GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 16

16

Formulae and changing the subject

Shared core with Higher extensions

A formula describes a relationship between quantities. Learn to evaluate it, build it from a context and rearrange it without losing its meaning. Higher extensions include powers and a subject appearing more than once.

$$y = 3x + 7 \rightarrow x = (y - 7)/3$$

Undo operations while preserving the relationship.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Your learning guide

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Before you begin

Substitute values, solve linear equations and factorise common terms.

Your route through this notebook

1. Using and constructing formulae
2. Rearranging with inverse operations
3. Powers, roots and repeated subjects — Higher

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Using and constructing formulae

A formula usually connects several quantities that can vary. Identify the subject, typically the isolated variable on the left, and the units of every input. Substitution must use compatible units: a speed in metres per second requires distance in metres and time in seconds.

To construct a formula, separate constants from variable relationships. Repeated equal groups suggest multiplication; a shared total may suggest division. State what each letter represents. A formula is not automatically valid for every real input: denominators cannot be zero and geometric dimensions normally must be positive.

A reliable method

1. Define letters and use consistent units.

2. Translate the relationship before substituting numbers.

3. Interpret the output in the required unit.

Worked example 1

Use v = d/t for d = 240 m and t = 30 s.

STEP-BY-STEP SOLUTION

v = 240/30 = 8 m/s. Both inputs are already in compatible units.

Worked example 2

A rental costs a fixed £18 and £6 per day. Write a formula and find the cost for 4 days.

STEP-BY-STEP SOLUTION

Let d be days and C be cost in pounds. C = 18 + 6d; C = 42 for d = 4.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The letters used in a formula are choices, not magic labels. A different author may use T instead of C for a cost. Definitions and units establish the meaning.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Use A = bh/2 for b = 12 cm and h = 7 cm.

1.2 Use v = d/t for d = 1.2 km and t = 4 minutes, giving m/s.

1.3 A tank holds 150 litres and fills at r litres per minute from empty. Express time t.

1.4 A rectangle has width w and length 2w + 3. Express its perimeter.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Rearranging with inverse operations

Changing the subject means isolating a different variable while retaining an equivalent relationship. Treat the other letters as quantities and apply the same balanced operations used in equations. Avoid moving symbols without naming the operation: division is not simply “moving a letter underneath”.

Grouped expressions must stay grouped. In $y = (x + a)/b$, multiply by b before subtracting a. If a variable is in a denominator, multiply first to remove that denominator, respecting its non-zero restriction. Rearranging does not remove any domain conditions of the original formula.

A reliable method

1. Identify the new subject and the operations surrounding it.

2. Undo operations on both sides in reverse order.

3. Use brackets for whole numerators and denominators.

Worked example 1

Make x the subject of $y = 3x + 7$.

STEP-BY-STEP SOLUTION

Subtract 7: $y - 7 = 3x$. Divide by 3: $x = (y - 7)/3$.

Worked example 2

Make t the subject of $v = d/t$.

STEP-BY-STEP SOLUTION

Multiply by t: $vt = d$. Divide by non-zero v: $t = d/v$. The original also requires $t \neq 0$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

From $y = 5x + 2$, $x = y/5 - 2$ is generally wrong because only y has been divided. The correct expression is $(y - 2)/5$. Substituting a simple original pair checks the rearrangement.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Make x the subject of $y = 4x - 9$.

2.2 Make h the subject of $A = bh/2$.

2.3 Make a the subject of $v = u + at$.

2.4 Make x the subject of $y = (x - 3)/5$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Powers, roots and repeated subjects — Higher

When the subject is squared, isolate its square before taking roots. Algebraically both signs may be needed; in a length formula the context often selects the positive root. For example, $x^2 = k$ gives $x = \pm\sqrt{k}$ for $k \geq 0$, but a radius is non-negative.

If the desired subject occurs in several terms, collect those terms and factorise. The subject can then be isolated by dividing by the remaining bracket. If that bracket could equal zero, the rearranged form has a restriction and the original relationship needs separate consideration in that case.

A reliable method

1. Isolate powers before taking roots and consider signs.

2. Collect all terms containing the subject on one side.

3. Factorise the subject, divide and state non-zero restrictions.

Worked example 1

Make r the subject of $A = \pi r^2$ for a circle.

STEP-BY-STEP SOLUTION

$r^2 = A/\pi$. Since r is a radius, $r = \sqrt{(A/\pi)}$, with $A \geq 0$.

Worked example 2

Make x the subject of $y = ax + bx + c$.

STEP-BY-STEP SOLUTION

$y - c = x(a + b)$. Thus $x = (y - c)/(a + b)$, provided $a + b \neq 0$. If $a + b = 0$, the original reduces to $y = c$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Taking a square root of a sum does not split it into separate roots. From $x^2 = p + q$, $x = \pm\sqrt{(p + q)}$, not $\pm(\sqrt{p} + \sqrt{q})$.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Make x the subject of $y = 2x^2 + 3$ over real numbers.

3.2 Make x the subject of $p = 5x - qx$.

3.3 Make x the subject of $y = (x + 2)/(x - 1)$.

3.4 Make r the subject of $V = \pi r^2 h$ for a cylinder with $h > 0$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Use $C = 5n + 12$ for $n = 9$. [2 marks]

R2. Make x the subject of $y = 7x + 4$. [2 marks]

R3. Make b the subject of $P = 2a + 2b$. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Higher: make x the subject of $y = 4x^2 - 5$. [3 marks]

R5. Higher: make x the subject of $p = ax - b(x + 1)$. [3 marks]

R6. Higher: make x the subject of $y = (2x + 3)/(x - 4)$. [4 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Using and constructing formulae

1.1 Use A = bh/2 for b = 12 cm and h = 7 cm.

A = 12 × 7 / 2 = 42 cm².

1.2 Use v = d/t for d = 1.2 km and t = 4 minutes, giving m/s.

1.2 km = 1200 m; 4 min = 240 s; v = 5 m/s.

1.3 A tank holds 150 litres and fills at r litres per minute from empty. Express time t.

t = 150/r minutes, for r > 0.

1.4 A rectangle has width w and length 2w + 3. Express its perimeter.

P = 2w + 2(2w + 3) = 6w + 6.

Lesson 2 • Rearranging with inverse operations

2.1 Make x the subject of y = 4x − 9.

x = (y + 9)/4.

2.2 Make h the subject of A = bh/2.

2A = bh, so h = 2A/b for b ≠ 0.

2.3 Make a the subject of v = u + at.

a = (v − u)/t, for t ≠ 0.

2.4 Make x the subject of y = (x − 3)/5.

5y = x − 3, so x = 5y + 3.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Powers, roots and repeated subjects — Higher

3.1 Make x the subject of $y = 2x^2 + 3$ over real numbers.

$x^2 = (y - 3)/2$; $x = \pm\sqrt{((y - 3)/2)}$, requiring $y \geq 3$.

3.2 Make x the subject of $p = 5x - qx$.

$p = x(5 - q)$, so $x = p/(5 - q)$, if $q \neq 5$.

3.3 Make x the subject of $y = (x + 2)/(x - 1)$.

$y(x - 1) = x + 2$; $x(y - 1) = y + 2$; $x = (y + 2)/(y - 1)$, with $y \neq 1$ and original $x \neq 1$.

3.4 Make r the subject of $V = \pi r^2 h$ for a cylinder with $h > 0$.

$r = \sqrt{(V/(\pi h))}$, taking the non-negative radius for $V \geq 0$.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Use $C = 5n + 12$ for $n = 9$. [2 marks]

$C = 45 + 12 = 57$. [1 for substitution; 1 for result.]

R2. Make x the subject of $y = 7x + 4$. [2 marks]

$x = (y - 4)/7$. [1 for subtraction; 1 for division.]

R3. Make b the subject of $P = 2a + 2b$. [2 marks]

$b = (P - 2a)/2$, or $P/2 - a$. [1 for isolation; 1 for division.]

R4. Higher: make x the subject of $y = 4x^2 - 5$. [3 marks]

$x = \pm\sqrt{((y + 5)/4)}$, for $y \geq -5$. [1 for isolating square; 1 for root; 1 for both signs/domain.]

R5. Higher: make x the subject of $p = ax - b(x + 1)$. [3 marks]

$p = (a - b)x - b$; $x = (p + b)/(a - b)$, for $a \neq b$. [1 for expansion; 1 for collecting; 1 for division/restriction.]

R6. Higher: make x the subject of $y = (2x + 3)/(x - 4)$. [4 marks]

$yx - 4y = 2x + 3$; $x(y - 2) = 4y + 3$; $x = (4y + 3)/(y - 2)$, with $y \neq 2$ and original $x \neq 4$. [1 for each algebraic stage; 1 for restrictions.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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