GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 14

14

Advanced expansion and factorisation

Higher-level GCSE material

Extend bracket methods to harder quadratics, differences of squares and products of three brackets. Choose a factorisation strategy by inspecting the structure first, then verify your answer by expansion.

$$a^2 - b^2 = (a - b)(a + b)$$

Look for structure before choosing a method.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

INDEPENDENT GCSE MATHS • WEBSITE EDITION • NOVEMBER 2026

© 2026 GCSE Online Tutoring. All rights reserved.

GCSE ONLINE TUTORING / ABOUT THIS BOOK

Your learning guide

© 2026 GCSE Online Tutoring. All rights reserved.

Free for personal and household educational use. Tutors and schools must obtain permission before copying or distributing these books to learners. Resale, paid redistribution, removal of branding and uploading to third-party websites are prohibited without written permission. For permissions, visit www.gcseonlinetutoring.uk.

Independent educational resource

All explanations, examples, diagrams, questions and marking guidance are independently written. No exam-board questions or mark schemes are reproduced. This book is not affiliated with, approved by or endorsed by any awarding organisation.

Before you begin

Expand double brackets, collect powers and factorise monic quadratics.

Your route through this notebook

1. Quadratics with a leading coefficient
2. Difference of squares and complete factorisation
3. Three brackets and algebraic structure

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 1 / UNDERSTAND

Quadratics with a leading coefficient

A quadratic $ax^2 + bx + c$ with $a \neq 1$ may factor into two linear brackets whose x coefficients multiply to a. A systematic method splits the middle term: find two numbers with product ac and sum b. Replace bx by those two terms, then factor by grouping.

The grouping works because each pair produces the same bracket. If the brackets do not match, check the selected numbers and signs. Always remove a common factor first. Integer factorisation is useful but not universal: some quadratics need other solving methods.

A reliable method

1. Take out any common factor.

2. Find a pair with product ac and sum b, then split bx.

3. Group into matching brackets and check by expansion.

Worked example 1

Factorise $6x^2 + 11x + 3$.

STEP-BY-STEP SOLUTION

ac = 18. Use 9 and 2: $6x^2 + 9x + 2x + 3 = 3x(2x + 3) + 1(2x + 3) = (3x + 1)(2x + 3)$.

Worked example 2

Factorise $2x^2 - 7x + 3$.

STEP-BY-STEP SOLUTION

ac = 6; use −6 and −1. Then $2x(x - 3) - 1(x - 3) = (2x - 1)(x - 3)$.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Finding numbers whose product is c alone works for monic quadratics, but not as a complete method when a ≠ 1. Check the coefficient of x^2 as well as the constant.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Factorise $2x^2 + 7x + 3$.

1.2 Factorise $3x^2 - 10x + 3$.

1.3 Factorise $4x^2 + 12x + 8$ fully.

1.4 Expand $(2x - 3)(3x + 4)$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 2 / UNDERSTAND

Difference of squares and complete factorisation

The identity $a^2 - b^2 = (a - b)(a + b)$ follows because the middle terms cancel on expansion. Recognise squared expressions, not just square numbers: $9x^2 - 25$ is $(3x)^2 - 5^2$. A sum of squares does not factor by this identity over the real numbers.

Full factorisation can require several stages. Remove a common factor, apply a structural identity, then inspect the factors again. For example, $x^4 - 16$ contains a difference of squares, and one resulting factor contains another. Stop when the factors cannot be broken down further using the intended GCSE real or integer methods.

A reliable method

1. Look for a common factor before applying an identity.
2. Identify both squares explicitly.
3. Inspect the resulting factors for another possible step.

Worked example 1

Factorise $25x^2 - 49$.

STEP-BY-STEP SOLUTION

This is $(5x)^2 - 7^2$, so $(5x - 7)(5x + 7)$.

Worked example 2

Factorise $3x^3 - 12x$ fully.

STEP-BY-STEP SOLUTION

Take out $3x$: $3x(x^2 - 4)$. Then difference of squares gives $3x(x - 2)(x + 2)$.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

$x^2 + 9$ is not $(x + 3)(x - 3)$, because that product is $x^2 - 9$. The sign between the squares is essential.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Factorise $x^2 - 64$.

2.2 Factorise $16a^2 - b^2$.

2.3 Factorise $2x^3 - 50x$.

2.4 Factorise $x^4 - 81$ over the real numbers using GCSE methods.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 3 / UNDERSTAND

Three brackets and algebraic structure

For three brackets, expand two first and collect like terms before multiplying by the third. Every term in the intermediate expression must multiply every term in the remaining bracket. Ordering powers in descending order helps prevent missing terms.

Sometimes a special pair makes the work shorter. Multiplying conjugate brackets such as $(x + 2)(x - 2)$ first removes the middle term. The constant term of a product is the product of the constants, and the leading term is the product of the leading terms. These observations give quick independent checks.

A reliable method

1. Choose the easiest pair of brackets first.

2. Multiply the collected result by every term in the final bracket.

3. Check the leading term, constant term and a simple input.

Worked example 1

Expand $(x + 1)(x + 2)(x + 3)$.

STEP-BY-STEP SOLUTION

First $(x + 1)(x + 2) = x^2 + 3x + 2$. Multiply by $x + 3$: $x^3 + 3x^2 + 3x^2 + 9x + 2x + 6 = x^3 + 6x^2 + 11x + 6$.

Worked example 2

Expand $(x - 2)(x + 2)(x - 5)$.

STEP-BY-STEP SOLUTION

First use difference of squares: $(x^2 - 4)(x - 5) = x^3 - 5x^2 - 4x + 20$.

© 2026 GCSE Online Tutoring. All rights reserved.

LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Substituting $x = 0$ checks only the constant term. It is useful but does not prove two expressions identical. Combine numerical checks with complete algebraic expansion.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Expand $(x + 1)(x - 1)(x + 4)$.

3.2 Expand $(x + 2)^3$.

3.3 Expand $(2x + 1)(x - 1)(x + 1)$.

3.4 Check the constant and leading terms of $(3x - 2)(x + 5)(x - 4)$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

© 2026 GCSE Online Tutoring. All rights reserved.

MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Factorise $6x^2 + 7x + 2$. [3 marks]

R2. Factorise $3x^2 - 8x + 4$. [3 marks]

R3. Factorise $49p^2 - 4q^2$. [2 marks]

© 2026 GCSE Online Tutoring. All rights reserved.

MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Factorise $5x^3 - 45x$ fully. [3 marks]

R5. Expand $(x - 1)(x + 2)(x + 5)$. [4 marks]

R6. Explain why $x^2 + 16$ cannot equal $(x - 4)(x + 4)$. [2 marks]

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Quadratics with a leading coefficient

1.1 Factorise $2x^2 + 7x + 3$.

Split $7x$ as $6x + x$: $(2x + 1)(x + 3)$.

1.2 Factorise $3x^2 - 10x + 3$.

Split $-10x$ as $-9x - x$: $(3x - 1)(x - 3)$.

1.3 Factorise $4x^2 + 12x + 8$ fully.

First take 4: $4(x^2 + 3x + 2) = 4(x + 1)(x + 2)$.

1.4 Expand $(2x - 3)(3x + 4)$.

$6x^2 + 8x - 9x - 12 = 6x^2 - x - 12$.

Lesson 2 • Difference of squares and complete factorisation

2.1 Factorise $x^2 - 64$.

$(x - 8)(x + 8)$.

2.2 Factorise $16a^2 - b^2$.

$(4a - b)(4a + b)$.

2.3 Factorise $2x^3 - 50x$.

$2x(x^2 - 25) = 2x(x - 5)(x + 5)$.

2.4 Factorise $x^4 - 81$ over the real numbers using GCSE methods.

$(x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9)$.

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Three brackets and algebraic structure

3.1 Expand $(x + 1)(x - 1)(x + 4)$.

$(x^2 - 1)(x + 4) = x^3 + 4x^2 - x - 4$.

3.2 Expand $(x + 2)^3$.

$(x^2 + 4x + 4)(x + 2) = x^3 + 6x^2 + 12x + 8$.

3.3 Expand $(2x + 1)(x - 1)(x + 1)$.

$(2x + 1)(x^2 - 1) = 2x^3 + x^2 - 2x - 1$.

3.4 Check the constant and leading terms of $(3x - 2)(x + 5)(x - 4)$.

Leading term is $3x^3$. Constant is $(-2) \times 5 \times (-4) = 40$.

© 2026 GCSE Online Tutoring. All rights reserved.

SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Factorise $6x^2 + 7x + 2$. [3 marks]

Split using 3 and 4: $(3x + 2)(2x + 1)$. [1 for pair; 1 for grouping; 1 for factors.]

R2. Factorise $3x^2 - 8x + 4$. [3 marks]

Split $-8x$ as $-6x - 2x$: $(3x - 2)(x - 2)$. [1 for split; 1 for grouping; 1 for result.]

R3. Factorise $49p^2 - 4q^2$. [2 marks]

$(7p - 2q)(7p + 2q)$. [1 for recognising squares; 1 for factors.]

R4. Factorise $5x^3 - 45x$ fully. [3 marks]

$5x(x^2 - 9) = 5x(x - 3)(x + 3)$. [1 for common factor; 1 for identity; 1 for full result.]

R5. Expand $(x - 1)(x + 2)(x + 5)$. [4 marks]

$(x^2 + x - 2)(x + 5) = x^3 + 6x^2 + 3x - 10$. [1 for first product; 2 for distribution; 1 for collection.]

R6. Explain why $x^2 + 16$ cannot equal $(x - 4)(x + 4)$. [2 marks]

The product is $x^2 - 16$. At $x = 0$ the claimed equality would say $16 = -16$. [1 for expansion; 1 for explanation/check.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

© 2026 GCSE Online Tutoring. All rights reserved.