GCSE ONLINE TUTORING

ALGEBRA & GRAPHS / TOPIC GUIDE 13

13

Expanding and factorising: foundations

Shared GCSE core

Expansion and factorisation are reverse processes. Learn to distribute multiplication over addition, remove common factors and connect simple quadratic factors to their expanded forms. Check transformations by reversing them.

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

Expansion and factorisation reverse each other.

UNDERSTAND THE IDEA. EXPLAIN THE METHOD.

Detailed teaching • Worked examples • Discussion • Original practice • Full solutions

Name: _____

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Before you begin

Collect like terms, multiply algebraic terms and identify common factors.

Your route through this notebook

1. Expanding single brackets
2. Factorising out a common factor
3. Double brackets and simple quadratics

How to use it

Read a lesson, cover the worked solutions and try the examples yourself. Complete its practice page before checking the answers at the back. Finish with the mixed review, then return to any method you could not explain.

YOUR PROGRESS

After each lesson, ask: Can I explain the idea? Can I choose a method? Can I check my result? Record any questions you want to revisit.

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LESSON 1 / UNDERSTAND

Expanding single brackets

A multiplier outside a bracket multiplies every term inside. This distributive property follows from splitting a quantity into parts: $3(x + 4) = 3x + 12$. A negative multiplier changes the signs of the products; it is safest to write each product explicitly before collecting terms.

If a bracket is preceded by a subtraction sign, treat the multiplier as -1. For example, $8 - (x - 2)$ becomes $8 - x + 2$. Expanding is not solving: there is no unknown value to find unless an equation is supplied. Equivalent expressions have the same value for every permitted input.

A reliable method

1. Multiply every term in the bracket.
2. Keep each term’s sign and the multiplier’s sign.
3. Collect like terms only after the bracket is removed.

Worked example 1

Expand $5(2x - 3)$.

STEP-BY-STEP SOLUTION

$5 \times 2x + 5 \times (-3) = 10x - 15$.

Worked example 2

Simplify $4x - 2(3x - 5)$.

STEP-BY-STEP SOLUTION

Expand the product as $-6x + 10$. Then $4x - 6x + 10 = -2x + 10$.

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LESSON 1 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

3(x + 4) does not equal 3x + 4: the 4 must also be multiplied. Test x = 0 to see the difference immediately: 12 versus 4.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

1.1 Expand 4(x + 7).

1.2 Expand −3(2a − 5).

1.3 Simplify 2(x + 3) + 5(x − 1).

1.4 Simplify 9 − (2y + 4).

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 2 / UNDERSTAND

Factorising out a common factor

Factorising rewrites a sum as a product. It reverses expansion. In $6x + 15$, both terms contain a factor 3, so $6x + 15 = 3(2x + 5)$. The bracket records what remains after dividing each term by the chosen common factor.

To factorise fully, take the greatest common numerical factor and every variable factor shared by all terms, using the smallest shared power. A term with no variable limits which letters can be taken outside. Negative factors are sometimes useful, but any proposed factorisation should be checked by expanding it back.

A reliable method

1. Find the highest common factor of all terms.
2. Divide every term by that factor to form the bracket.
3. Expand the result to check that every original term returns.

Worked example 1

Factorise $12x^2 + 18x$ fully.

STEP-BY-STEP SOLUTION

The HCF is $6x$. Dividing gives $2x$ and 3, so $6x(2x + 3)$.

Worked example 2

Factorise $8a - 20$.

STEP-BY-STEP SOLUTION

The common numerical factor is 4. Result: $4(2a - 5)$.

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LESSON 2 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

Writing $12x^2 + 18x$ as $3x(4x + 6)$ is correct but not fully factorised because the bracket still has a factor 2. Full factorisation removes every common factor.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

2.1 Factorise $10x + 25$.

2.2 Factorise $15y^2 - 5y$.

2.3 Factorise $14ab + 21a$.

2.4 Check whether $6(x + 4)$ factorises $6x + 4$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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LESSON 3 / UNDERSTAND

Double brackets and simple quadratics

To expand two brackets, every term in one bracket must multiply every term in the other. In $(x + a)(x + b)$, the products give $x^2 + ax + bx + ab$, which collect to $x^2 + (a + b)x + ab$. This connects factorisation to finding two numbers with a required sum and product.

For a monic quadratic $x^2 + bx + c$, seek two integers whose sum is b and whose product is c. If c is negative, the integers have opposite signs. If c is positive and b is negative, both integers are negative. Not every quadratic has integer factors, so a failed search does not justify inventing brackets.

A reliable method

1. For expansion, list all four products before collecting.

2. For factorisation, find the required sum and product.

3. Expand the factors to check both the middle and constant terms.

Worked example 1

Expand $(x + 4)(x - 3)$.

STEP-BY-STEP SOLUTION

Products: $x^2 - 3x + 4x - 12$. Collect: $x^2 + x - 12$.

Worked example 2

Factorise $x^2 - 7x + 12$.

STEP-BY-STEP SOLUTION

Find numbers with product 12 and sum -7: -3 and -4. Therefore $(x - 3)(x - 4)$.

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LESSON 3 / DISCUSS & PRACTISE

Explain it, then try it

DISCUSSION • TEST THE IDEA

$(x + 5)^2$ is not $x^2 + 25$. It is $(x + 5)(x + 5) = x^2 + 10x + 25$. The two middle products do not disappear.

Say it in your own words

Explain the key idea to someone else. Use a small example to support your explanation. If you disagree with a claim, show why it fails.

3.1 Expand $(x + 2)(x + 7)$.

3.2 Expand $(x - 5)(x + 3)$.

3.3 Factorise $x^2 + 8x + 15$.

3.4 Factorise $x^2 - x - 12$.

Check before turning to the answers: is the size sensible, is the notation clear, and have you included any required units?

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 1

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R1. Expand $-4(3x - 2)$. [2 marks]

R2. Simplify $3(2x + 1) - 2(x - 4)$. [3 marks]

R3. Factorise $18a^2 - 12a$ fully. [2 marks]

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MIXED REVIEW / ORIGINAL EXAM-STYLE PRACTICE

Apply your learning • 2

These questions are independently authored, not taken or adapted from past papers. Marks are suggested for self-assessment. Show your method and explain decisions. Use a calculator only where the arithmetic requires it.

R4. Expand $(x - 6)(x - 2)$. [3 marks]

R5. Factorise $x^2 + 2x - 24$. [2 marks]

R6. A rectangle has width $x + 3$ and length $x + 8$. Express its area in expanded form. [3 marks]

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SOLUTIONS / LESSON PRACTICE

Check the reasoning

Your layout may differ. Compare the mathematical reasoning as well as the final answer. Correct any error, then reattempt the question without looking.

Lesson 1 • Expanding single brackets

1.1 Expand $4(x + 7)$.

$4x + 28$.

1.2 Expand $-3(2a - 5)$.

$-6a + 15$.

1.3 Simplify $2(x + 3) + 5(x - 1)$.

$2x + 6 + 5x - 5 = 7x + 1$.

1.4 Simplify $9 - (2y + 4)$.

$9 - 2y - 4 = 5 - 2y$.

Lesson 2 • Factorising out a common factor

2.1 Factorise $10x + 25$.

$5(2x + 5)$.

2.2 Factorise $15y^2 - 5y$.

$5y(3y - 1)$.

2.3 Factorise $14ab + 21a$.

$7a(2b + 3)$.

2.4 Check whether $6(x + 4)$ factorises $6x + 4$.

No. Expanding gives $6x + 24$. A correct factorisation is $2(3x + 2)$.

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SOLUTIONS / LESSON PRACTICE

Check the reasoning • continued

Lesson 3 • Double brackets and simple quadratics

3.1 Expand $(x + 2)(x + 7)$.

$x^2 + 9x + 14$.

3.2 Expand $(x - 5)(x + 3)$.

$x^2 - 2x - 15$.

3.3 Factorise $x^2 + 8x + 15$.

$3 + 5 = 8$ and $3 \times 5 = 15$, so $(x + 3)(x + 5)$.

3.4 Factorise $x^2 - x - 12$.

$-4 + 3 = -1$ and $-4 \times 3 = -12$, so $(x - 4)(x + 3)$.

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SOLUTIONS / MIXED REVIEW

Answers and marking guidance

Award the suggested method marks for valid working, even where a later arithmetic slip occurs. Do not count the same step twice. These are independent teaching allocations, not official exam-board mark schemes.

R1. Expand $-4(3x - 2)$. [2 marks]

$-12x + 8$. [1 for each product.]

R2. Simplify $3(2x + 1) - 2(x - 4)$. [3 marks]

$6x + 3 - 2x + 8 = 4x + 11$. [1 for each expansion; 1 for collection.]

R3. Factorise $18a^2 - 12a$ fully. [2 marks]

$6a(3a - 2)$. [1 for 6a; 1 for bracket.]

R4. Expand $(x - 6)(x - 2)$. [3 marks]

$x^2 - 2x - 6x + 12 = x^2 - 8x + 12$. [1 for four products; 1 for signs; 1 for collection.]

R5. Factorise $x^2 + 2x - 24$. [2 marks]

$(x + 6)(x - 4)$. [1 for suitable pair; 1 for factors.]

R6. A rectangle has width $x + 3$ and length $x + 8$. Express its area in expanded form. [3 marks]

$A = (x + 3)(x + 8) = x^2 + 11x + 24$. [1 for product model; 1 for expansion; 1 for simplification.]

Reflect, repair and revisit

Record one secure method, one mistake you corrected and one question to retry. Reattempt the questions you missed in a few days. Explain why your new method works before you compare your answer.

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