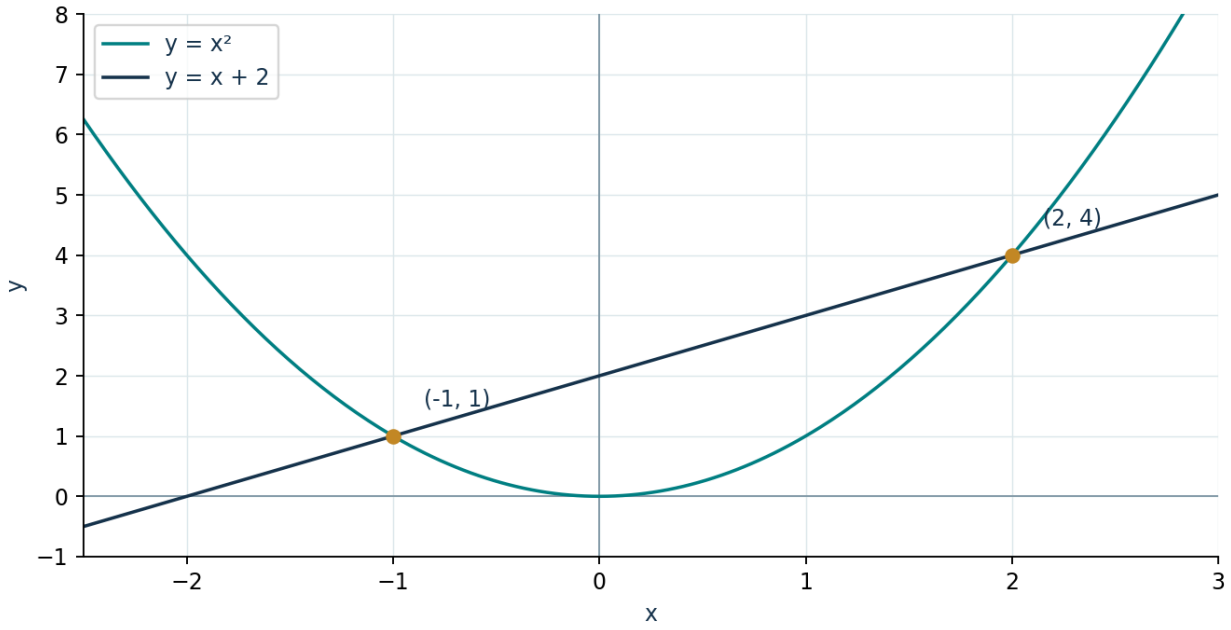
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

Intersections solve both equations



WHAT THE FIGURE SHOWS

The two intersections are (−1,1) and (2,4). Each pair satisfies $y = x^2$ and $y = x + 2$. Algebra confirms them by solving $x^2 - x - 2 = 0$. The graph does not pair $x = -1$ with $y = 4$.

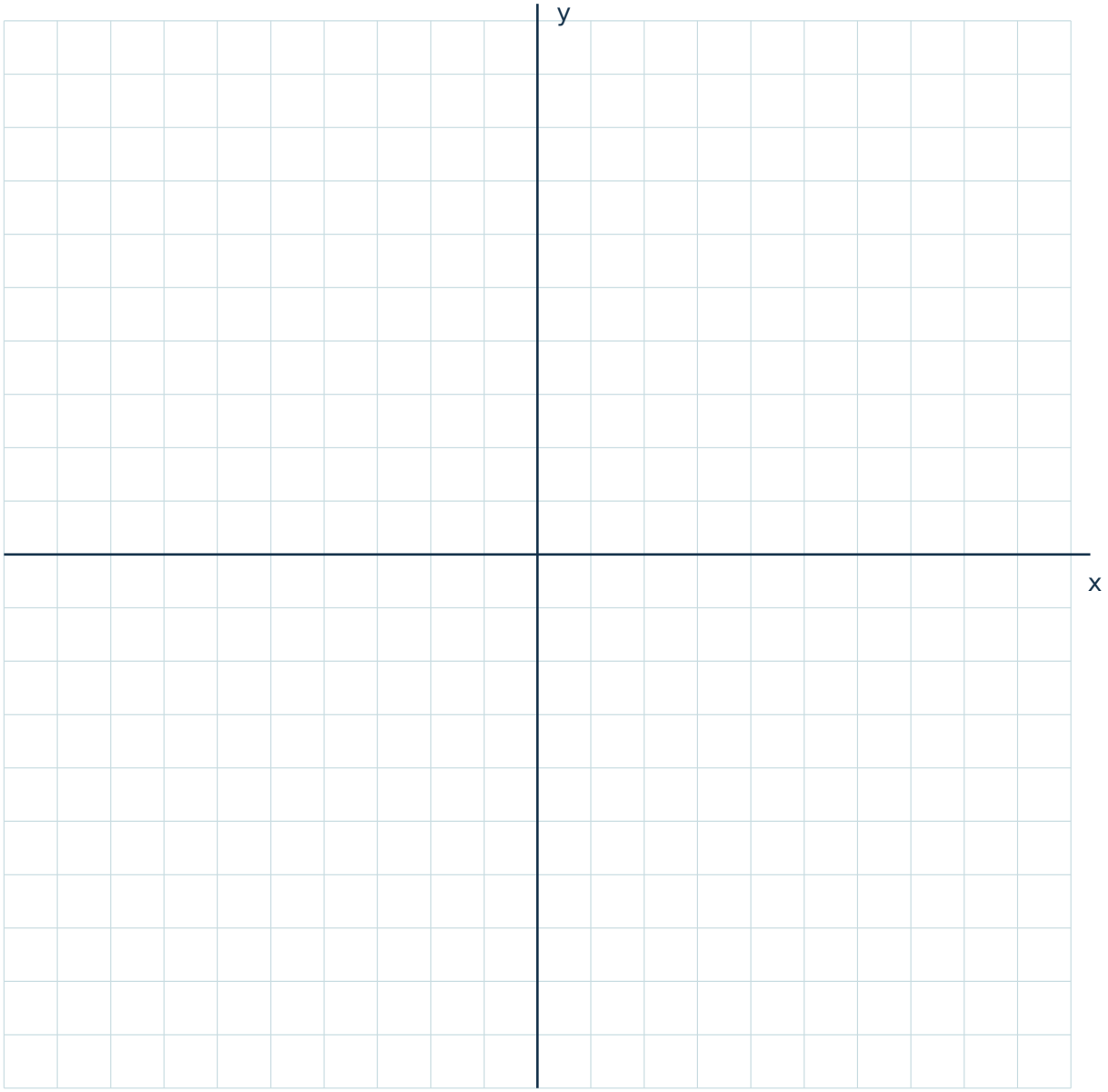
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Solve $x + y = 14$ and $x - y = 4$. [3 marks]

R2. Solve $3x + 2y = 19$ and $x + 2y = 9$. [3 marks]

R3. Solve $y = 3x - 2$ and $x + y = 14$. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Two pens and one book cost £9. One pen and one book cost £7. Find each price. [3 marks]

R5. Higher: solve $y = x + 12$ and $y = x^2$. [4 marks]

R6. Higher: solve $x + y = 7$ and $xy = 12$. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Elimination with linear equations

1.1 Solve $x + y = 11$ and $x - y = 5$.

Add: $2x = 16$, so $x = 8$; $y = 3$.

1.2 Solve $2x + y = 10$ and $x + y = 7$.

Subtract: $x = 3$; then $y = 4$.

1.3 Solve $3x + 2y = 16$ and $x + 2y = 8$.

Subtract: $2x = 8$, so $x = 4$; $y = 2$.

1.4 Solve $2x + 5y = 9$ and $3x - 5y = 11$.

Add: $5x = 20$, so $x = 4$; then $8 + 5y = 9$, so $y = 1/5$.

Lesson 2 • Substitution, models and graphical meaning

2.1 Solve $y = x + 4$ and $2x + y = 13$.

$3x + 4 = 13$, so $x = 3$ and $y = 7$.

2.2 Solve $x = 3y$ and $x + y = 20$.

$4y = 20$, so $y = 5$ and $x = 15$.

2.3 How many solutions have $y = 2x + 1$ and $y = 2x - 3$?

None: equal gradients and different intercepts give parallel distinct lines.

2.4 How many solutions have $x + y = 6$ and $2x + 2y = 12$?

Infinitely many: the second equation is twice the first and represents the same line.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Linear–quadratic systems — Higher

3.1 Solve $y = x + 6$ and $y = x^2$.

$x^2 - x - 6 = 0$ gives $x = 3$ or -2. Pairs: (3,9), (−2,4).

3.2 Solve $y = 2x$ and $y = x^2$.

$x^2 = 2x$ gives $x(x - 2) = 0$. Pairs: (0,0), (2,4).

3.3 Solve $y = 2x - 1$ and $y = x^2$.

$x^2 - 2x + 1 = 0$, so $x = 1$ and $y = 1$. One solution: (1,1).

3.4 Solve $y = -1$ and $y = x^2$ over real numbers.

No real solution, because x^2 cannot equal -1.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Solve $x + y = 14$ and $x - y = 4$. [3 marks]

$2x = 18$, so $x = 9$ and $y = 5$. [1 for elimination; 1 for each value.]

R2. Solve $3x + 2y = 19$ and $x + 2y = 9$. [3 marks]

$2x = 10$; $x = 5$ and $y = 2$. [1 for elimination; 1 for each value.]

R3. Solve $y = 3x - 2$ and $x + y = 14$. [3 marks]

$4x - 2 = 14$, so $x = 4$ and $y = 10$. [1 for substitution; 1 for each value.]

R4. Two pens and one book cost £9. One pen and one book cost £7. Find each price. [3 marks]

$2p + b = 9$ and $p + b = 7$. Subtract: $p = 2$; then $b = 5$. [1 for model; 1 for each price.]

R5. Higher: solve $y = x + 12$ and $y = x^2$. [4 marks]

$x^2 - x - 12 = 0$ gives $(x - 4)(x + 3) = 0$. Pairs: $(4,16)$, $(-3,9)$. [1 for quadratic; 1 for roots; 1 for each pair.]

R6. Higher: solve $x + y = 7$ and $xy = 12$. [4 marks]

$x(7 - x) = 12$ leads to $x^2 - 7x + 12 = 0$. Pairs: $(3,4)$, $(4,3)$. [1 for substitution; 1 for quadratic; 1 for roots; 1 for pairing.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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