GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 17

17

Inequalities

Shared core with Higher extensions

Inequalities describe sets of possible values rather than a single answer. Learn number-line notation and balanced solving, then extend to shaded coordinate regions and quadratic inequalities.

$$-2x > 8 \rightarrow x < -4$$

Dividing by a negative reverses the inequality.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Solve linear equations, use signed arithmetic and plot straight lines.

Your route through this notebook

1. Number lines and solving linear inequalities
2. Combined inequalities and graphical regions — extension
3. Quadratic inequalities — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Number lines and solving linear inequalities

The symbols < and > exclude equality; ≤ and ≥ include it. On a number line an open circle excludes an endpoint and a filled circle includes it. An arrow or line shows all allowed values, not only integers. If integer solutions are requested, select the integers within the solution interval.

Adding or subtracting the same quantity preserves an inequality. Multiplying or dividing by a negative number reverses its direction: $2 < 5$ but $-2 > -5$. This reversal follows from reversing the order on the number line. Division by zero is not permitted.

A reliable method

1. Solve using balanced operations.

2. Reverse the inequality when multiplying or dividing by a negative.

3. Represent the entire solution set, checking endpoint inclusion.

Worked example 1

Solve $3x - 4 \leq 11$.

STEP-BY-STEP SOLUTION

$3x \leq 15$, so $x \leq 5$. A filled circle at 5 and shading left represent all solutions.

Worked example 2

Solve $-2x + 7 > 15$.

STEP-BY-STEP SOLUTION

$-2x > 8$. Dividing by -2 reverses the sign: $x < -4$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

$x < 4$ does not mean $x = 3$. It includes 3.9, zero and every real value below 4. A list of integers is a narrower answer and needs an integer condition.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Solve $x + 6 > 10$.

1.2 Solve $5x \leq 20$.

1.3 Solve $-3x \geq 12$.

1.4 List integer values satisfying $-2 < x \leq 3$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Combined inequalities and graphical regions — extension

A double inequality can constrain a value from both sides. Apply additions, subtractions or positive divisions to all three parts. For more complicated combinations, solve each inequality separately and take their intersection: values must satisfy both.

In two variables, an inequality selects a region of the coordinate plane. Draw the boundary equation first. A strict inequality has a dashed boundary; an inclusive inequality has a solid boundary. Test a point not on the line to decide which side satisfies the condition. A system of inequalities uses the overlap of all valid regions.

A reliable method

1. Solve both parts of a combined condition.

2. Draw each boundary with the correct solid or dashed style.

3. Test a point, then retain the overlap.

Worked example 1

Solve $2 < 3x + 5 \leq 14$.

STEP-BY-STEP SOLUTION

Subtract 5 throughout: $-3 < 3x \leq 9$. Divide by 3: $-1 < x \leq 3$.

Worked example 2

Describe $y \geq 2x + 1$.

STEP-BY-STEP SOLUTION

The boundary $y = 2x + 1$ is solid. At (0,0), $0 \geq 1$ is false. Keep the region above the line, including the line.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

“Shade the region” can mean either shade the wanted region or shade out unwanted regions depending on the instructions. State your convention and label the final valid region clearly.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Solve $-5 \leq 2x + 1 < 7$.

2.2 Solve $x > 1$ and $x \leq 6$.

2.3 Describe the boundary and region for $x < 3$.

2.4 Does (2,5) satisfy $y \geq x + 2$ and $x \geq 0$?

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Quadratic inequalities — Higher

A quadratic inequality asks where a quadratic expression is positive or negative. Find its roots and examine the sign between and outside them. For a positive leading coefficient, a parabola opens upwards: it is below the axis between two distinct roots and above it outside. A negative leading coefficient reverses that pattern.

A sketch or test values make the reasoning visible. Include roots for ≤ or ≥, and exclude them for < or >. The roots alone are boundary values, not the entire solution set. Repeated roots and quadratics with no real roots need a separate sign check.

A reliable method

1. Rearrange to compare a quadratic with zero.

2. Find roots and determine the opening direction.

3. Select intervals of the required sign, with correct endpoint inclusion.

Worked example 1

Solve $x^2 - 5x + 6 < 0$.

STEP-BY-STEP SOLUTION

Factorise $(x - 2)(x - 3)$. Roots are 2 and 3; the upward parabola is negative between them. Thus $2 < x < 3$.

Worked example 2

Solve $x^2 - 9 \geq 0$.

STEP-BY-STEP SOLUTION

Roots are -3 and 3. The expression is non-negative outside the interval, including endpoints: $x \leq -3$ or $x \geq 3$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Do not write $x < 2$ and $x > 3$ for an outside region: no number satisfies both. Outside intervals usually require “or”; an inside interval requires “and”.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Solve $(x - 1)(x - 4) \leq 0$.

3.2 Solve $x^2 - 16 > 0$.

3.3 Solve $x^2 + 1 > 0$ over real numbers.

3.4 Solve $-x^2 + 4 \geq 0$.

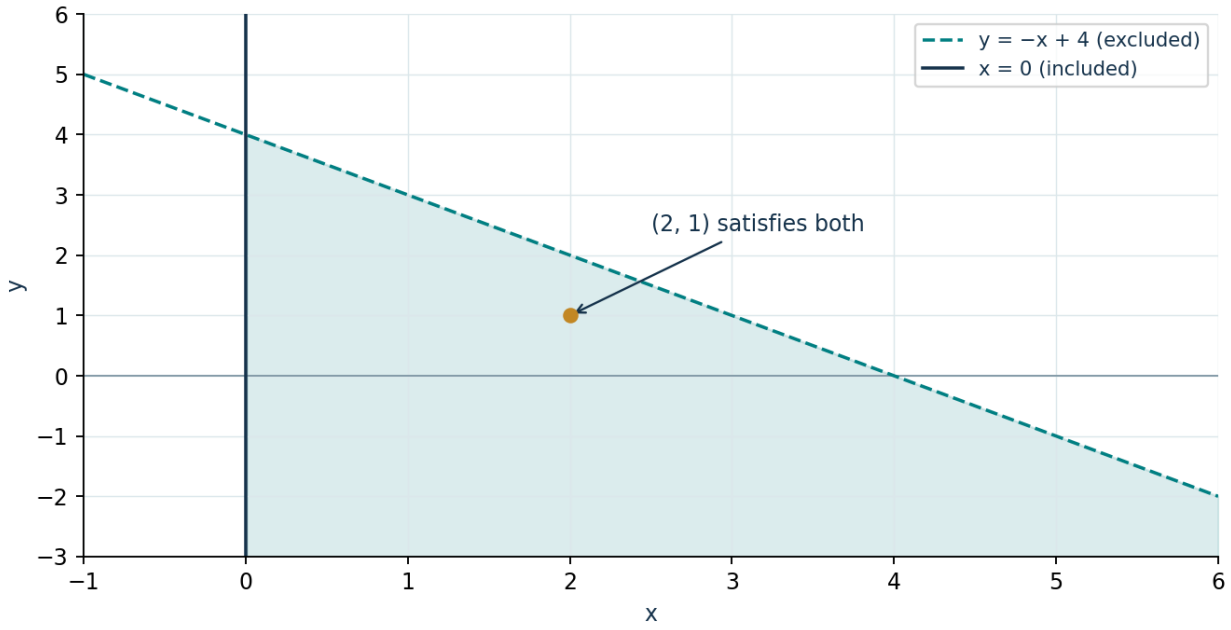
Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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VISUAL EXPLAINER / CONNECT THE RULE TO THE GRAPH

Read the mathematics

An inequality selects a whole region



WHAT THE FIGURE SHOWS

The shaded overlap satisfies $y < -x + 4$ and $x \geq 0$. The dashed sloping boundary is excluded; the vertical boundary is included wherever the other inequality holds. The point (2,1) is an example, not the whole solution set.

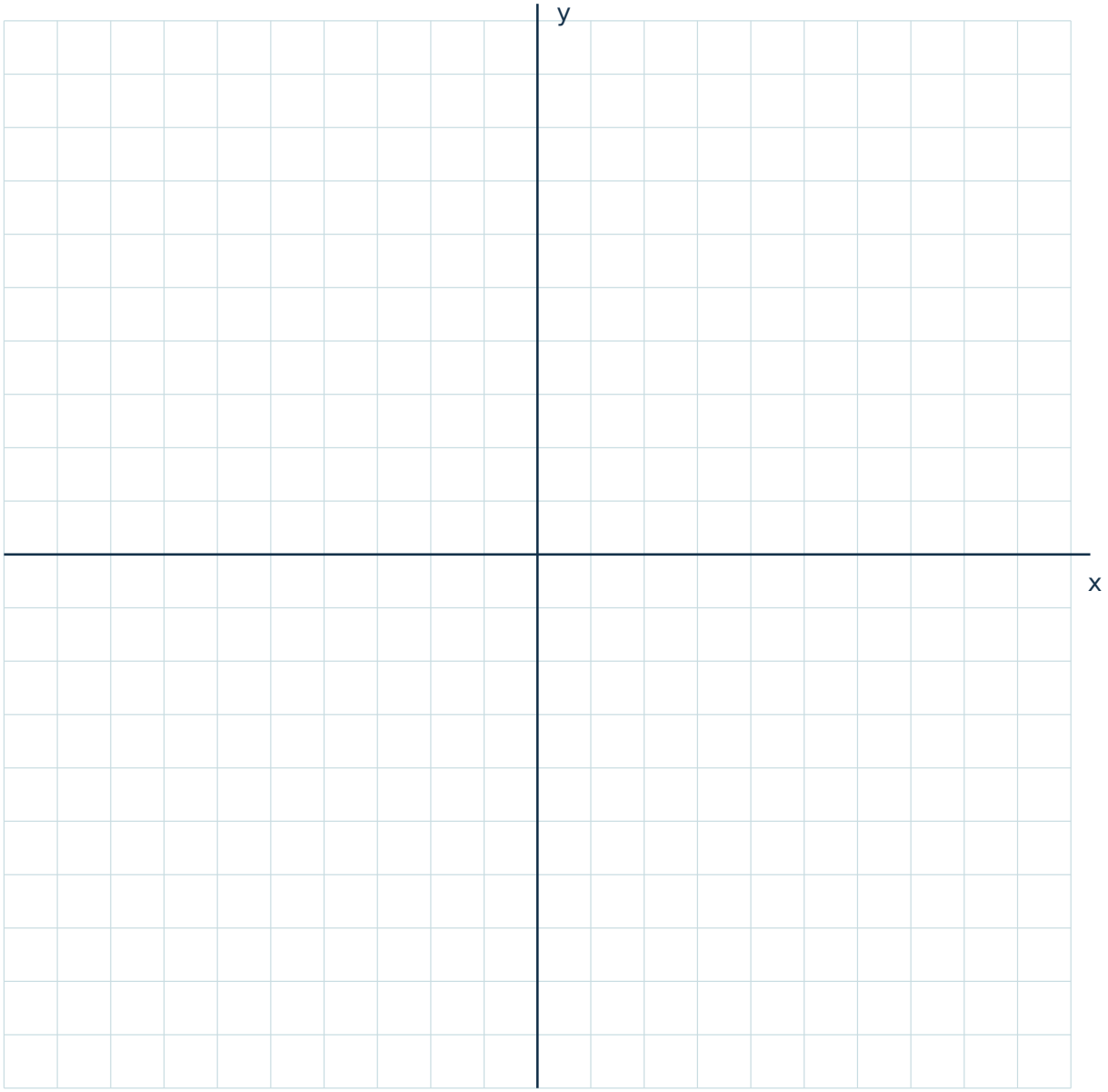
Explain how the labelled features follow from the equations. Check a point by substitution or recalculate a gradient or area before relying on the picture.

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GRAPH WORKSPACE / RECREATE AND EXPLAIN

Build the picture yourself

Choose one graph from the preceding visual explainer and reconstruct it on this grid. Choose and label a suitable scale on each axis; one square need not represent one unit. Include the displayed domain, key points and units. For a real-life graph, relabel x and y with the measured quantities.



SELF-CHECK

Compare with the preceding plotted figure and its explanation. Check at least three points, any intercepts and the direction or shape of the graph. Explain how the equation or context produces those features.

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Solve $4x - 3 < 17$. [2 marks]

R2. Solve $8 - 2x \leq 14$. [2 marks]

R3. List integer solutions of $-1 \leq x < 4$. [1 mark]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Solve $1 < 2x + 3 \leq 11$. [3 marks]

R5. Higher: describe the region $y < -x + 4$, $x \geq 0$. [3 marks]

R6. Higher: solve $x^2 - x - 6 \geq 0$. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Number lines and solving linear inequalities

1.1 Solve $x + 6 > 10$.

$x > 4$.

1.2 Solve $5x \leq 20$.

$x \leq 4$.

1.3 Solve $-3x \geq 12$.

Divide by -3 and reverse: $x \leq -4$.

1.4 List integer values satisfying $-2 < x \leq 3$.

$-1, 0, 1, 2, 3$.

Lesson 2 • Combined inequalities and graphical regions — extension

2.1 Solve $-5 \leq 2x + 1 < 7$.

Subtract 1 and divide by 2: $-3 \leq x < 3$.

2.2 Solve $x > 1$ and $x \leq 6$.

$1 < x \leq 6$.

2.3 Describe the boundary and region for $x < 3$.

A dashed vertical line at $x = 3$; the valid region is to its left.

2.4 Does (2,5) satisfy $y \geq x + 2$ and $x \geq 0$?

Yes: $5 \geq 4$ and $2 \geq 0$. Both conditions hold.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Quadratic inequalities — Higher

3.1 Solve $(x - 1)(x - 4) \leq 0$.

$1 \leq x \leq 4$.

3.2 Solve $x^2 - 16 > 0$.

$x < -4$ or $x > 4$.

3.3 Solve $x^2 + 1 > 0$ over real numbers.

All real x, because $x^2 \geq 0$ so $x^2 + 1$ is always positive.

3.4 Solve $-x^2 + 4 \geq 0$.

$x^2 \leq 4$, so $-2 \leq x \leq 2$.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Solve $4x - 3 < 17$. [2 marks]

$x < 5$. [1 for isolation; 1 for division.]

R2. Solve $8 - 2x \leq 14$. [2 marks]

$-2x \leq 6$, so $x \geq -3$. [1 for isolation; 1 for reversal/result.]

R3. List integer solutions of $-1 \leq x < 4$. [1 mark]

$-1, 0, 1, 2, 3$. [1 for complete list.]

R4. Solve $1 < 2x + 3 \leq 11$. [3 marks]

$-2 < 2x \leq 8$, so $-1 < x \leq 4$. [1 for each balanced stage; 1 for endpoints.]

R5. Higher: describe the region $y < -x + 4$, $x \geq 0$. [3 marks]

Below the dashed line $y = -x + 4$ and on or to the right of the solid y-axis. [1 for each boundary; 1 for overlap.]

R6. Higher: solve $x^2 - x - 6 \geq 0$. [3 marks]

$(x - 3)(x + 2) \geq 0$, so $x \leq -2$ or $x \geq 3$. [1 for factors; 1 for sign regions; 1 for inclusivity.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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