GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 15

15

Linear equations

Shared GCSE core

An equation is a balance: equivalent operations on both sides preserve its solutions. Progress from one-step equations to brackets, fractions and unknowns on both sides, then form equations from short problems.

$4x + 7 = 31 \rightarrow x = 6$

Balance both sides and verify by substitution.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Use inverse operations, expand brackets and calculate with signed numbers.

Your route through this notebook

1. Balancing and inverse operations
2. Brackets and unknowns on both sides
3. Fractions and forming equations

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Balancing and inverse operations

Solving means finding values that make an equation true. Adding or subtracting the same quantity on both sides preserves equality. Multiplying or dividing both sides by the same non-zero constant also preserves equality. These operations undo the operations on the unknown.

Work backwards through a calculation. In $3x + 5 = 20$, the final addition is undone before the multiplication. Show a new equation after every step instead of moving symbols without explanation. Check a solution by substituting it into the original equation, not only your final line.

A reliable method

1. Identify the operations applied to the unknown.

2. Undo them in reverse order on both sides.

3. Substitute the result into the original equation.

Worked example 1

Solve $4x + 7 = 31$.

STEP-BY-STEP SOLUTION

Subtract 7: $4x = 24$. Divide by 4: $x = 6$. Check: $4 \times 6 + 7 = 31$.

Worked example 2

Solve $x/5 - 3 = 4$.

STEP-BY-STEP SOLUTION

Add 3: $x/5 = 7$. Multiply by 5: $x = 35$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

The equals sign connects expressions with the same value. Do not write $4x + 7 = 31 - 7 = 24$, because the first expression equals 31, not 24. Each balanced stage should be its own equation.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Solve $x + 9 = 17$.

1.2 Solve $6x = -42$.

1.3 Solve $5x - 8 = 22$.

1.4 Solve $x/4 + 2 = -1$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Brackets and unknowns on both sides

When x appears on both sides, collect all x terms on one side and constants on the other by balanced addition or subtraction. Either direction can work; choosing a positive final coefficient often simplifies the arithmetic. Expand brackets before collecting, taking care with negative multipliers.

An equation may have one solution, no solutions or infinitely many solutions. If all variable terms cancel and leave a false statement, no input works. If they leave an identity such as 5 = 5, every input in the original domain works. A cancelled x does not automatically mean x = 0.

A reliable method

1. Expand and simplify each side.

2. Subtract variable terms from both sides, then isolate the constant.

3. If variables disappear, interpret the remaining statement.

Worked example 1

Solve 5x + 4 = 2x + 19.

STEP-BY-STEP SOLUTION

Subtract 2x: 3x + 4 = 19. Subtract 4: 3x = 15. Therefore x = 5.

Worked example 2

Solve 3(2x − 1) = 4x + 9.

STEP-BY-STEP SOLUTION

Expand: 6x − 3 = 4x + 9. Then 2x = 12, so x = 6.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

For 2(x + 3) = 2x + 6, expansion leaves the same expression on both sides, so every real x works. For 2(x + 3) = 2x + 7, cancellation gives 6 = 7, so no real x works.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Solve $7x - 5 = 3x + 11$.

2.2 Solve $4(x + 2) = 3x + 13$.

2.3 Solve $5 - 2(x - 3) = x + 2$.

2.4 Solve $3x + 8 = 3x - 1$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Fractions and forming equations

For equations with fractional terms, multiplying every term on both sides by a common denominator can remove the fractions. The multiplier must apply to entire numerators. Brackets preserve this structure and reduce sign errors. When denominators are fixed non-zero numbers, this step preserves the solution set.

For a word problem, define the unknown, express each relevant quantity and state the relationship as an equation. After solving, check whether the answer fits the context: lengths must be positive and a count may need an integer. Writing an equation is part of the solution, not just a route to a number.

A reliable method

1. Use the LCM of fixed denominators to clear fractions.

2. Apply the multiplier to every term, preserving brackets.

3. For a model, define x and interpret its solution.

Worked example 1

Solve $(x + 2)/3 = (x - 1)/2$.

STEP-BY-STEP SOLUTION

Multiply by 6: $2(x + 2) = 3(x - 1)$. Thus $2x + 4 = 3x - 3$ and $x = 7$.

Worked example 2

A rectangle has width x cm and length $x + 4$ cm. Its perimeter is 36 cm. Find both sides.

STEP-BY-STEP SOLUTION

$2x + 2(x + 4) = 36$. Then $4x + 8 = 36$, so $x = 7$. Width 7 cm and length 11 cm.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Multiplying (x + 2)/3 by 6 gives 2(x + 2), not 2x + 2. The original fraction divides the entire numerator, so the inverse step must act on that entire group.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Solve $x/3 + 2 = 7$.

3.2 Solve $(2x - 1)/5 = 3$.

3.3 Solve $x/2 + x/3 = 10$.

3.4 Three consecutive integers sum to 72. Find them.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Solve $7x + 2 = 44$. [2 marks]

R2. Solve $9 - 3x = 24$. [2 marks]

R3. Solve $5(x - 2) = 2x + 8$. [3 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Solve $(x + 1)/4 + x/2 = 4$. [3 marks]

R5. A number is doubled and increased by 9 to give 35. Find it. [3 marks]

R6. Solve $4(x + 2) = 4x + 8$ and explain. [2 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Balancing and inverse operations

1.1 Solve $x + 9 = 17$.

Subtract 9: $x = 8$.

1.2 Solve $6x = -42$.

Divide by 6: $x = -7$.

1.3 Solve $5x - 8 = 22$.

$5x = 30$, so $x = 6$.

1.4 Solve $x/4 + 2 = -1$.

$x/4 = -3$, so $x = -12$.

Lesson 2 • Brackets and unknowns on both sides

2.1 Solve $7x - 5 = 3x + 11$.

$4x = 16$, so $x = 4$.

2.2 Solve $4(x + 2) = 3x + 13$.

$4x + 8 = 3x + 13$, so $x = 5$.

2.3 Solve $5 - 2(x - 3) = x + 2$.

$11 - 2x = x + 2$; $9 = 3x$, so $x = 3$.

2.4 Solve $3x + 8 = 3x - 1$.

Subtract $3x$: $8 = -1$ is false, so there is no solution.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Fractions and forming equations

3.1 Solve x/3 + 2 = 7.

x/3 = 5, so x = 15.

3.2 Solve (2x − 1)/5 = 3.

2x − 1 = 15; x = 8.

3.3 Solve x/2 + x/3 = 10.

Multiply by 6: 3x + 2x = 60. Thus x = 12.

3.4 Three consecutive integers sum to 72. Find them.

Let the first be n. n + (n + 1) + (n + 2) = 72 gives 3n = 69, so 23, 24, 25.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Solve $7x + 2 = 44$. [2 marks]

$7x = 42$, so $x = 6$. [1 for subtracting 2; 1 for division.]

R2. Solve $9 - 3x = 24$. [2 marks]

$-3x = 15$, so $x = -5$. [1 for isolation; 1 for result.]

R3. Solve $5(x - 2) = 2x + 8$. [3 marks]

$5x - 10 = 2x + 8$; $3x = 18$; $x = 6$. [1 for expansion; 1 for collection; 1 for result.]

R4. Solve $(x + 1)/4 + x/2 = 4$. [3 marks]

Multiply by 4: $x + 1 + 2x = 16$. Then $3x = 15$ and $x = 5$. [1 for clearing fractions; 1 for collection; 1 for result.]

R5. A number is doubled and increased by 9 to give 35. Find it. [3 marks]

$2n + 9 = 35$; $2n = 26$; $n = 13$. [1 for model; 1 for balancing; 1 for result.]

R6. Solve $4(x + 2) = 4x + 8$ and explain. [2 marks]

Expansion gives $4x + 8 = 4x + 8$, true for every real x. [1 for expansion; 1 for all-real conclusion.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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